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Title: Inclusive and Diffractive Dijet Photoproduction at the Electron–Ion Collider in NLO QCD

Year: 2023

Version: Published version

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Please cite the original version:

Guzey, V., & Klasen, M. (2023). Inclusive and Diffractive Dijet Photoproduction at the Electron–Ion Collider in NLO QCD. In J. Chwastowski, K. Golec-Biernat, M. Praszalowicz, M. Przybycien, & R. Staszewski (Eds.), 29th Cracow Epiphany Conference on Physics at the Electron–Ion Collider and Future Facilities : Cracow, Poland, 16–19 January, 2023 (16, Article 7-A11). Jagiellonian University. Acta Physica Polonica B : Proceedings Supplement. <https://doi.org/10.5506/APhysPolBSupp.16.7-A11>

INCLUSIVE AND DIFFRACTIVE DIJET PHOTOPRODUCTION AT THE ELECTRON–ION COLLIDER IN NLO QCD*

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*Received 8 March 2023, accepted 21 March 2023,
published online 6 September 2023*

In the framework of collinear factorization and next-to-leading order (NLO) perturbative QCD, we make predictions for inclusive and diffractive dijet photoproduction in electron–proton and electron–nucleus scattering in the EIC kinematics. We establish kinematic ranges in the \bar{p}_T , $\bar{\eta}$, x_A^{obs} , and x_γ^{obs} variables, quantify sensitivity to small- x nuclear PDFs, and analyze various scenarios of factorization breaking in the case of diffractive scattering.

DOI:10.5506/APhysPolBSupp.16.7-A11

1. Introduction

All currently available information on jet photoproduction on hadrons comes from electron (positron)–proton scattering at the Hadron–Electron Ring Accelerator (HERA), for reviews, see [1–3]. Provided that the jet transverse momenta p_T are sufficiently large, this process allows one to probe the microscopic quark–gluon structure of the proton and the real photon in quantum chromodynamics (QCD) as well as the strong interaction dynamics in the regime of perturbative QCD (pQCD). The predictions of next-to-leading order (NLO) pQCD provide a good description of the dijet photoproduction

* Presented by V. Guzey at the 29th Cracow Epiphany Conference on *Physics at the Electron–Ion Collider and Future Facilities*, Cracow, Poland, 16–19 January, 2023.

cross section measured at HERA as a function of various jet observables in a wide range of p_T [4–7]. This serves as an important test of the QCD factorization and universality of parton distribution functions (PDFs).

A related important incentive to study photoproduction of jets is that the cross section of this process has enhanced sensitivity to the gluon distribution. As a result, QCD analyses of the combined data on the dijet cross section and the total cross section of lepton–proton deep inelastic scattering (DIS) provide additional constraints on the gluon PDF of the proton, see, *e.g.* [8]. Similarly, the combination with the available data on the $F_2^\gamma(x, Q^2)$ photon structure function measured in electron–positron annihilation enables one to better constrain the gluon PDF of the real photon [9]. Also, in the case of diffractive dijet photoproduction, one can use this process to analyze the poorly understood mechanism of the QCD factorization breaking in diffractive scattering observed experimentally [10–13].

It is expected that studies of photoproduction of jets will be continued at the future Electron–Ion Collider (EIC) in the U.S. [14] and the Large Hadron Electron Collider (LHeC) [15] and/or a Future Circular Collider (FCC) [16] at CERN. It will allow one not only to measure this process in a kinematic region complementary to that covered by HERA and with much higher precision, but will also give for the first time the access to novel nuclear diffractive PDFs in the case of nuclear beams.

Note that first results on inclusive dijet photoproduction on heavy nuclei have recently been obtained by ATLAS [17] by analyzing lead–lead ultra-peripheral collisions (UPCs) at the Large Hadron Collider (LHC). It was shown in [18] that NLO pQCD provides a good description of these data.

2. Inclusive dijet photoproduction in eA scattering at EIC

As we explained in Introduction, photoproduction of jets provides complementary information on the partonic structure of hadrons and photons in QCD. In particular, the process of inclusive dijet photoproduction in lepton–nucleus (eA) scattering, $e + A \rightarrow e' + 2 \text{ jets} + X$, is expected to yield new constraints on nuclear PDFs. Typical leading order (LO) Feynman graphs for this process are shown in Fig. 1: graphs (a) and (b) represent the so-called direct-photon and the resolved-photon contributions, respectively. In graph (a), the photon enters the hard process of the photon–gluon fusion directly as an elementary particle. In contrast, in graph (b), the photon participates in hard scattering by means of its partonic content, which is hence revealed (resolved) in this process.

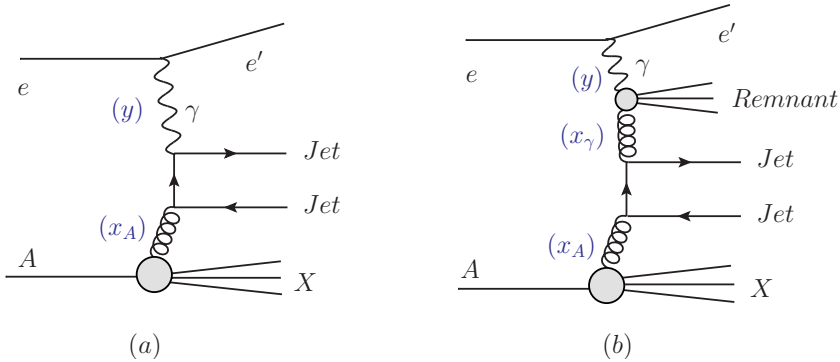


Fig. 1. Typical LO direct-photon (left) and resolved-photon (right) contributions to dijet photoproduction in eA scattering. The involved momentum fractions y , x_A , and x_γ are shown in parentheses.

In the framework of collinear factorization and NLO pQCD, the $e + A \rightarrow e' + 2 \text{ jets} + X$ cross section can be written as the following convolution [19, 20]

$$\begin{aligned} d\sigma(e + A \rightarrow e' + 2 \text{ jets} + X) = & \sum_{a,b} \int dy \int dx_\gamma \int dx_A f_{\gamma/e}(y) \\ & \times f_{a/\gamma}(x_\gamma, \mu^2) f_{b/A}(x_A, \mu^2) d\hat{\sigma}(ab \rightarrow \text{jets}), \end{aligned} \quad (1)$$

where $f_{\gamma/e}(y)$ is the photon flux of the electron with y being the momentum fraction carried by the photon; $f_{a/\gamma}(x_\gamma, \mu^2)$ are the photon PDFs in the resolved-photon case, which depend on the parton-in-photon momentum fraction x_γ and the scale μ ; $f_{b/A}(x_A, \mu^2)$ are nuclear PDFs depending on the parton momentum fraction x_A and the scale μ ; $d\hat{\sigma}(ab \rightarrow \text{jets})$ is the cross section of hard scattering of partons a and b into jets. In the direct-photon case, parton a corresponds to the photon leading to $f_{\gamma/\gamma}(x_\gamma, \mu^2) = \delta(1 - x_\gamma)$ at LO. In Eq. (1), all involved hard scales have been set to be equal. In our analysis, we identify them with the mean transverse momentum of the two jets, $\mu = \bar{p}_T = (p_{T,1} + p_{T,2})/2$. Note that while the separation between the direct and resolved photons is not unique beyond LO, it is still useful since the direct-photon contribution peaks in the $x_\gamma \rightarrow 1$ limit.

Our predictions [21] for the dijet photoproduction in eA scattering at the EIC are based on the numerical implementation of Eq. (1) combined with the anti- k_T jet clustering algorithm with at most 2 partons in a jet, which was developed in [22–24]. While the parton momentum fractions x_A and x_γ are not directly measurable, they can be approximated using the following hadron-level estimates based on the jet transverse momenta $p_{T,1}$ and $p_{T,2}$ and the jet (pseudo)rapidities η_1 and η_2

$$\begin{aligned}
 x_A^{\text{obs}} &= \frac{p_{T,1} e^{\eta_1} + p_{T,2} e^{\eta_2}}{2E_A}, \\
 x_\gamma^{\text{obs}} &= \frac{p_{T,1} e^{-\eta_1} + p_{T,2} e^{-\eta_2}}{2yE_e},
 \end{aligned}
 \tag{2}$$

where E_A and E_e are the energies of the nucleus and electron beams, respectively. For definiteness, we take $E_A = 100$ GeV per nucleon and $E_e = 21$ GeV corresponding to $\sqrt{s} = 92$ GeV [14]. For final-state jets, we assume generic conditions based on the HERA experience: the leading jet has $p_{T,1} > 5$ GeV and the subleading jets carry $p_{T,i \neq 1} > 4.5$ GeV; all jets have $\eta_{1,2} < 4$; the jet cone parameter is $R = 0.4$. Finally, we use the GRV HO photon PDFs [25] and the nCTEQ15 nuclear PDFs [26].

The resulting distributions in the dijet average transverse momentum $\bar{p}_T = (p_{T,1} + p_{T,2})/2$, the dijet average rapidity $\bar{\eta} = (\eta_1 + \eta_2)/2$, and the observed nucleus and photon momentum fractions, x_A^{obs} and x_γ^{obs} , are shown in Fig. 2. One can see from the figure that at the EIC, the kinematic reach in these variables is $5 < \bar{p}_T < 20$ GeV, $-1 < \bar{\eta} < 2$, $0.01 < x_A^{\text{obs}} < 1$, and $0.03 < x_\gamma^{\text{obs}} < 1$.

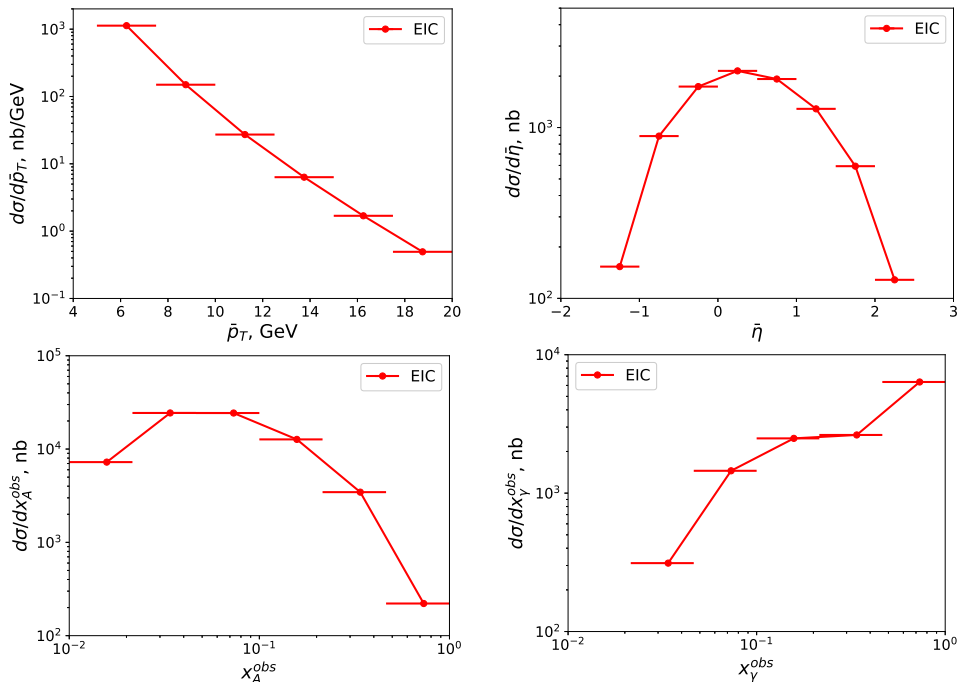


Fig. 2. NLO pQCD predictions for the $e + A \rightarrow e' + 2 \text{ jets} + X$ dijet photoproduction cross section in eA scattering at the EIC as a function of the average dijet transverse momentum \bar{p}_T , the average rapidity $\bar{\eta}$, and the momentum fractions x_A^{obs} and x_γ^{obs} .

Going from the EIC to LHeC and further to FCC, the collision energy increases, which subsequently dramatically expands the kinematic coverage. In particular, it was shown in [21] that dijet photoproduction in eA scattering can be probed there at $5 < \bar{p}_T < 60$ GeV, $-2 < \bar{\eta} < 4$, 10^{-5} – $10^{-4} < x_A^{\text{obs}} < 1$, and $10^{-3} < x_\gamma^{\text{obs}} < 1$.

While the nucleus momentum fraction x_A^{obs} at the EIC has a modest kinematic reach in the small- x region, the dijet cross section is nevertheless sensitive to nuclear modifications of PDFs: the ratio of the cross sections on the nucleus and the proton as a function of x_A^{obs} exhibits a 10–20% suppression (nuclear shadowing) at small x_A^{obs} followed by a 10–20% enhancement at $x_A^{\text{obs}} \sim 0.1$ (nuclear antishadowing), which are characteristic for the gluon nuclear PDFs. Note, however, that the magnitude of the observed effects is compatible with sizable uncertainties of the nuclear PDFs. The similar behavior is also obtained when we use the EPPS16 nPDFs [27] as an input for our calculations.

3. Diffractive dijet photoproduction in lepton–proton and lepton–nucleus scattering at EIC

One of the major HERA physics results is the unexpected observation that diffraction makes up approximately 10–15% of the total electron–proton (ep) DIS cross section [2, 3]. Similarly to the case of inclusive scattering, one can define diffractive PDFs in the framework of collinear QCD factorization [28], extract them from the HERA data on the proton diffractive structure functions [29, 30], and test their universality in diffractive dijet and open charm production in DIS [31, 32]. At the same time, it was found that NLO pQCD overestimates the measured cross section of diffractive dijet photoproduction by approximately a factor of 2 [10–13], which indicates breaking of the QCD factorization. The mechanism of it remains unknown: the theory and the data can be made consistent by introducing either the global suppression factor of $R_{\text{glob}} = 0.5$ or the suppression factor of $R_{\text{dir}} = 0.34$ for the resolved-photon contribution only or the x_γ -dependent suppression factor interpolating between these two scenarios [33].

Diffractive dijet photoproduction corresponds to the situation, when one requires that the target hadron (proton, nucleus) in Fig. 1 stays intact or dissociates into a low-mass excitation. In the proton target case, the $e + p \rightarrow e' + 2 \text{jets} + X' + Y$ cross section of diffractive dijet photoproduction in NLO pQCD reads [compare to Eq. (1)]

$$\begin{aligned} d\sigma (e + p \rightarrow e' + 2 \text{jets} + X' + Y) &= \sum_{a,b} \int dy \int dx_\gamma \int dt \int dx_{\mathbb{P}} \int dz_{\mathbb{P}} f_{\gamma/e}(y) \\ &\times f_{a/\gamma}(x_\gamma, \mu^2) f_{b/p}^{\text{D}(4)}(z_{\mathbb{P}}, \mu^2, x_{\mathbb{P}}, t) d\hat{\sigma}(ab \rightarrow \text{jets}), \end{aligned} \quad (3)$$

where $f_{b/p}^{\text{D}(4)}(z_{\mathbb{P}}, \mu^2, x_{\mathbb{P}}, t)$ is the so-called diffractive PDF of the proton. It is a conditional probability to find parton b with the momentum fraction $z_{\mathbb{P}}$ with respect to the diffractive exchange carrying the momentum fraction $x_{\mathbb{P}}$ (often called the Pomeron) provided that the final-state proton (or its low-mass excitation Y) receives the momentum transfer squared t . To further illustrate this concept, it is convenient to assume the so-called Regge factorization for diffractive PDFs, where they are given as a product of the Pomeron flux $f_{\mathbb{P}/p}(x_{\mathbb{P}}, t)$ and the PDFs of the Pomeron $f_{b/\mathbb{P}}(z_{\mathbb{P}}, \mu^2)$,

$$f_{b/p}^{\text{D}(4)}(z_{\mathbb{P}}, \mu^2, x_{\mathbb{P}}, t) = f_{\mathbb{P}/p}(x_{\mathbb{P}}, t) f_{b/\mathbb{P}}(z_{\mathbb{P}}, \mu^2) + f_{\mathbb{R}/p}(x_{\mathbb{P}}, t) f_{b/\mathbb{R}}(z_{\mathbb{P}}, \mu^2). \quad (4)$$

In Eq. (4), the second term gives the sub-leading Reggeon contribution, which becomes important only for large $x_{\mathbb{P}} > 0.03$ [29].

Using the numerical implementation of Eq. (3) discussed above, we make predictions for diffractive dijet photoproduction in ep scattering at the EIC [34]. In addition to the generic cuts and the energy configuration ($E_p = 100$ GeV, $E_e = 21$ GeV) discussed in Section 2, we take $|t| < 1$ GeV², $M_Y < 1.6$ GeV, and $x_{\mathbb{P}} \leq 0.03$, and use H1 2006 Fit B for proton diffractive PDFs [29].

An example of our predictions is presented in Fig. 3 showing the distributions in the dijet average transverse momentum \bar{p}_T (left) and the photon momentum fraction x_{γ}^{obs} (right). The red solid curves give the full result, where we use only the Pomeron contribution in Eq. (4), the blue dashed curves show the contribution of the gluon diffractive PDF, and the green dotted curves are the direct-photon contribution. One can see from the figure that the coverage in both \bar{p}_T and x_{γ}^{obs} is rather limited. In the accessible range of $x_{\gamma}^{\text{obs}} > 0.5$, the cross section is dominated by the contributions of direct photons and point-like quark-antiquark pairs, which makes it dif-

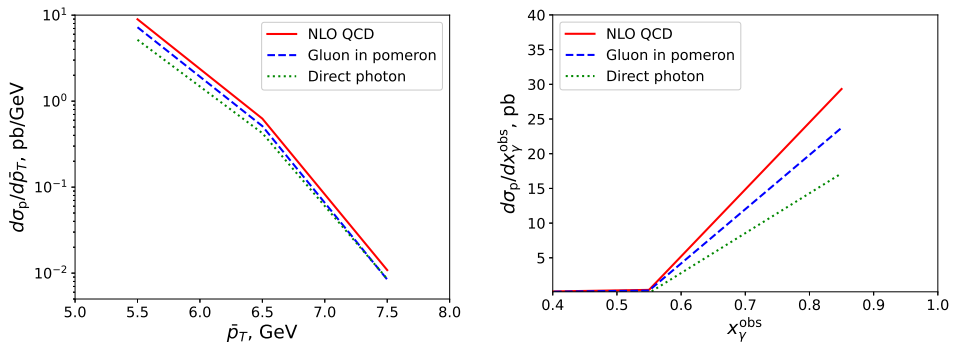


Fig. 3. NLO pQCD predictions for the $e + p \rightarrow e' + 2 \text{ jets} + X' + Y$ cross section of diffractive dijet photoproduction in ep scattering at the EIC as a function of the average dijet transverse momentum \bar{p}_T and the photon momentum fraction x_{γ}^{obs} .

ficult to study the mechanism of factorization breaking mentioned above. Also, the cross section probes large values of $x_{\mathbb{P}}$ and $z_{\mathbb{P}}$, which results in the dominance of the gluon diffractive PDF.

To extend the kinematic coverage, we repeated our analysis using a larger range in $x_{\mathbb{P}}$ up to $x_{\mathbb{P}} < 0.1$. The results for the \bar{p}_{T} and x_{γ}^{obs} distributions are presented in Fig. 4. The red solid and blue dashed curves correspond to the Pomeron and Reggeon contributions, respectively, see Eq. (4); the green dotted curves give the direct-photon contribution. A comparison to Fig. 3 demonstrates that the use of the $x_{\mathbb{P}} < 0.1$ range extends the coverage up to $\bar{p}_{\text{T}} < 14$ GeV and down to $x_{\gamma}^{\text{obs}} > 0.1$. In addition, it brings about the sub-leading Reggeon trajectory, which now contributes at the level of 10–35% for $x_{\mathbb{P}} > 0.06$.

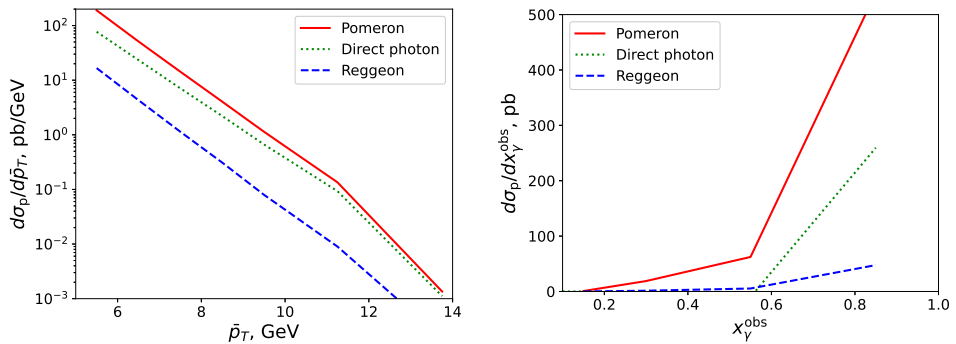


Fig. 4. The same as Fig. 3, but now with an extended range in $x_{\mathbb{P}} < 0.1$. The sub-leading Reggeon contribution is shown by the blue dashed lines.

We discussed above that NLO pQCD predictions for diffractive dijet photoproduction should be in general supplemented by the factor accounting for the QCD factorization breaking. Since its mechanism involves an interplay of the direct-photon and resolved-photon contributions, the most sensitive observable is the x_{γ}^{obs} distribution. To disentangle competing scenarios of the factorization breaking, one needs a sufficiently large range in x_{γ}^{obs} , which in turn requires the highest proton beam energy, and high precision since the cross section falls by two orders of magnitude. Our analysis [34] demonstrated that the assumed pattern of factorization breaking affects mostly the normalization of the \bar{p}_{T} distribution (and other kinematic distributions) and only rather moderately the shape of the x_{γ}^{obs} distribution.

To better differentiate among different schemes of factorization breaking, one can study diffractive dijet photoproduction in electron–nucleus (eA) scattering at the EIC, $e + A \rightarrow e' + 2 \text{ jets} + X' + A$, where nuclei play the role of “filters” for different components of the photon in photon–nucleus scattering. In addition, it will allow one to probe the novel nuclear diffractive PDFs.

At small values of $x_{\mathbb{P}}$ relevant for diffraction, nuclear diffractive PDFs are expected to be suppressed compared to their free proton counterparts due to nuclear shadowing. In the leading twist approach [35], t -integrated nuclear diffractive PDFs $f_{i/A}^{\text{D}(3)}(z_{\mathbb{P}}, \mu^2, x_{\mathbb{P}})$ are obtained by summing the diagrams corresponding to coherent diffractive scattering on 1, 2, ..., A nucleons of the nuclear target

$$f_{i/A}^{\text{D}(3)}(z_{\mathbb{P}}, \mu^2, x_{\mathbb{P}}) = 16\pi f_{i/p}^{\text{D}(4)}(z_{\mathbb{P}}, \mu^2, x_{\mathbb{P}}, t=0) \times \int d^2\vec{b} \left| \frac{1 - e^{-\frac{1}{2}(1-i\eta)\sigma_{\text{soft}}^i(x, \mu^2)T_A(b)}}}{(1-i\eta)\sigma_{\text{soft}}^i(x, \mu^2)} \right|^2. \quad (5)$$

Here, $T_A(b) = \int dz \rho_A(b, z)$ is the nuclear optical density, where $\rho_A(b, z)$ is the nuclear density and \vec{b} is the transverse position of the interacting nucleon; $\sigma_{\text{soft}}^i(x, \mu^2)$ is the effective soft cross section controlling the strength of the interaction with the target nucleons and $\eta = 0.15$ is the ratio of the real-to-imaginary parts of the corresponding scattering amplitude. One can see from Eq. (5) that nuclear shadowing explicitly violates the Regge factorization for nuclear diffractive PDFs [compare to the proton case in Eq. (4)].

In practice, to estimate yields and kinematic distributions, one can use the numerical observation that the effect of nuclear shadowing in Eq. (5) in most of the kinematics weakly depends on the parton flavor i , the momentum fractions $z_{\mathbb{P}}$ and $x_{\mathbb{P}}$, and scale μ . In this case, the nuclear diffractive PDFs are given by the following simple expression:

$$f_{i/A}^{\text{D}(3)}(z_{\mathbb{P}}, \mu^2, x_{\mathbb{P}}) = AR(x, A)f_{i/p}^{\text{D}(3)}(z_{\mathbb{P}}, \mu^2, x_{\mathbb{P}}), \quad (6)$$

where A is the nucleus atomic mass number and $R(x, A) \approx 0.65$ is a weak function of x and A calculated using Eq. (5). Replacing proton diffractive PDFs by nuclear diffractive PDFs in Eq. (3), one can readily make predictions for the $e + A \rightarrow e' + 2\text{jets} + X' + A$ cross section of coherent dijet photoproduction on nuclei in the EIC kinematics.

Figure 5 shows the x_{γ}^{obs} distribution for the gold nucleus (Au-197) and contrasts two scenarios of the QCD factorization breaking in diffraction: the red solid curve corresponds to the global suppression factor of $R_{\text{glob}} = 0.5$ as in the proton case and the blue dashed curve is obtained by applying the $R_{\text{res}} = 0.04$ suppression factor to the resolved-photon contribution. One can see from the figure that the two scenarios lead to sufficiently different predictions for $x_{\gamma}^{\text{obs}} < 0.5$.

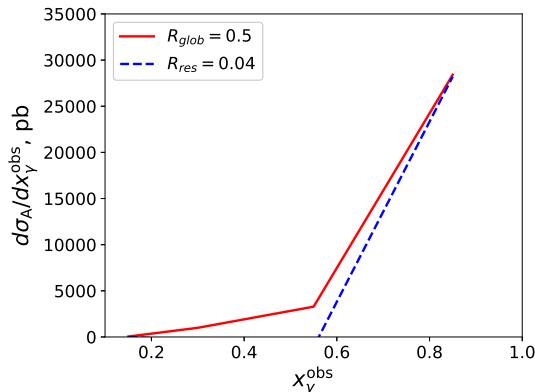


Fig. 5. NLO pQCD predictions for the $e + A \rightarrow e' + 2\text{jets} + X' + A$ cross section of coherent diffractive dijet photoproduction on Au-197 at the EIC as a function of the photon momentum fraction x_γ^{obs} . The red solid and blue dashed curves correspond to the two assumed schemes of factorization breaking, see the text for details.

4. Conclusions

Photoproduction of dijets is a standard tool of QCD. Its theory is well-established in NLO pQCD, whose predictions compare very well to HERA data. Inclusive and diffractive dijet photoproduction at the EIC are complementary to respective DIS measurements and can help constrain proton and nucleus usual and diffractive PDFs. In addition, diffractive dijet photoproduction at the EIC may shed some light on the outstanding problem of factorization breaking. This requires a wide coverage in x_γ^{obs} , which is provided by the highest proton beam energy and a large range in $x_{\mathbb{P}}$, and will benefit from the use of nuclear beams.

The research of V.G. was funded by the Academy of Finland project 330448, the Center of Excellence in Quark Matter of the Academy of Finland (projects 346325 and 346326), and the European Research Council project ERC-2018-ADG-835105 YoctoLHC. The work of M.K. was also funded by the DFG through the Research Training Group 2149 “Strong and Weak Interactions — from Hadrons to Dark Matter” and the SFB 1225 “Isoquant”, project-id 273811115.

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