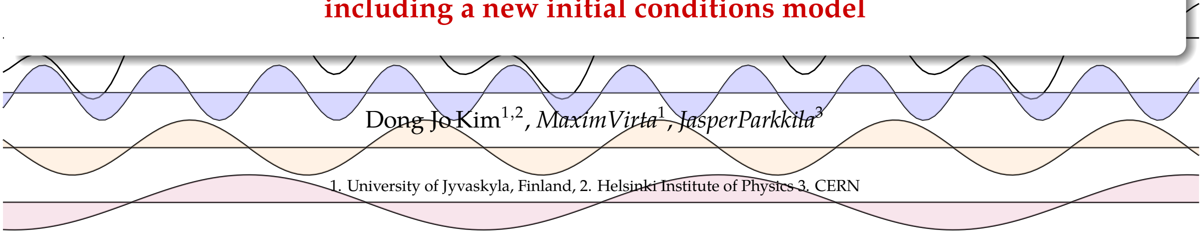
Improving Bayesian parameter estimation with the latest RHIC and LHC data including a new initial conditions model



Dong Jo Kim[1,2], *MaximVirta*[1], *JasperParkkila*[3]

1. University of Jyvaskyla, Finland, 2. Helsinki Institute of Physics 3. CERN

Wednesday 8th February, 2023

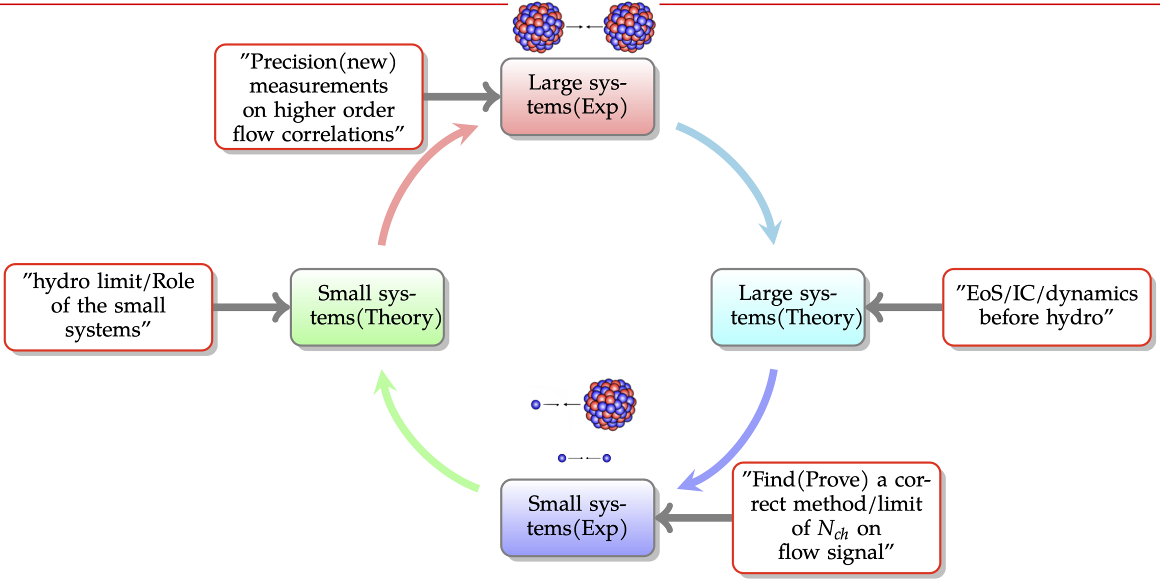
The 38th Winter Workshop on Nuclear Dynamics, Puerto Vallarta, Mexico

UNIVERSITY OF JYVÄSKYLÄ
Centre Of Excellence in Quark Matter

HELSINKI INSTITUTE OF PHYSICS

CERN

CHALLENGES - UNDERSTANDING THE EXPERIMENTAL/THEORY UNCERTAINTIES



"Precision(new) measurements on higher order flow correlations"

Large systems(Exp)

Large systems(Theory)

"EoS/IC/dynamics before hydro"

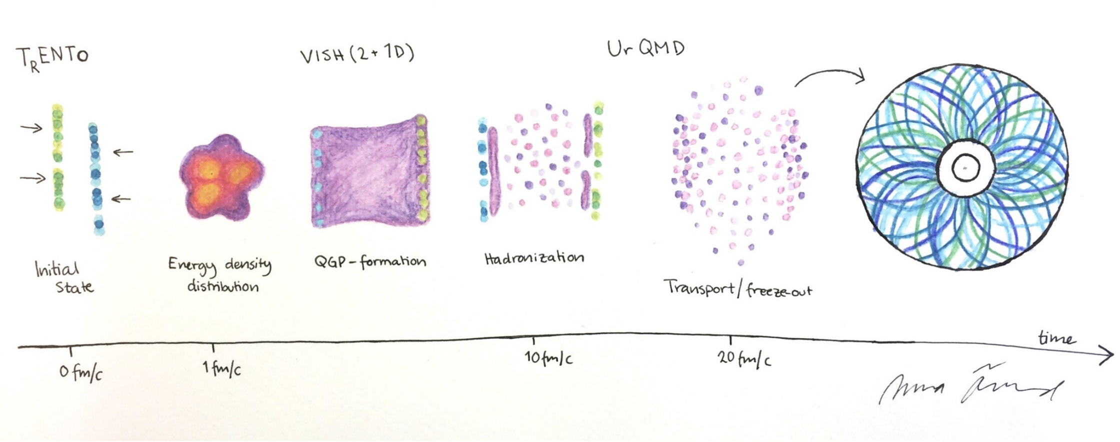
Small systems(Exp)

"Find(Prove) a correct method/limit of N_{ch} on flow signal"

Small systems(Theory)

"hydro limit/Role of the small systems"

THE DIFFERENT STAGES OF HEAVY-ION COLLISIONS



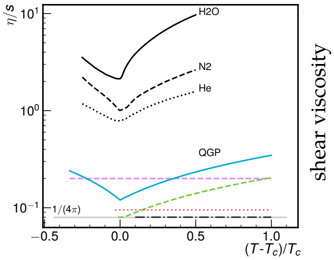
Credits to Anna Onnerstad

$$\boxed{T^{\mu\nu} = e u^{\mu} u^{\nu} - (P + \Pi)\Delta_{\mu\nu} + \pi^{\mu\nu}, \qquad \delta_{\mu} T^{\mu\nu} = 0}$$

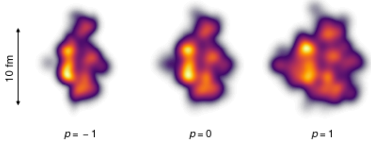
based on https://github.com/Duke-QCD

8 X 7 PARAMETERS, JYVASKYLA (2022)

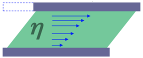
Parameter	Description
T_c	Temperature of const. $\eta/s(T)$, $T < T_c$
$\eta/s(T_c)$	Minimum $\eta/s(T)$
$(\eta/s)_{\text{slope}}$	Slope of $\eta/s(T)$ above T_c
$(\eta/s)_{\text{curve}}$	Curvature of $\eta/s(T)$ above T_c
$(\zeta/s)_{\text{peak}}$	Temperature of $\zeta/s(T)$ maximum
$(\zeta/s)_{\max}$	Maximum $\zeta/s(T)$
$(\zeta/s)_{\text{width}}$	Width of $\zeta/s(T)$ peak
T_{switch}	Switching / particlization temperature
N(2.76 TeV)	Overall normalization (2.76 TeV)
N(5.02 TeV)	Overall normalization (5.02 TeV)
p	Entropy deposition parameter
w	Nucleon width
σ_k	Std. dev. of nucleon multiplicity fluctuations
$d^3_{\min}$	Minimum volume per nucleon
τ_{fs}	Free-streaming time

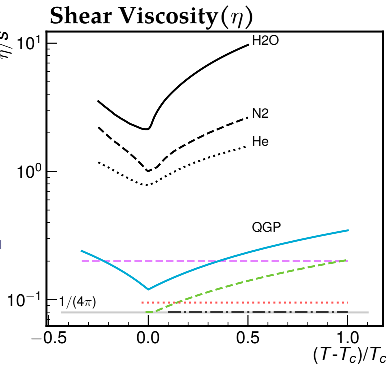


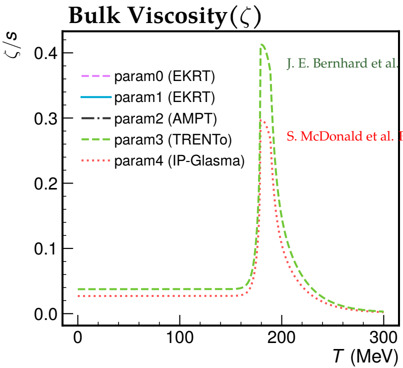
Trento p-value, http://qcd.phy.duke.edu/trento/



TRANSPORT PROPERTIES IN HEAVY-ION COLLISIONS

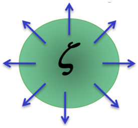






J. E. Bernhard et al. PRC 94 024907 (2016)

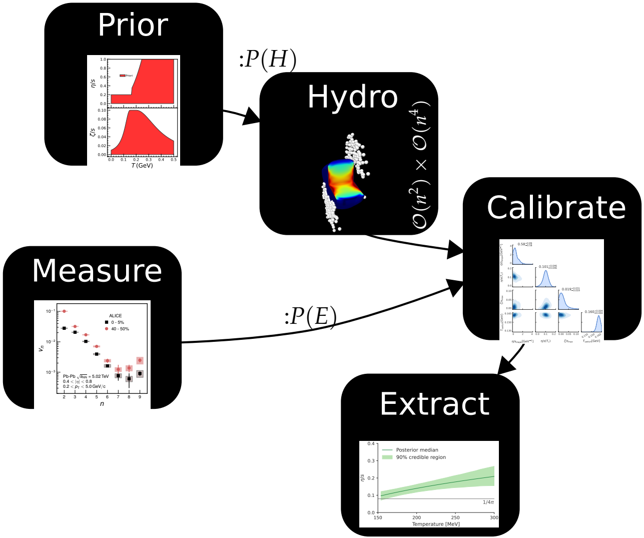
S. McDonald et al. PRC 95, 064913 (2017)



ALICE, JHEP05 (2020) 085

$$(\eta/s)(T) = (\eta/s)(T_c) + (\eta/s)_{\text{slope}}(T - T_c)\left(\frac{T}{T_c}\right)^{(\eta/s)_{\text{curve}}}, (\zeta/s)(T) = \frac{(\zeta/s)_{\max}}{1 + \left(\frac{T - (\zeta/s)_{T_{\text{peak}}}}{(\zeta/s)_{\text{width}}}\right)^2}$$

BAYESIAN PARAMETER ESTIMATION



Prior

$:P(H)$

Hydro

$\mathcal{O}(n^2) \times \mathcal{O}(n^4)$

Measure

$:P(E)$

Calibrate

Extract

Bayes' theorem:

$$\boxed{P(H|E) = \frac{P(E|H) \cdot P(H)}{P(E)}}$$

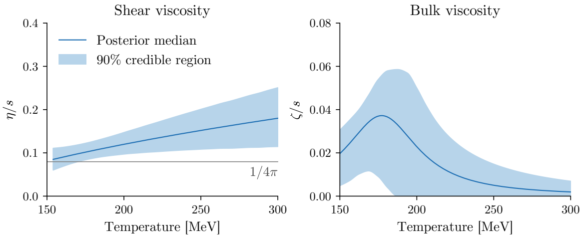
$$, P(E) = \sum_{i=1}^{n} P(E|H_i) P(H_i)$$

- Find optimal set of model parameters that best reproduce the experimental data
- Utilize constraints, such as flow observables, to help narrow down the $\eta/s(T)$ and such.

Testing a single set of parameters requires $\mathcal{O}(10^4)$ hydro events, and evaluating eight different parameters five times each requires $5^8 \times 10^4 \approx 10^9$ hydro events.
That's roughly 10^5 CPU years!

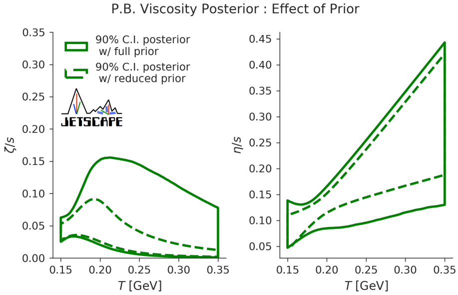
BAYESIAN PARAMETER ESTIMATION

Duke `TRENTo+VISH(2+1D)+UrQMD`



Steffen A. Bass *et. al*, Nature Physics (2019)

JETSCAPE `TRENTo+MUSIC+SMASH`

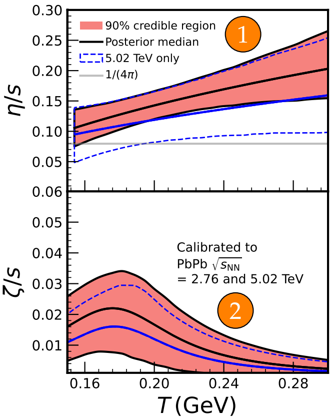


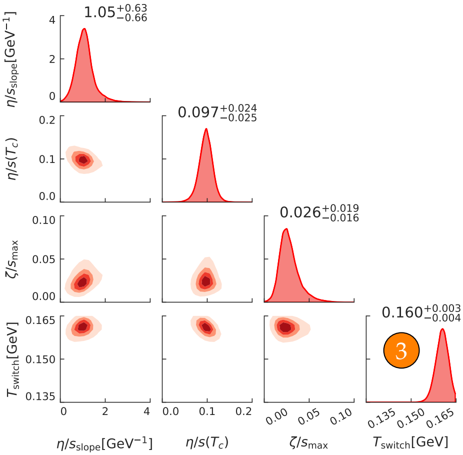
JETSCAPE Collaboration, PRC **103** (2021) 054904

- Large uncertainty for both $\eta/s(T)$ and $\zeta/s(T)$.
- Subsequent studies with still limited observables:
  - J. Auvinen *et al.* PRC. **102**, 044911 (2020)
  - G. Nijs *et al.* PRL. **126**, 202301 (2021)

Uncertainties need to and can be further improved.
Only low-order harmonic v_n was used, including a *limited set of mostly 2.76 TeV* observables.

RESULT: JYVASKYLA (2022) – COMBINED COLLISION ENERGY ANALYSIS (2.76 + 5.02 TEV)





① Significantly improved $\eta/s(T)$ uncertainty

② Non-zero $\zeta/s(T)$

③ Higher switching temperature $T_{\rm switch}$

④ Overall better convergence for parameter components

- Together with two collision energies and added observables, the uncertainty has reduced!

J.E. Parkkila, A. Onnerstad, M. Virta, S.F. Taghavi, C. Mordasini, A. Bilandzic, D.J. Kim, Phys. Lett. B 835 (2022) 137485

FIRST WHY THE UNCERTAINTY WAS REDUCED?

	Duke (2019) [3]	Jyväskylä (2021) [1]	Jyväskylä (2022) [2]
2.76 TeV	PID[1] mult. N_{ch} PID[1] $\langle p_T \rangle$ Transverse energy E_T $\delta p_T / \langle p_T \rangle$ v_2 to v_4		N_{ch} PID[1] $\langle p_T \rangle$ v_2 to v_4 NSC(3,2), NSC(4,2) NSC(2,3,4), NSC(2,3,5) $\rho_{4,22}$ to $\rho_{6,mk}$ $\chi_{4,22}$ to $\chi_{6,mk}$
5.02 TeV	N_{ch} v_2 to v_4	PID[2] mult. N_{ch} PID[1] $\langle p_T \rangle$ v_2 to v_4 v_5 to v_7 NSC(3,2) to NSC(4,2) $\chi_{4,22}$ to $\chi_{6,mk}$	PID[2] mult. N_{ch} PID[1] $\langle p_T \rangle$ v_2 to v_4 v_5 to v_7 NSC(3,2) to NSC(4,2) $\rho_{4,22}$ to $\rho_{6,mk}$ $\chi_{4,22}$ to $\chi_{6,mk}$

All reference data based on ALICE measurements.

Red: Missing from other group(Duke, etc)
Blue: New since our PRC.
Orange: Not used in our studies.

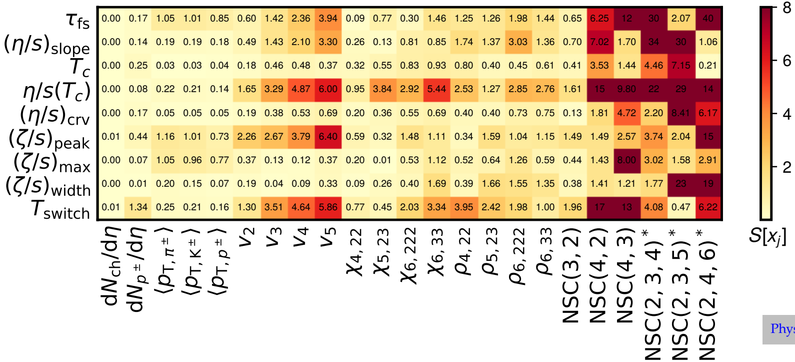
---

[1] $\pi^{\pm}$, K$^{\pm}$ and p$^{\pm}$
[2] p$^{\pm}$

[1]. J.E. Parkkila, A. Onnerstad, D.J. Kim, PRC **104** (2021) 054904
[2]. J.E. Parkkila, A. Onnerstad, M. Virta, S.F. Taghavi, C. Mordasini, A. Bilandzic, D.J. Kim, Phys. Lett. B 835 (2022) 137485
[3]. J.E. Bernhard et al, Nature Phys. 15, 1113-1117 (2019)

SECOND WHY THE UNCERTAINTY WAS REDUCED?

Sensitivity of the observables: $S[x_j] = \Delta/\delta.$, where $\Delta = \frac{|\hat{O}(\vec{x}') - \hat{O}(\vec{x})|}{|\hat{O}(\vec{x})|}$.

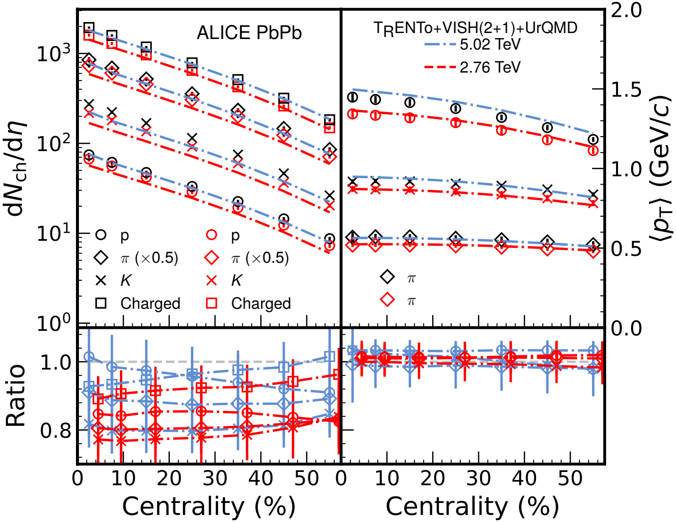


	$dN_{ch}/d\eta$	$dN_{p^\pm}/d\eta$	$\langle p_{T,\pi^\pm}\rangle$	$\langle p_{T,K^\pm}\rangle$	$\langle p_{T,p^\pm}\rangle$	v_2	v_3	v_4	v_5	$\chi_{4,22}$	$\chi_{5,23}$	$\chi_{6,222}$	$\chi_{6,33}$	$\rho_{4,22}$	$\rho_{5,23}$	$\rho_{6,222}$	$\rho_{6,33}$	NSC(3, 2)	NSC(4, 2)	NSC(4, 3)	NSC(2, 3, 4)*	NSC(2, 3, 5)*	NSC(2, 4, 6)*
τ_{fs}	0.00	0.17	1.05	1.01	0.85	0.60	1.42	2.36	3.94	0.09	0.77	0.30	1.46	1.25	1.26	1.98	1.44	0.65	6.25	12	30	2.07	40
$(\eta/s)_{slope}$	0.00	0.14	0.19	0.19	0.18	0.49	1.43	2.10	3.30	0.26	0.13	0.81	0.85	1.74	1.37	3.03	1.36	0.70	7.02	1.70	34	30	1.06
T_c	0.00	0.25	0.03	0.03	0.04	0.18	0.46	0.48	0.37	0.32	0.55	0.83	0.93	0.80	0.40	0.45	0.61	0.41	3.53	1.44	4.46	7.15	0.21
$\eta/s(T_c)$	0.00	0.08	0.22	0.21	0.14	1.65	3.29	4.87	6.00	0.95	3.84	2.92	5.44	2.53	1.27	2.85	2.76	1.61	15	9.80	22	29	14
$(\eta/s)_{crv}$	0.00	0.17	0.05	0.05	0.05	0.19	0.38	0.53	0.69	0.20	0.36	0.55	0.69	0.40	0.40	0.73	0.75	0.13	1.81	4.72	2.20	8.41	6.17
$(\zeta/s)_{peak}$	0.01	0.44	1.16	1.01	0.73	2.26	2.67	3.79	6.40	0.59	0.32	1.48	1.11	0.34	1.59	1.04	1.15	1.49	1.49	2.57	3.74	2.04	15
$(\zeta/s)_{max}$	0.00	0.07	1.05	0.96	0.77	0.37	0.13	0.12	0.37	0.20	0.01	0.53	1.12	0.52	0.64	1.26	0.59	0.44	1.43	8.00	3.02	1.58	2.91
$(\zeta/s)_{width}$	0.00	0.01	0.20	0.15	0.07	0.19	0.04	0.09	0.33	0.09	0.26	0.40	1.69	0.39	1.66	1.55	1.35	0.38	1.41	1.21	1.77	23	19
T_{switch}	0.01	1.34	0.25	0.21	0.16	1.30	3.51	4.64	5.86	0.77	0.45	2.03	3.34	3.95	2.42	1.98	1.00	1.96	17	13	4.08	0.47	6.22

Phys. Lett. B 835 (2022) 137485

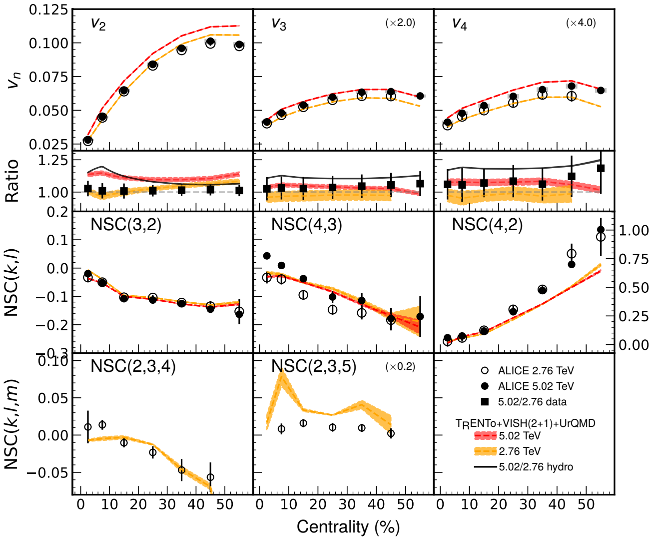
- $N_{p^\pm}/d\eta$ is sensitive to T_{switch} and $\langle p_T\rangle$ is sensitive to τ_{fs}.
- NSC(m,n) and NSC(k,l,m) are among the most sensitive observables followed by v_n and $\chi_{n,mk}$.
- The precision measurements of observables, reflecting mostly non-linear responses, are crucial.

PID MULTIPLICITY AND $\langle p_T \rangle$



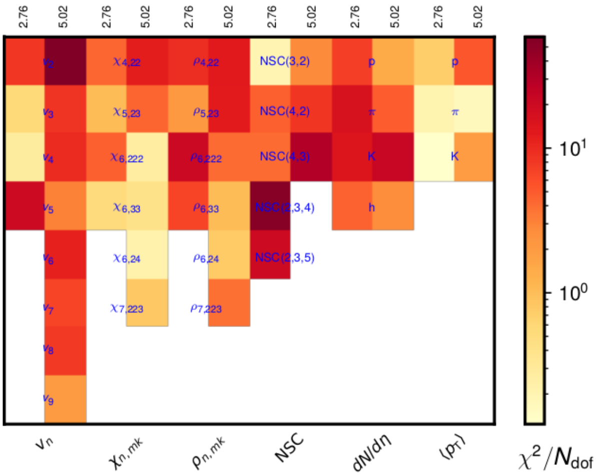
- Agreement for charged particle yield in 2.76 TeV and 5.02 TeV
- 10–20% difference for PID multiplicity
- Qualitative agreement for $\langle p_T \rangle_{\pi,K}$

v_n AND SYMMETRIC CUMULANTS



- Good agreement for 2.76 TeV v_n, overestimated v_2 for 5.02 TeV by $\sim 10\%$
- Magnitude and centrality dependence of NSC well captured. Further improved estimate for NSC(4,2).
- Good agreement for NSC(2,3,4). NSC(2,3,5) overestimated.

REMAINING CONCERNS? χ^2-TEST



- overestimated v_n for 5.02 TeV by ~ 10%
- still underestimated NSC(4,2)
- overestimated NSC(2,3,5)
- PID multiplicity (especially $\pi^{\pm}$)

New developments on experiments

The precision measurements of observables, reflecting mostly non-linear responses, are crucial
New and Independent information!

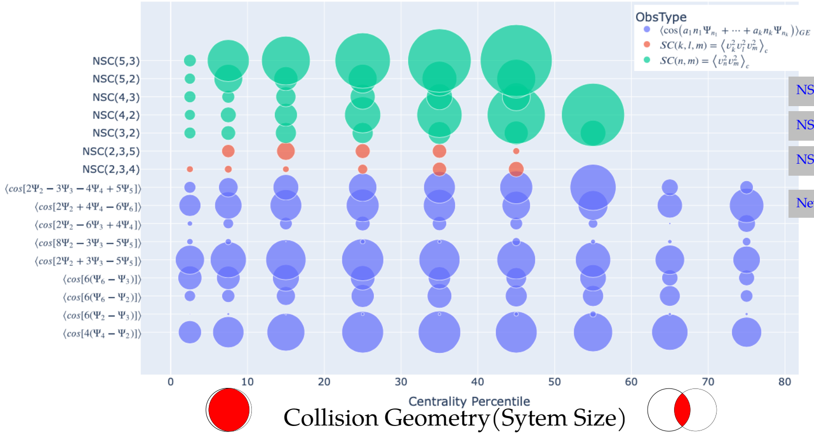
$$V_n \equiv v_n\{\Psi_n\} e^{in(\Psi_n - \phi)}$$

① Symmetric cumulants	$\langle v_m^2 v_n^2 \rangle_c$, $\langle v_k^2 v_l^2 v_m^2 \rangle_c$, "SC$(m,n)$"	additonal information
② Non-linear flow modes	relations of flow obs, "$\chi_{n,kl}$"	decomposition
③ Asymmetric cumulants	$\langle v_m^{2\cdot a} v_n^{2\cdot b} \rangle_c$, "AC$_{a,b}(m,n)$"	new moments
④ Symmetric plane Corr.	$\langle \cos\left(a_1 n_1 \Psi_{n_1} + \cdots + a_k n_k \Psi_{n_k}\right)\rangle$	independent
⑤ Small systems	v_n only from 2PC **Su-Jeong Ji On Thur**	jets are dominant

- Thanks to the higher order flow measurements! ALICE, Phys.Rev.Lett.116 (2016) 132302, JHEP05 (2020) 085
- Thank you: Flow Fluctuation! ALICE, JHEP 07(2018) 103, 2018

RECENT FLOW OBSERVABLES IN ONE FIGURE, ① AND ④

Pb–Pb 2.76 TeV



Collision Geometry(Sytem Size)

NSC: Phys.Rev.Lett.117(2016) 182301

NSC: Phys.Rev.C 97(2018) 024906

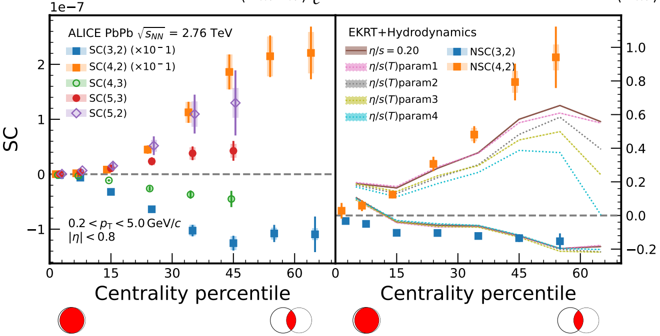
NSC: Phys.Rev.Lett.127(2021) 092302

New SPC: ALICE, arXiv:2302.01234, Feb.2 2023

- Bubble size ∝ correlation strength : few selected observables from 2.76TeV
- Varies with different harmonic orders and system size - Different sensitivites with independent information!
- Pb–Pb 5.02 TeV, NL mode SC: JHEP05 (2020) 085, Phys.Lett.B818 (2021) 136354

High precision flow results and New developments- ① Symmetric Cumulants

$$\mathrm{SC}(m,n) = \left\langle v_m^2 v_n^2 \right\rangle_c, \ \mathrm{NSC}(m,n) = \mathrm{SC}(m,n) / \left\langle v_m^2 \right\rangle \left\langle v_n^2 \right\rangle$$



$$\mathrm{SC}(k,l,m) = \left\langle v_k^2 v_l^2 v_m^2 \right\rangle_c$$

ALICE, Phys. Rev. Lett. 127 (2021) 092302

- $\eta/s(T)$ and accessing $\zeta/s(T)$

- Very challenging measurements because of their required high precisions (i.e 10^{-6} SC(m,n), 10^{-12} for SC(k,l,m)) and difficulties in correcting experimental biases.

- Accessing the temperature dependence of $\eta/s(T)$

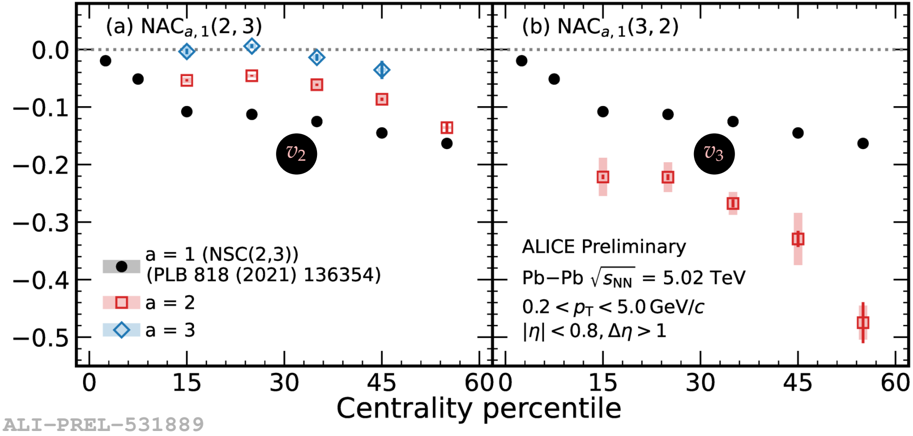
ALICE, Phys. Rev. Lett. 117 (2016) 182301, Editors' Suggestion

ALICE, Phys. Rev. C 97 no. 2, (2018) 024906

ASYMMETRIC CUMULANTS: ③ WHAT ABOUT DIFFERENT MOMENTS OF v_n?

- Generalized symmetric cumulants with different moments - additional information!(Skewness,Kurtosis)
- First measurements in Pb-Pb collisions at 5.02 TeV.

$$\mathrm{NAC}_{a,1}(m,n) = \langle v_m^{2\cdot a} v_n^{2\cdot 1}\rangle_c / \langle v_m^2\rangle^a \langle v_n^2\rangle^1, \ \mathrm{NAC}_{1,1}(m,n) = \mathrm{NSC}(m,n)$$



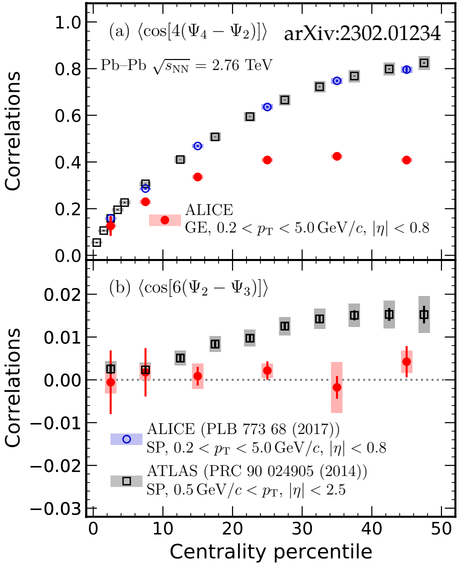
4 Improved Symmetry planes correlations (SPC)

- Paper for 2.76 TeV ALICE, arXiv:2302.01234, Feb.2 2023
- Previously done with scalar product (SP) method (ATLAS: PRC 90, 024905 (2014), ALICE, PLB 773: 68 (2017) [J.E. Parkkila, D.J. Kim])
- Seperating effects from v_m-v_n correlations

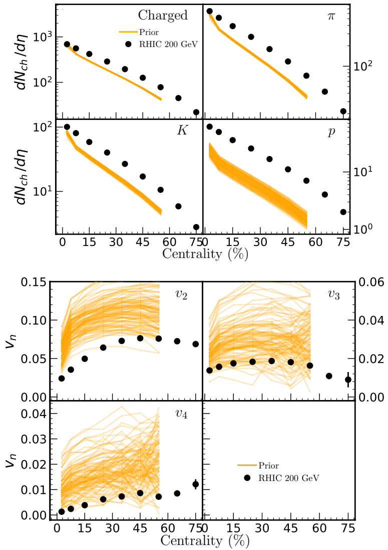
$$\langle\cos\left(a_1 n_1 \Psi_{n_1} + \cdots + a_k n_k \Psi_{n_k}\right)\rangle_{\mathrm{GE},\mathrm{SP}}$$

$$\simeq \sqrt{\frac{\pi}{4}} \frac{\langle v_{n_1}^{a_1} \cdots v_{n_k}^{a_k} \cos\left(a_1 n_1 \Psi_{n_1} + \cdots + a_k n_k \Psi_{n_k}\right)\rangle}{\sqrt{\langle v_{n_1}^{2a_1} \cdots v_{n_k}^{2a_k}\rangle}},$$

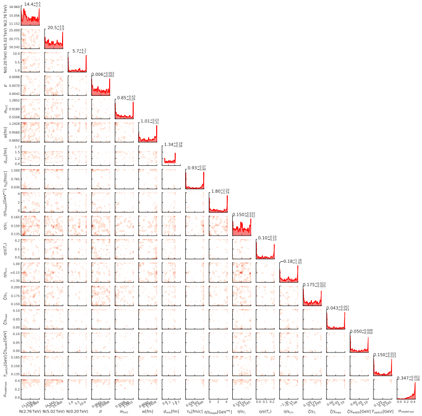
$$= \frac{\langle v_{n_1}^{a_1} \cdots v_{n_k}^{a_k} \cos\left(a_1 n_1 \Psi_{n_1} + \cdots + a_k n_k \Psi_{n_k}\right)\rangle}{\sqrt{\langle v_{n_1}^{2a_1}\rangle \cdots \langle v_{n_k}^{2a_k}\rangle}},$$



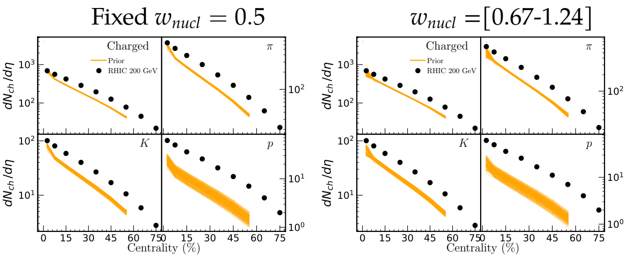
INCLUSION OF RHIC DATA

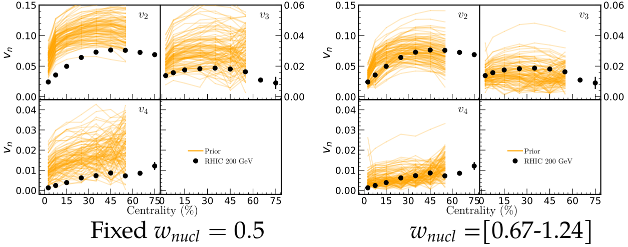


- Included observables : v_2 - v_4, $\langle p_T \rangle$ and N_{ch} for charged and identified particles
- Prior doesn't extend well over the data points
- Posterior distributions don't converge

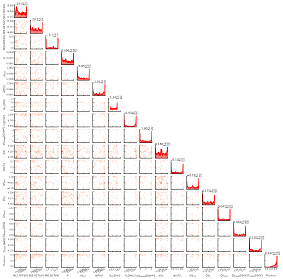


INCLUSION OF RHIC DATA

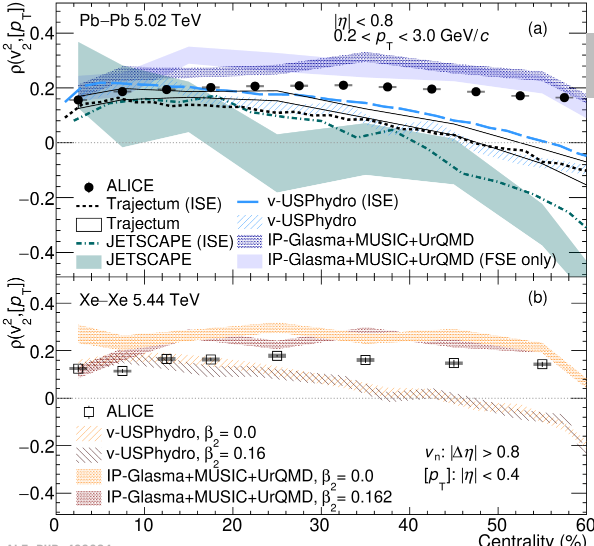




- Included observables : v_2 - v_4, $\langle p_T \rangle$ and N_{ch} for charged and identified particles
- Fixed nuclen width $w = 0.5$ and now relaxed (0.67-1.24)
- small improvements for N_{ch} and better for v_n
- Posterior distributions don't converge still



v_n, $[p_T]$ CORRELATION - SHORTAGE OF TRENTo MODEL



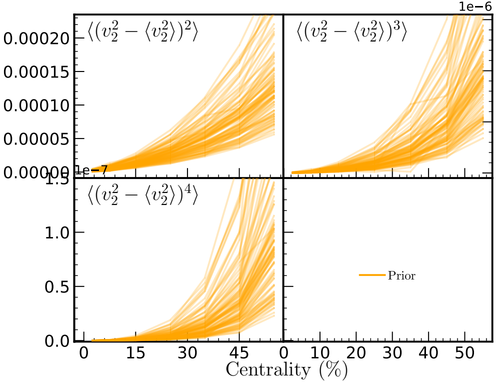
ALI-PUB-499024

P. Bozek, R. Samanta, Phys. Rev. C 102, 034905
B. Schenke, C. Shen, D. Teaney, Phys. Rev. C 102, 034905
ALICE, Phys.Lett.B 834 (2022) 137393

$$\rho(v_2^2,[p_T]) = \frac{\langle \delta v_2^2\, \delta[p_T]\rangle}{\sqrt{\langle(\delta v_2^2)^2\rangle\langle(\delta[p_T])^2\rangle}} \qquad (1)$$

- Correlation between $[p_T]$ and v_2:
  - can be used to differentiate initial state models
  - More peripheral → best described by models with IP-Glasma
  - strong centrality dependence on the models with Trento
- Ongoing progress:
  - Calculate sensitivity
  - Adapt it to the Bayesian Analysis

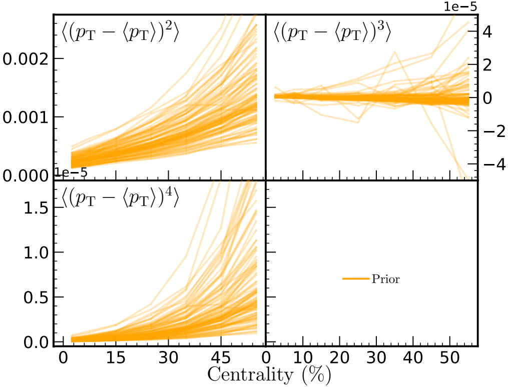
MOMENTS OF δv_n AND δp_{T}



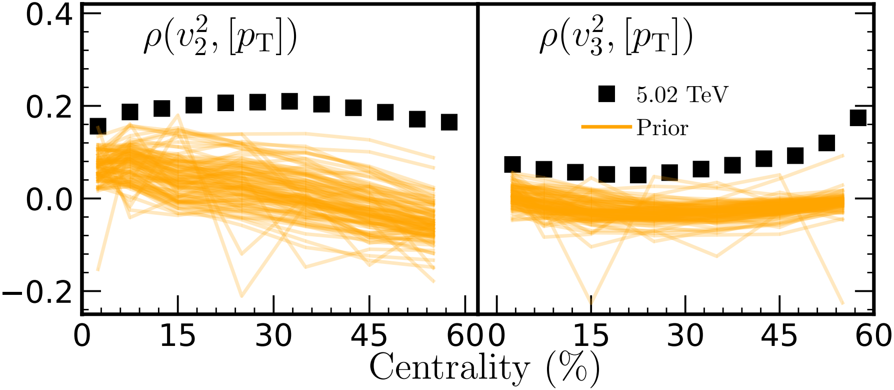
- Characterizes the fluctuation of different order
- $|\eta| < 0.8$ and $0.2\,\mathrm{GeV} < p_{\mathrm{T}} < 5.0\,\mathrm{GeV}$

Moments of a distribution

Variance: $(X-\mu)^2$, **skewness:** $(X-\mu)^3$ and **kurtosis:** $(X-\mu)^4$



PRIOR DISTRIBUTION OF $\rho(v_n^2, [p_\mathrm{T}])$



$v_n^2, [p_\mathrm{T}]$ correlation

$$\rho(v_2^2, [p_T]) = \frac{\langle (v_2^2 - \langle v_2^2 \rangle)\,([p_T] - \langle [p_T] \rangle) \rangle}{\sqrt{\langle (v_2^2 - \langle v_2^2 \rangle)^2 \rangle \langle ([p_T] - \langle [p_T] \rangle)^2 \rangle}}$$

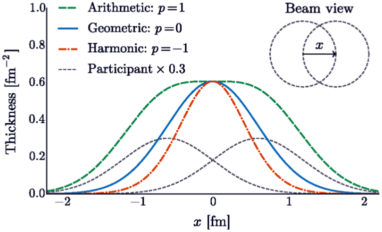
- Clear centrality dependence
- Gains negative values - what is going on?

ALICE, Phys. Lett. B 834 (2022) 137393

Initial state models, TRENTo vs. EKRT: Ongoing

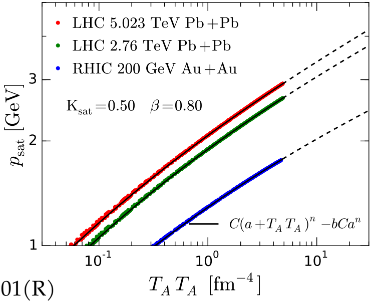
TRENTo[1]

- Flexibility to produce some other models
- Unable to predict ($\sqrt{s_{NN}}$ - Cent) dependence
- Has six free parameters



EKRT [2]

- Only two free parameters, K_{sat} and β_{sat}
- ($\sqrt{s_{NN}}$ - Cent) dependence comes automatically from the gluon saturation and mini-jet production
- Computationally a bit heavier → much improved via ML recently

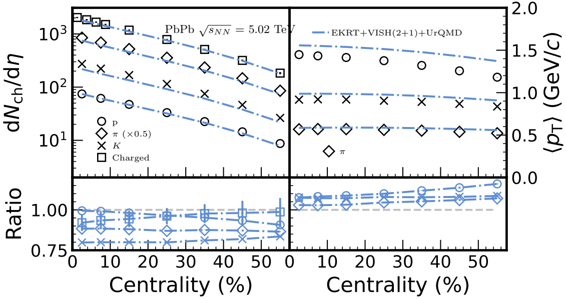


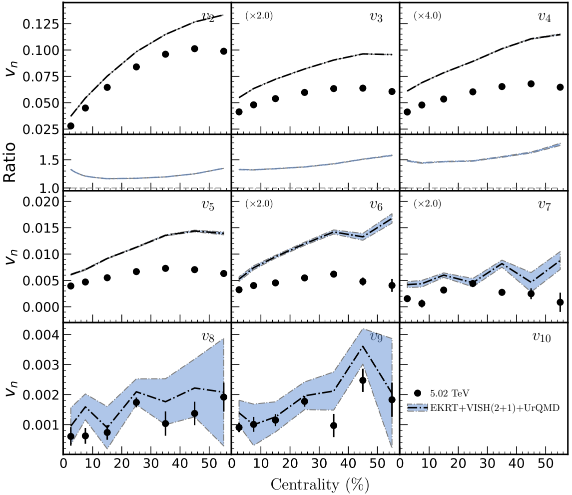
Phys. Rev. C 93, 014912

[1]. J. S. Moreland, J. E. Bernhard, and S. A. Bass, PRC **92** 011901(R)
[2]. H. Niemi, K. J. Eskola, and R. Paatelainen, PRC **93** 024907

EKRT PbPb 5.02 TeV results

- Particle distributions relatively well described
- $\langle p_T \rangle$ a bit overestimated
- Large overestimation in flow harmonics





SUMMARY

Success:

- More Beam energies and new sensitive flow data → better understanding of QCD matter
- Importance of measuring new independent observables - good progress

Challenges:

- Current status
  - 10% difference for v_2 (5.02 TeV) and $\rho(v_2^2, [p_T])$
  - still lacking for NSC(4,2)
  - Remaining discrepancy for PID multiplicity (especially $\pi^{\pm}$) ...
  - better understanding/constraints on initial conditions are challenged!
- Improving the initial conditions with
  - EKRT, IP+Glasma
  - or nucleon size
  - better understading of proton

THANKS

Thank you for your attention!

TOUGH QUESTIONS TO ANSWER BUT WE DO HAVE ANSWERS

1. Q — What is the use of measuring correlations?

   A — Additional and independent information than v_n (event averaged) because of the flow fluctuations

2. Q — Even if each observable provide independent information, how do we know it is useful information?

   A — Based on the known constraints, sensitivities are quantified. More sensitive, better constraints!

3. Q — Is there a limit on how many independent observables we can add into the Bayesian analysis?

   A — Please measure as many observables as you can as long as they are independent. Even though there are overlapping information, it is okay to include

SUMMARY AND OUTLOOK

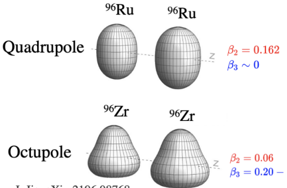
Experiments

- RHIC data (AuAu collisions) - Energy and system size dependence
- LHC pPb and pp data - System size dependence but with improved method

  A. Önnerstad
- Use new observables
  - Higher order ($n > 5$) Symmetric cumulants
  - Improved Symmetric Plane Correlation (SPC) : independent from flow magnitude correlations and Asymmetric Cumulants (AC) **C. Mordasini**
- Soft-Hard interation **M. Virta**
- What about isobar runs in LHC? (WCPF2022, J, Jia)

Theory

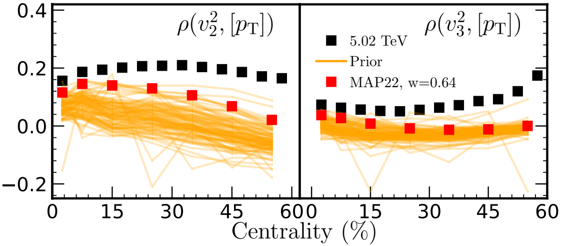
- Improving the initial conditions with
  - EKRT, IP+Glasma
  - or nucleon size
  - better understading of proton
- Testing hydro limit of small systems?
- Role of the small system for further



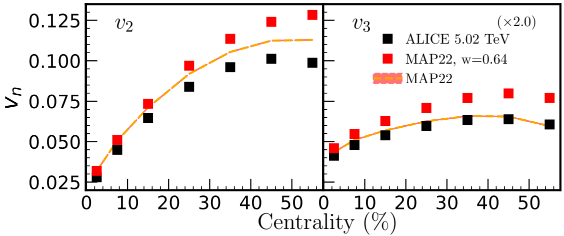
J. Jia, aXiv:2106.08768

THE EFFECTS OF NUCLEON WIDTH PARAMETER

- Nucleon width should be less than 0.8
- Jyväskylä 2022 estimates $w > 0.8$
- Use Jyväskylä 2022 MAP with nucleon width $w = 0.64$, PhysRevLett.129.232301



Validation of the parametrization

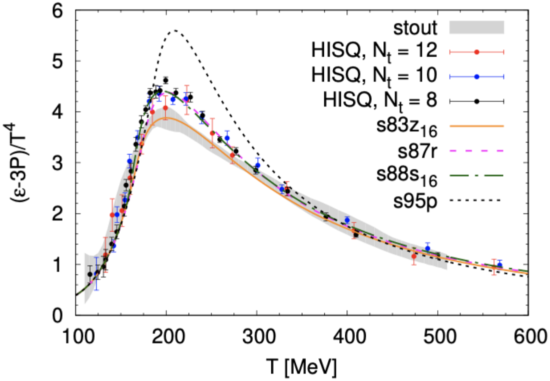


- Reduction only in w guides towards correct ρ but drifts away from experimental v_ns
- $|\eta| < 0.8$ and $0.2\,\mathrm{GeV} < p_T < 5.0\,\mathrm{GeV}$

QUESTIONS TO THINK ABOUT DURING THIS CONFERENCE?

Effect of EoS to Bayesian analysis

- Uncertainties from the equation of state?

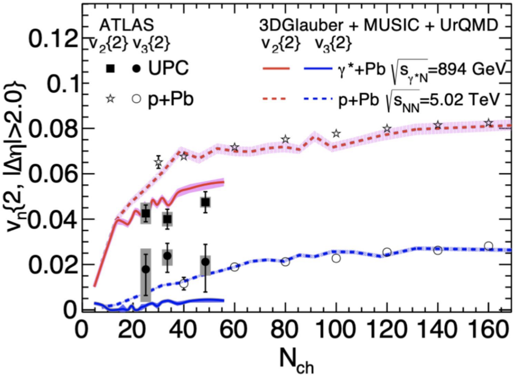


P. Huovinen, P. Petreczky, Nucl.Phys.A837:26-53,2010

PoS(Confinement2018)135

Zero flow at zero multiplicity?

- What does it mean by seeing non-zero flow at zero multiplicity?

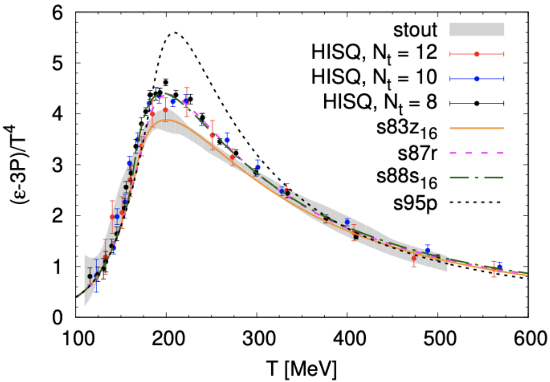


W. Zhao, C. Shen, B. Schenke, arXiv:2203.06094

TWO QUESTIONS TO THINK ABOUT DURING THIS CONFERENCE?

Effect of EoS to Bayesian analysis

- Uncertainties from the equation of state?



P. Huovinen, P. Petreczky, Nucl.Phys.A837:26-53,2010

PoS(Confinement2018)135

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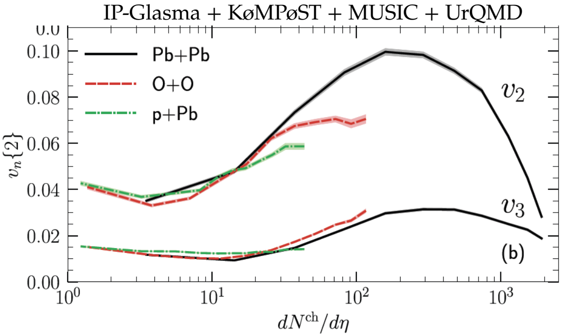
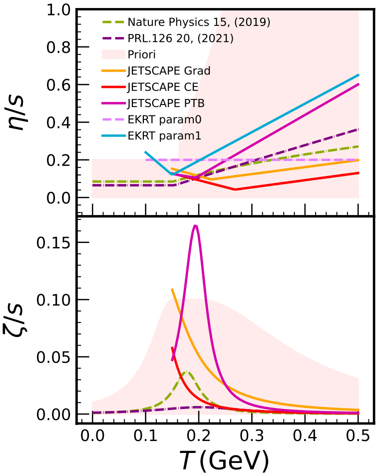


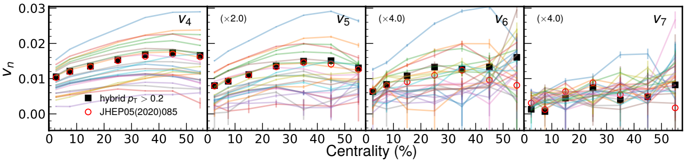
B. Schenke et. al, PRC, 105, 014909 (2022)

ANALYSIS STEPS AND PRIORI

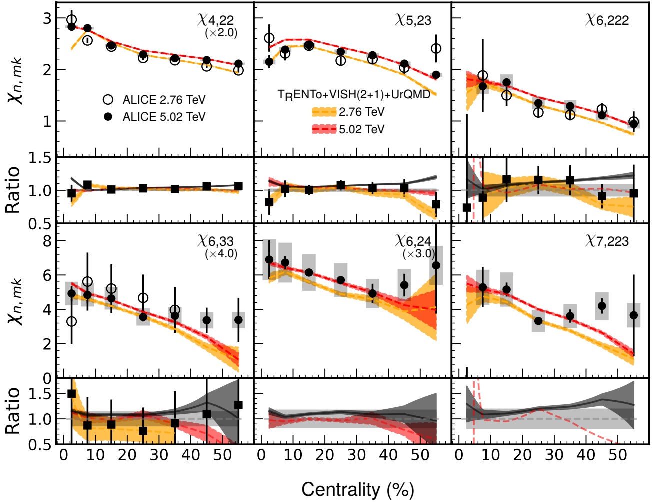


PRC 104 (2021) 054904

1. Choose prior parameter range based on results from 2019
2. Run hydro T$_{\mathrm{R}}$ENTo+VISH(2+1D)+UrQMD for 500 parameterizations, 3-5 million events ($\times$100 previous).
3. Calculate observables using our experimental framework
4. Train emulator and setup/run Bayesian analysis



Non-linear flow coefficients



- Qualitative agreement in both beam energies for all mode coupling coefficients.
- See arXiv:2111.08145 for all graphs.