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Influence of dissipative tunneling on the photodielectric effect associated with the excitation of impurity complexes $A^+ + e$ in a quasi-zero-dimensional structure

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ABSTRACT Effect of tunneling decay for the quasi-stationary A^+ -state, in an impurity complex $A^+ + e$ (a hole, localized on a neutral acceptor, interacting with an electron, localized in the ground state of a quantum dot) on the photodielectric effect, associated with the excitation of impurity complexes $A^+ + e$ in a quasi-zero-dimensional structure, has been studied in the zero-radius potential model in the one-instanton approximation. Calculation of the binding energy of a hole in an impurity complex $A^+ + e$ was performed in the zero radius potential model in the adiabatic approximation. It is shown that as the probability of dissipative tunneling increases, the binding energy of a hole in a complex $A^+ + e$ decreases, which is accompanied by an increase in the effective localization radius of the impurity complex and, accordingly, an increase in the magnitude of the photodielectric effect. The spectral dependence of the photodielectric effect has been calculated in the dipole approximation taking into account the dispersion of the quantum dot radius. A high sensitivity of the magnitude of the photodielectric effect to such parameters of dissipative tunneling as the frequency of the phonon mode, temperature, and coupling constant with the contact medium, has been revealed.

KEYWORDS photodielectric effect, quantum dot, adiabatic approximation, impurity complex, relative permittivity, dissipative tunneling, adiabatic potential, quasi-zero-dimensional structure

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1. Introduction

Dielectric permittivity, as a measure of polarizability, is a fundamental parameter that determines the properties of various materials. Reversible control of the permittivity of materials in a wide range is an urgent problem. In this regard, of interest is the photodielectric effect (PDE), associated with a change in the permittivity of a material under the action of photoirradiation. In recent years, interest in PDE in various materials has increased due to a wide range of practical applications [1–8]. Thus, in article [1], a photo sensor developed on the basis of PDE in CdS is presented, which changes its electrical capacitance upon photoirradiation. The significant change in photocapacitance discovered by the authors of [1] opens up certain prospects for photosensor applications. In [2], PDE was experimentally studied in the compound $\text{LaAl}_{0.99}\text{Zn}_{0.01}\text{O}_{3-\delta}$. It was found that photoirradiation significantly increases the dielectric permittivity in the compound under study in a wide frequency range. It is expected that the results of [2] will contribute to the development of photocapacitors that allow remote control of the dielectric response using photoirradiation. In article [3], the systematic tuning of the PDE in $\text{Ba}(\text{Al}_{1-x}\text{Zn}_x)_2\text{O}_{4-\delta}$ was experimentally studied by varying the Zn concentration (x) and the intensity of the incident light. Results of the study in [3] can form the basis for the development of innovative phototunable functional devices. From the point of view of the possibility of controlling the PDE in a wide range, semiconductor nanostructures with impurity centers are promising. The presence of the quantum size effect makes it possible to control the binding energy of impurity centers, as well as the energy spectrum of the nanostructure, by varying its characteristic size.

Interest in the photodielectric effect (PDE) associated with the excitation of impurity complexes $A^+ + e$ in quantum dots (QDs) (a hole, localized at the A^+ -center, forms a quasi-stationary A^+ -state; an electron is localized in the ground

state of the QD) is due to the possibility of its usage as a method for spectroscopic studies of impurities in quasi-zero-dimensional semiconductor structures [9]. Spectral studies of impurities using PDE, in contrast to photothermal ionization, do not require thermal ionization of excited states. Therefore, measurements can be carried out at arbitrarily low temperatures, which eliminates the effect of temperature on the width of the optical absorption line. From a fundamental point of view, PDE is of interest as a nonlinear optical effect with a lower threshold than conventional nonlinear optical effects, and in the case of QD, this threshold can be controlled by changing of its (QD's) characteristic size. The resonant frequencies ν_0 , which characterize the PDE dispersion, can be in the submillimeter range, for example, as estimates show, for InSb QDs with radius $R_0 \approx 70$ nm, $\nu_0 \sim 10^{11}$ c⁻¹. Consequently, when a semiconductor quasi-zero-dimensional structure is irradiated with energy quanta $h\nu_0$, the refractive index of submillimeter waves should change. Therefore, PDE is an effective mechanism for the influence of IR-radiation on the propagation of submillimeter waves in semiconductor quasi-zero-dimensional structures with impurity complexes $A^+ + e$. As will be shown below, the presence of tunneling decay of quasi-stationary A^+ - states in a QD provides additional degrees of freedom for controlling of the PDE. The purpose of this work is to study theoretically influence of the tunnel decay of a quasi-stationary A^+ -state on the PDE, associated with the excitation of impurity complexes $A^+ + e$ in a quasi-zero-dimensional structure with semiconductor QDs.

2. The binding energy of a quasi-stationary A^+ -state in an impurity complex $A^+ + e$ in a semiconductor quantum dot

Figure 1 shows the energy structure of the “quantum dot with an impurity complex $A^+ + e$ – external matrix” model under consideration. Quasi-stationary A^+ -state can be formed by attaching an additional hole to a neutral acceptor. The finite lifetime of a hole is associated with the decay of the A^+ -state.

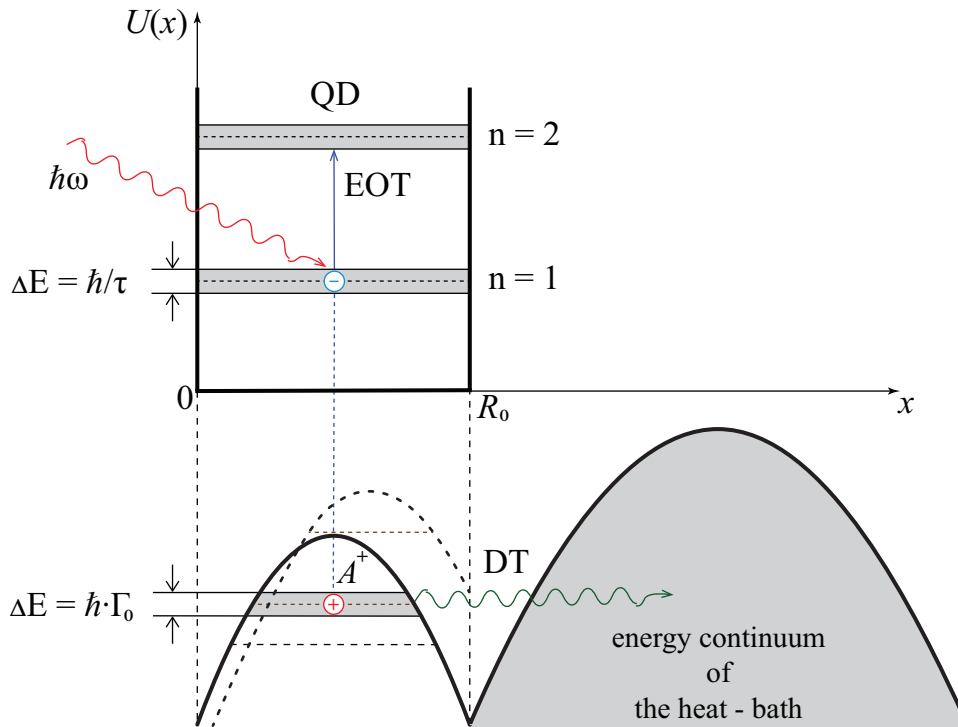


FIG. 1. Energy structure of the “quantum dot with an impurity complex $A^+ + e$ – external matrix” model. Electron is localized in the ground state of the quantum dot, the hole is in the quasi-stationary state of A^+ -center; $\Delta E = \hbar/\tau$ – magnitude of the broadening of the electronic energy level of the quantum dot, this broadening is associated with the radiative lifetime τ ; $\Delta E = \hbar\Gamma_0$ is the broadening of the impurity level, which is associated with the tunneling decay of the A^+ -state; DT – dissipative tunneling; the electron optical transition (EOT) is the optical transition of an electron from the ground state of the QD to the excited state; the dotted line shows the change in the potential profile of the adiabatic potential of the electron due to the change in its quantum state; R_0 is the QD radius.

We will assume that the process of decay of the quasi-stationary A^+ -state is mainly due to the dissipative tunneling of the hole. In the one-instanton approximation, the tunnel decay probability Γ_0 can be represented as $\Gamma_0 = B \exp(-S)$, where the action S and the pre-exponential factor B are defined as (in the Bohr units) [10]:

$$B = \frac{2\sqrt{U_0^*} E_h (1+b^*) \sqrt{\varepsilon_T^*}}{\sqrt{\pi} \hbar} \times \left\{ \frac{A^* \left[\beta_1^* \cosh\left(\frac{\beta_1^*}{2}\right) - 1 \right] + D^* \left[\beta_2^* \cosh\left(\frac{\beta_2^*}{2}\right) - 1 \right]}{\left\{ A^* \left[\frac{\beta_1^*}{2} \frac{\cosh\left(\frac{\beta_1^*}{2} - \tilde{\tau}_{01}^*\right)}{\sinh\left(\frac{\beta_1^*}{2}\right)} - 1 \right] + D^* \left[\frac{\beta_2^*}{2} \frac{\cosh\left(\frac{\beta_2^*}{2} - \tilde{\tau}_{02}^*\right)}{\sinh\left(\frac{\beta_2^*}{2}\right)} - 1 \right] \right\}^{\frac{1}{2}} + \frac{A^* \left[1 - \frac{\beta_1^*}{2} \frac{\cosh\left[\frac{\beta_1^*}{2} - \tilde{\tau}_{01}^*\right]}{\sinh\left[\frac{\beta_1^*}{2}\right]} \right] + D^* \left(\frac{\beta_2^*}{2} \frac{\cosh\left(\frac{\beta_2^*}{2} - \tilde{\tau}_{02}^*\right)}{\sinh\left(\frac{\beta_2^*}{2}\right)} - 1 \right)}{\left\{ A^* \left[\frac{\beta_1^*}{2} \frac{\cosh\left[\frac{\beta_1^*}{2} - \tilde{\tau}_{01}^*\right]}{\sinh\left[\frac{\beta_1^*}{2}\right]} - 1 \right] + D^* \left(\frac{\beta_2^*}{2} \frac{\cosh\left(\frac{\beta_2^*}{2} - \tilde{\tau}_{02}^*\right)}{\sinh\left(\frac{\beta_2^*}{2}\right)} - 1 \right) \right\}^{\frac{1}{2}}} \right\}, \quad (1)$$

and

$$S = a^* \sqrt{U_0^*} \left\{ \frac{1}{2} (b^* + 1)(3 - b^*) \tau_0^* - \frac{(b^* + 1)^2 (\tau_0^*)^2}{2\beta^*} - \frac{(b^* + 1)^2}{2\tilde{\gamma}'} \left\{ \frac{(1 - \tilde{x}'_2)}{\sqrt{\tilde{x}'_1}} \left[\coth(\beta^* \sqrt{\tilde{x}'_1}) - \frac{1}{\sinh(\beta^* \sqrt{\tilde{x}'_1})} \left\{ \cosh((\beta^* - \tau_0^*) \sqrt{\tilde{x}'_1}) - \cosh(\beta^* \sqrt{\tilde{x}'_1}) \right\} + \cosh((\beta^* - \tau_0^*) \sqrt{\tilde{x}'_1}) \right] - \frac{1 - \tilde{x}'_1}{\sqrt{\tilde{x}'_2}} \left[\coth(\beta^* \sqrt{\tilde{x}'_2}) - \frac{1}{\sinh(\beta^* \sqrt{\tilde{x}'_2})} \left\{ \cosh(\beta^* - \tau_0^*) \sqrt{\tilde{x}'_2} - \cosh(\beta^* \sqrt{\tilde{x}'_2}) \right\} + \cosh((\beta^* - \tau_0^*) \sqrt{\tilde{x}'_2}) \right] \right\} \right\}, \quad (2)$$

where

$$A^* = \left(2\varepsilon_L^* a^{*2} - \gamma_{\delta.p.}^{(-)} \right) / \left(\gamma_{\delta.p.}^{(-)} - \gamma_{\delta.p.}^{(+)} \right) \gamma_{\delta.p.}^{(-)},$$

$$D^* = \left(2\varepsilon_L^* a^{*2} - \gamma_{\delta.p.}^{(+)} \right) / \left(\gamma_{\delta.p.}^{(-)} - \gamma_{\delta.p.}^{(+)} \right) \gamma_{\delta.p.}^{(+)},$$

$$\beta_1^* = \sqrt{\gamma_1} \beta = \sqrt{2U_0^* \gamma_{\delta.p.}^{(-)}} / a^* \varepsilon_T^*,$$

$$\beta_2^* = \sqrt{\gamma_2} \beta = \sqrt{2U_0^* \gamma_{\delta.p.}^{(+)}} / a^* \varepsilon_T^*,$$

$$\tilde{\tau}_{01}^* = 2\sqrt{\gamma_1} \tau_0 = \sqrt{\gamma_{\delta.p.}^{(-)} \tau_0'^*} / \sqrt{2},$$

$$\tilde{\tau}_{02}^* = 2\sqrt{\gamma_2} \tau_0 = \sqrt{\gamma_{\delta.p.}^{(+)} \tau_0'^*} / \sqrt{2},$$

$$b^* = b/a,$$

$$\beta^* = \sqrt{U_0^*} / a^* \varepsilon_T^*,$$

$$\tau_0'^* = \operatorname{arcsinh} \left((1 - b^*) \sinh \beta^* / (1 + b^*) \right) + \beta^*,$$

$$\tilde{\gamma}' = \sqrt{\left(\varepsilon_L^* a^{*2} / 4U_0^* + 1 + \varepsilon_c^4 a^{*2} / 4\varepsilon_L^* U_0^* \right)^2 - \varepsilon_L^* a^{*2} / U_0^*},$$

$$\varepsilon_T^* = kT / E_h,$$

$$\varepsilon_c^* = \hbar \sqrt{C} / E_h,$$

$$\varepsilon_L^* = \hbar \omega_L / E_h,$$

$$\tilde{x}'_1 = \frac{1}{2} \left[\varepsilon_L^* a^{*2} / 4U_0^* + 1 + \varepsilon_c^4 a^{*2} / 4\varepsilon_L^* U_0^* - \sqrt{\left(\varepsilon_L^* a^{*2} / 4U_0^* + 1 + \varepsilon_c^4 a^{*2} / 4\varepsilon_L^* U_0^* \right)^2 - \varepsilon_L^* a^{*2} / 4U_0^*} \right],$$

$$\tilde{x}'_2 = \frac{1}{2} \left[\varepsilon_L^* a^{*2} / 4U_0^* + 1 + \varepsilon_c^4 a^{*2} / 4\varepsilon_L^* U_0^* + \sqrt{\left(\varepsilon_L^* a^{*2} / 4U_0^* + 1 + \varepsilon_c^4 a^{*2} / 4\varepsilon_L^* U_0^* \right)^2 - \varepsilon_L^* a^{*2} / 4U_0^*} \right].$$

Figure 2 shows dependence of the probability of dissipative tunneling on the parameters ε_L^* , ε_T^* and ε_c^* , which characterize, respectively, frequency of the phonon mode, temperature, and constant of interaction with the contact medium. As it can be seen from Fig. 2, that, as the parameters ε_L^* and ε_T^* increase, the probability of dissipative tunneling of a hole increases too (due to approaching to the top of the potential barrier), and an increase in the parameter ε_c^* leads to blocking of the tunneling decay (due to an increase in the degree of viscosity of the contact medium (or of the heat-bath)).

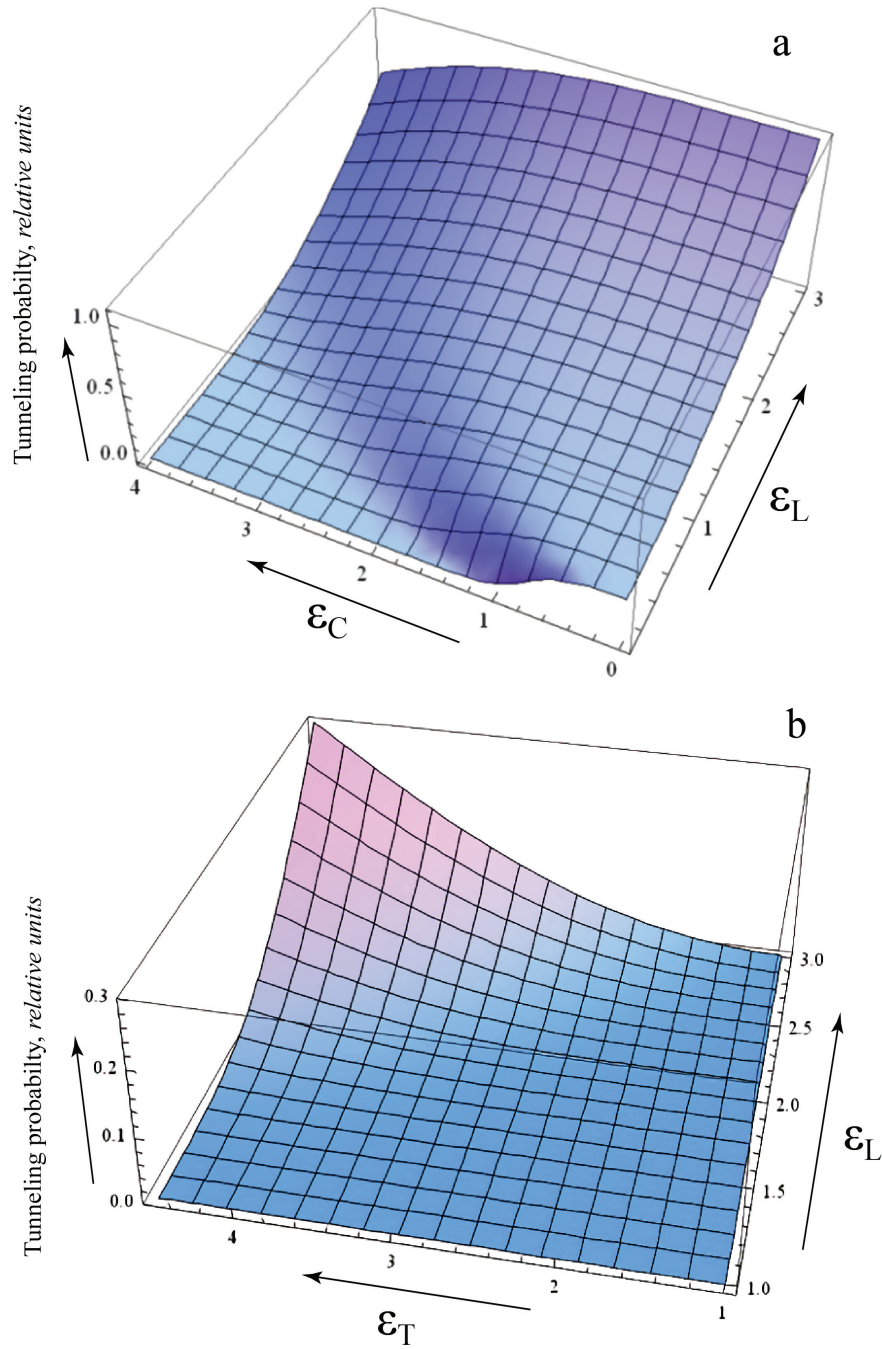


FIG. 2. Dependence of the probability of dissipative tunneling at $R_0 = 70$ nm, $\eta_i = 3.5$ on the next parameters (in relative units): a – ε_L and ε_C , under $\varepsilon_T = 1$; b – ε_L and ε_T , under $\varepsilon_C = 3$

Let the electron be initially in the s-state ($l = m = 0$), then the hole is acted upon by a spherically symmetric potential:

$$V_{n,l,m}(\vec{r}) = -\frac{e^2}{4\pi\varepsilon\varepsilon_0} \int_0^{R_0} \frac{|\psi_{n,l,m}(\vec{r}_e)|^2}{|\vec{r} - \vec{r}_e|} d\vec{r}_e, \quad (3)$$

where e – the electron charge; ε – permittivity of the QD material; ε_0 is the electric constant; $\Psi_{n,l,m}(\vec{r}_e)$ is the electron wave function in QD; n is the radial quantum number of an electron; $m = 0, \pm 1, \pm 2 \dots$ – magnetic quantum number; $l = 0, 1, 2 \dots$ is the orbital quantum number. In the first order of perturbation theory, for the ground state of an electron

($m = 0, l = 0$), potential (3) can be represented as:

$$V_{n,0,0}(r) = -\frac{e^2\beta_n^s}{4\pi\epsilon\epsilon_0R_0} + \frac{m_h^*\omega_n^2r^2}{2}, \quad (4)$$

where $\beta_n^s = \gamma_0 - Ci(2\pi n) + \ln(2\pi n)$, $\gamma_0 = 1.781$ is the Euler constant; $Ci(x)$ is the integral cosine; the frequency ω_n is determined by the following relation:

$$\hbar\omega_n = [(2\hbar^2\pi^2n^2e^2)/(3m_h^*R_0^3\epsilon)]^{1/2}. \quad (5)$$

The energy levels of such an oscillator are given in the form

$$E_{n_1, n_2, n_3}^{n, 0, 0} = -\frac{e^2}{4\pi\epsilon\epsilon_0R_0}\beta_n^s + \hbar\omega_n \left(n_1 + n_2 + n_3 + \frac{3}{2} \right), \quad (6)$$

and the corresponding one-particle wave functions are written as

$$\Psi_{n_1, n_2, n_3}^n(x, y, z) = C_n \exp\left(-\frac{x^2 + y^2 + z^2}{2a_n^2}\right) H_{n_1}\left(\frac{x}{a_n}\right) H_{n_2}\left(\frac{y}{a_n}\right) H_{n_3}\left(\frac{z}{a_n}\right), \quad (7)$$

where $C_n = [2^{n_1+n_2+n_3}n_1!n_2!n_3!\pi^{3/2}a_n^3]^{-1/2}$; $a_n = \sqrt{\hbar/(m_h^*\omega_n)}$; $H_n(x)$ are the Hermite polynomials; n_1, n_2, n_3 – quantum numbers, corresponding to the energy levels of a harmonic oscillator (6).

Using, further, the zero-radius potential procedure (see, for example, [11]), we obtain an equation, that determines dependence of the binding energy of a hole in a complex $A^+ + e$ on the QD and dissipative tunneling parameters.

The short-range impurity perturbation potential can be written correctly in the form of a pseudopotential [12] as The short-range potential can be represented as a boundary condition at the point (x_a, y_a, z_a) given by a pseudo-potential of the form

$$V_\delta(x, y, z; x_a, y_a, z_a) = \gamma\delta(x - x_a)\delta(y - y_a)\delta(z - z_a) \times \left[1 + (x - x_a)\frac{\partial}{\partial x} + (y - y_a)\frac{\partial}{\partial y} + (z - z_a)\frac{\partial}{\partial z} \right], \quad (8)$$

where $\gamma = 2\pi\hbar^2/(\alpha m_h^*)$ is the zero-range potential intensity; α is determined by the energy E_i of the bound state of the same A^+ -center in the bulk semiconductor; ρ_a, z_a are the coordinates of the A^+ -center in QD.

To determine the binding energy of a hole in the complex $A^+ + e$, it is necessary to construct a one-particle Green's function $G(r, R_a; E_{\lambda n})$ to the Schrödinger equation with a Hamiltonian, containing potential (8):

$$G(r, R_a; E_{\lambda n}) = - \sum_{n_1, n_2, n_3} \frac{\Psi_{n_1, n_2, n_3}^{n*}(x_a, y_a, z_a) \Psi_{n_1, n_2, n_3}^n(x, y, z)}{-E_{\lambda n} - \frac{e^2\beta_n^s}{\epsilon R_0} + \frac{i\Gamma_0}{\hbar\omega_n} + \hbar\omega_n \left(n_1 + n_2 + n_3 + \frac{3}{2} \right)}, \quad (9)$$

where $E_{\lambda n} = -\hbar^2\lambda_n^2/(2m_h^*)$.

For the Green's function, taking into account (7), we obtain

$$G(\vec{r}, \vec{R}_a, \varepsilon_{\lambda n}) = -\frac{1}{\pi^{3/2}a_n^2} e^{-\frac{r^2 + R_a^2}{2a_n^2}} \sum_{n_1, n_2, n_3} \frac{H_{n_1}\left(\frac{x}{a_n}\right) H_{n_1}\left(\frac{x_a}{a_n}\right) H_{n_2}\left(\frac{y}{a_n}\right) H_{n_2}\left(\frac{y_a}{a_n}\right)}{2^{n_1}n_1!} \frac{H_{n_2}\left(\frac{y}{a_n}\right) H_{n_2}\left(\frac{y_a}{a_n}\right)}{2^{n_2}n_2!} \times \\ \times \frac{H_{n_3}\left(\frac{z}{a_n}\right) H_{n_3}\left(\frac{z_a}{a_n}\right)}{2^{n_3}n_3!} \frac{1}{\hbar\omega_n \left(-\varepsilon_{\lambda n} - \frac{e^2\beta_n^s}{\epsilon R_0\hbar\omega_n} + \frac{i\Gamma_0}{\hbar\omega_n} + \frac{3}{2} + n_1 + n_2 + n_3 \right)}, \quad (10)$$

where $\varepsilon_{\lambda n} = E_{\lambda n}/\hbar\omega_n$.

Further, considering that

$$\frac{1}{-\varepsilon_{\lambda n} - \frac{e^2\beta_n^s}{\epsilon R_0\hbar\omega_n} + \frac{i\Gamma_0}{\hbar\omega_n} + \frac{3}{2} + n_1 + n_2 + n_3} = \\ = \int_0^\infty dt \exp \left[-t \left(-\varepsilon_{\lambda n} - \frac{e^2\beta_n^s}{\epsilon R_0\hbar\omega_n} + \frac{i\Gamma_0}{\hbar\omega_n} + \frac{3}{2} + n_1 + n_2 + n_3 \right) \right], \quad (11)$$

the expression for the Green's function will take the following form

$$G(\vec{r}, \vec{R}_a, \varepsilon_{\lambda n}) = -\frac{1}{\pi^{3/2} a_n^3 \hbar \omega_n} e^{-\frac{r^2 + R_a^2}{2a_n}} \int_0^\infty dt \exp \left[-t \left(-\varepsilon_{\lambda n} - \frac{e^2 \beta_n^s}{\varepsilon R_0 \hbar \omega_n} + \frac{i\Gamma_0}{\hbar \omega_n} + \frac{3}{2} \right) \right] \times \\ \times \sum_{n_1} \left(\frac{e^{-tn_1}}{2} \right) \frac{H_{n_1} \left(\frac{x}{a_n} \right) H_{n_1} \left(\frac{x_a}{a_n} \right)}{n_1!} \sum_{n_2} \left(\frac{e^{-tn_2}}{2} \right) \frac{H_{n_2} \left(\frac{y}{a_n} \right) H_{n_2} \left(\frac{y_a}{a_n} \right)}{n_2!} \times \\ \times \sum_{n_3} \left(\frac{e^{-tn_3}}{2} \right) \frac{H_{n_3} \left(\frac{z}{a_n} \right) H_{n_3} \left(\frac{z_a}{a_n} \right)}{n_3!}. \quad (12)$$

The summation in (12) over quantum numbers can be performed using the Moeller formula for the generating function of Hermite polynomials [13]

$$\sum_{k=0}^{\infty} \left(\frac{z}{2} \right)^k \frac{H_k(x) H_k(y)}{k!} = \frac{1}{\sqrt{1-z^2}} \exp \left(\frac{2xyz - (x^2 + y^2)z^2}{1-z^2} \right). \quad (13)$$

Finally, for the Green's function we will have

$$G(\vec{r}, \vec{R}_a, \varepsilon_{\lambda n}) = -\frac{1}{\pi^{3/2} a_n^3 \hbar \omega_n} \exp \left[-\frac{r^2 + R_a^2}{2a_n} \right] \int_0^\infty dt \exp \left[-t \left(-\varepsilon_{\lambda n} - \frac{e^2 \beta_n^s}{\varepsilon R_0 \hbar \omega_n} + \frac{i\Gamma_0}{\hbar \omega_n} + \frac{3}{2} \right) \right] \times \\ \times (1 - e^{-2t})^{-3/2} \exp \left[\frac{2e^{-t} (\vec{r} \vec{R}_a) - e^{-2t} (r^2 + R_a^2)}{a_n^2 (1 - e^{-2t})} \right]. \quad (14)$$

After separating the divergent part in (14) (see, for example, [11]), we obtain

$$G(\vec{r}, \vec{R}_a, \varepsilon_{\lambda n}) = \frac{1}{\pi^{3/2} a_n^3 \hbar \omega_n} \left\{ -\frac{a_n}{2\pi} \frac{\exp \left[\sqrt{2 \left(-\varepsilon_{\lambda n} - \frac{e^2 \beta_n^s}{\varepsilon R_0 \hbar \omega_n} + \frac{i\Gamma_0}{\hbar \omega_n} + \frac{3}{2} \right)} \frac{|\vec{r} - \vec{R}_a|}{a_n} \right]}{|\vec{r} - \vec{R}_a|} - \right. \\ \left. - \frac{1}{(2\pi)^{3/2}} \int_0^\infty dt \exp \left[-t \left(-\varepsilon_{\lambda n} - \frac{e^2 \beta_n^s}{\varepsilon R_0 \hbar \omega_n} + \frac{i\Gamma_0}{\hbar \omega_n} + \frac{3}{2} \right) \right] \times \right. \\ \left. \times \left[\exp \left\{ \frac{2e^{-t} (\vec{r} \vec{R}_a) - e^{-2t} (r^2 + R_a^2)}{a_n^2 (1 - e^{-2t})} \right\} e^{-\frac{r^2 + R_a^2}{2a_n}} \frac{(1 - e^{-2t})^{-3/2}}{2^{-3/2}} - \frac{1}{t\sqrt{t}} e^{-\frac{(\vec{r} - \vec{R}_a)^2}{2a_n^2 t}} \right] \right\}. \quad (15)$$

Acting on both sides of relation (15) by the operator \hat{T} , we obtain an equation, that determines dependence of the binding energy $E_{\lambda n}$ of a hole in a complex $A^+ + e$ on the parameters of the QD, as well as on the parameters of dissipative tunneling and the quantum number n .

$$\eta_i = \sqrt{-\eta_n^2 - \frac{2\beta_n^s}{R_0^*} + 4i\Gamma_0^* + 3\gamma_n^{s-1}} + \frac{2}{\sqrt{\gamma_n^s \pi}} \int_0^\infty dt \exp \left[-\frac{t}{2} \left(-\gamma_n^s \eta_n^2 + 4i\gamma_n^s \Gamma_0^* - \frac{2\gamma_n^s \beta_n^s}{R_0^*} + 3 \right) \right] \times \\ \times \left[\frac{1}{2t\sqrt{2t}} - \frac{\exp \left(-\frac{R_0^{*2} R_a^{*2}}{\gamma_n^s} \left(\frac{1-e^{-t}}{1+e^{-t}} \right) \right)}{(1 - e^{-2t})^{3/2}} \right], \quad (16)$$

where $\gamma_n^s = \sqrt{6R_0^* R_0^* / (2\pi n)}$; $\eta_i = \sqrt{|E_i|/E_h}$; $\eta_n = \sqrt{E_{\lambda n}/E_h}$; $\Gamma_0^* = \hbar\Gamma_0/(4E_h)$.

Further, it is necessary to find the explicit form of the potential $V_{n,l,m}(\vec{r}_h)$, defined by formula (3) in the case, when the electron is in the excited p-state ($l = 1, m = 0$) of QD. Expression (3) can be represented as

$$V_{n,l,m}(r_h) = \frac{-e^2}{4\pi\varepsilon\varepsilon_0} \int_0^\pi \sin\theta d\theta \int_0^{2\pi} d\varphi \left(\int_0^{r_h} \frac{|\Psi_{n,l,m}(r_e, \theta, \varphi)|^2 r_e^2 dr_e}{\sqrt{r_h^2 + r_e^2 - 2r_h r_e \cos\theta}} + \int_{r_h}^{R_0} \frac{|\Psi_{n,l,m}(r_e, \theta, \varphi)|^2 r_e^2 dr_e}{\sqrt{r_h^2 + r_e^2 - 2r_h r_e \cos\theta}} \right). \quad (17)$$

The wave function of an electron, in the p-state with $m = 0$, is given by the next expression

$$\psi_{n,1,0}(r_e, \theta, \varphi) = \frac{J_{\frac{3}{2}}\left(\frac{X_{n,1}r_e}{R_0}\right)}{\sqrt{\pi r_e R_0} J_{\frac{5}{2}}(X_{n,1})} \cos \theta, \quad (18)$$

then, after integration in (17) we obtain

$$\begin{aligned} V_{n,1,0}(r_h) = & \frac{e^2 X_{n,1}}{120\pi R_0^3 r_h^3 \varepsilon \varepsilon_0 (3X_{n,1} \cos(X_{n,1}) + (X_{n,1}^2 - 3) \sin(X_{n,1}))^2} \left[15R_0^3 (R_0^2 + X_{n,1}^2 r_h^2) \times \right. \\ & \times \sin\left(\frac{2X_{n,1}r_h}{R_0}\right) - 30X_{n,1}R_0^4 r_h \cos\left(\frac{2X_{n,1}r_h}{R_0}\right) + X_{n,1}r_h^3 \left(3r_h^2 (1 + 2X_{n,1}^2) - 5R_0^2 (8X_{n,1}^2 - 3) - \right. \\ & - 3 \cos(2X_{n,1}) (5R_0^2 + r_h^2) + 6X_{n,1} \left[5R_0^2 X_{n,1} \left(\text{Ci}(2X_{n,1}) - \text{Ci}\left(\frac{2X_{n,1}r_h}{R_0}\right) - \ln\left(\frac{R_0}{r_h}\right) \right) - \right. \\ & \left. \left. - \sin(2X_{n,1}) (5R_0^2 + r_h^2) \right] \right] \left. \right]. \quad (19) \end{aligned}$$

Note, that in the case of p-state with $m = 0$, the minimum of potential (19) is shifted relative to the QD center (see Fig. 1). Position of the minimum is determined from the solution of the following transcendental equation, which is obtained from the equality to zero of the derivative of expression (19) with respect to r_h :

$$\begin{aligned} 2X_{n,1}r_h \min \left[15R_0^4 \cos\left(\frac{2X_{n,1}r_h \min}{R_0}\right) + 5R_0^2 X_{n,1}^2 r_h \min^2 - \right. \\ \left. - \left(\cos(2X_{n,1}) + 2X_{n,1} \sin(2X_{n,1}) - 1 - 2X_{n,1}^2 \right) r_h \min^4 \right] - \\ - 15R_0^3 \sin\left(\frac{2X_{n,1}r_h \min}{R_0}\right) (R_0^2 - X_{n,1}^2 r_h \min^2) = 0. \quad (20) \end{aligned}$$

Expanding expression (19) into a Taylor series near $r_{h\min}$ and restricting ourselves to the quadratic term, we obtain

$$V_{n,1,0}(r_h) = -\frac{e^2}{\varepsilon R_0} \beta_n^{p,0} - \frac{m_h \omega_n^{p,0,2} (r_h - r_{h\min})^2}{2}, \quad (21)$$

where the quantities $\beta_n^{p,0}$ and $\omega_n^{p,0}$ are defined as follows

$$\begin{aligned} \beta_n^{p,0} = & \frac{X_{n,1}}{30R_0^2 r_h^3 \min (3X_{n,1} \cos(X_{n,1}) + (X_{n,1}^2 - 3) \sin(X_{n,1}))^2} \left[6r_h^5 \min X_{n,1}^3 - 3r_h^5 \min X_{n,1} - \right. \\ & - 15r_h^3 \min X_{n,1} R_0^2 + 40r_h^3 \min X_{n,1}^3 R_0^2 + 3r_h^5 \min X_{n,1} \cos(2X_{n,1}) + 15r_h^3 \min X_{n,1} R_0^2 \cos(2X_{n,1}) + \\ & + 30r_h \min X_{n,1} R_0^4 \cos\left(\frac{2r_h \min X_{n,1}}{R_0}\right) - 30r_h^3 \min X_{n,1}^3 R_0^2 \text{Ci}(2X_{n,1}) + 30r_h^3 \min X_{n,1}^3 R_0^2 \text{Ci}\left(\frac{2r_h \min X_{n,1}}{R_0}\right) - \\ & - 30r_h^3 \min X_{n,1}^3 R_0^2 \ln\left(\frac{r_h \min}{R_0}\right) + 6r_h^5 \min X_{n,1}^2 \sin(2X_{n,1}) + 30r_h^3 \min X_{n,1}^2 R_0^2 \sin(2X_{n,1}) - \\ & \left. - 15r_h^2 \min X_{n,1}^2 R_0^3 \sin\left(\frac{2r_h \min X_{n,1}}{R_0}\right) - 15R_0^5 \sin\left(\frac{2r_h \min X_{n,1}}{R_0}\right) \right]. \quad (22) \end{aligned}$$

The binding energy of a hole localized at the A^+ center, in the case when the electron is in the p-state with $m = 0$, is determined from the solution of the transcendental equation

$$\begin{aligned} \eta_i = & \sqrt{-\eta_n^p - \frac{2\beta_n^{p,0}}{R_0^*} + 4i\Gamma_0^* + 3\gamma_n^{p-1} +} \\ & + \frac{2}{\sqrt{\gamma_n^{p,0} \pi}} \int_0^\infty dt \exp \left[-\frac{\gamma_n^p t}{2} \left(-(\eta_n^p)^2 + 4i\Gamma_0^* - \frac{2\beta_n^p}{R_0^*} + \frac{3}{\gamma_n^p} \right) \right] \left[\frac{1}{2t\sqrt{2t}} - \frac{\exp\left(-\frac{R_0^{*2} R_a^{*2}}{\gamma_n^p} \left(\frac{1-e^{-t}}{1+e^{-t}}\right)\right)}{(1-e^{-2t})^{3/2}} \right], \quad (23) \end{aligned}$$

where

$$\gamma_n^{p,0} = \left[\frac{5r_{h \min}^{*5} R_0^{*3} (3X_{n,1} \cos(X_{n,1}) + (X_{n,1}^3 - 3) \sin(X_{n,1}))^2}{X_{n,1}} \left(r_{h \min}^{*5} X_{n,1} + 2r_{h \min}^{*5} X_{n,1}^3 - \right. \right. \\ \left. \left. - 5r_{h \min}^{*3} X_{n,1}^3 R_0^{*2} - r_{h \min}^{*5} X_{n,1} \cos(2X_{n,1}) + 15r_{h \min}^* X_{n,1} R_0^{*2} (r_{h \min}^{*2} X_{n,1}^2 - 4R_0^{*2}) \cos\left(\frac{2r_{h \min}^* X_{n,1}}{R_0^*}\right) - \right. \right. \\ \left. \left. - 2r_{h \min}^{*5} X_{n,1}^2 \sin(2X_{n,1}) - 15R_0^{*3} \sin\left(\frac{2r_{h \min}^* X_{n,1}}{R_0^*}\right) (3r_{h \min}^{*2} X_{n,1}^2 + 2R_0^{*2}) \right)^{-1} \right]^{1/2} \quad (24)$$

and correspondingly $r_{h \min}^*/r_{h \min}/a_h, \eta_n^{p,0} = \sqrt{|E^{p,0}, \lambda_n|/E_h}$.

The wave function of a hole localized at the A^+ -center in the p-state is defined as:

$$\Psi_{\lambda; n}^p(\vec{r}_h) = \sqrt{\frac{\Gamma\left(\frac{3}{4} + \frac{\varepsilon_{\lambda n}^p}{2}\right) \Gamma\left(\frac{1}{4} + \frac{\varepsilon_{\lambda n}^p}{2}\right)}{2\pi^2 a_{p,n}^3 \left(\psi\left(\frac{3}{4} + \frac{\varepsilon_{\lambda n}^p}{2}\right) - \psi\left(\frac{3}{4} + \frac{\varepsilon_{\lambda n}^p}{2}\right)\right)}} \left(\frac{r_h^2}{a_{p,n}^2}\right)^{-3/4} W_{-\frac{\varepsilon_{\lambda n}^p}{2}, \frac{1}{4}}\left(\frac{r_h^2}{a_{p,n}^2}\right), \quad (25)$$

where

$$\varepsilon_{\lambda n}^p = \left[\frac{5r_{h \min}^{*5} R_0^* (3X_{n,1} \cos(X_{n,1}) + (X_{n,1}^3 - 3) \sin(X_{n,1}))^2}{X_{n,1}} \times \right. \\ \times \left(r_{h \min}^{*5} X_{n,1} + 2r_{h \min}^{*5} X_{n,1}^3 - 5r_{h \min}^{*3} X_{n,1}^3 R_0^{*2} - r_{h \min}^{*5} X_{n,1} \cos(2X_{n,1}) + \right. \\ \left. + 15r_{h \min}^* X_{n,1} R_0^{*2} (r_{h \min}^{*2} X_{n,1}^2 - 4R_0^{*2}) \cos\left(\frac{2r_{h \min}^* X_{n,1}}{R_0^*}\right) - 2r_{h \min}^{*5} X_{n,1}^2 \sin(2X_{n,1}) - \right. \\ \left. \left. - 45r_{h \min}^{*2} X_{n,1}^2 R_0^{*3} \sin\left(\frac{2r_{h \min}^* X_{n,1}}{R_0^*}\right) + 30R_0^{*5} \sin\left(\frac{2r_{h \min}^* X_{n,1}}{R_0^*}\right) \right)^{-1} \right]^{1/2} \left(\frac{R_0^* \eta_i^2 - 2\beta_{p,n}^{p,0}}{2}\right), \quad (26)$$

where $a_{p,n}^2 = a_h^2 \gamma_n^p$.

Thus, the wave function of the excited state of the electron-hole pair will have the following form:

$$\Phi_{out}(r_e, r_h) = \psi_{n,1}^p(r_e) \Psi_{\lambda; n}^p(r_h). \quad (27)$$

Figures 3(a) and 3(b) show dependence of the binding energy of the quasi-stationary A^+ -state in the impurity complex $A^+ + e$ on the dissipative tunneling parameters $\varepsilon_L, \varepsilon_c$ and ε_T for InSb-based QDs at $R_0 = 70$ nm, calculated using formula (23).

It can be seen, that with an increase in the parameters ε_L and ε_T , which characterize the frequency of the phonon mode and temperature, respectively, the binding energy of the quasi-stationary A^+ -state decreases (see Fig. 3(b)) due to an increase in the probability of dissipative tunneling (see Fig. 2(a,b)), as a result, the wave function of the quasi-stationary A^+ -state ‘‘spreads’’, which leads to an increase in the effective radius of the localized A^+ -state. An increase in the parameter ε_C , which characterizes the constant of interaction with the contact medium (with the heat-bath), is accompanied by the blocking of tunnel decay and a corresponding increase in the binding energy of the quasi-stationary A^+ -state (Fig. 3(a)).

The obtained relations (16), (23), and (25) will be used in the next section to calculate the process of photoexcitation of impurity complexes $A^+ + e$ in a quasi-zero-dimensional structure.

3. Photodielectric effect in a quasi-zero-dimensional structure with impurity complexes $A^+ + e$ in the presence of dissipative tunneling

Let us consider the process of photoexcitation for complexes $A^+ + e$ in QDs under conditions of dissipative tunneling. We will assume that the A^+ -center is localized at the point $\vec{R}_a = (0, 0, 0)$, (see Fig. 1). The energy spectrum of an electron in a size-quantized band will be determined by an expression of the next form

$$E_{n,l} = \frac{\tilde{X}_{n,l}^2 E_h}{R_0^{*2}}. \quad (28)$$

Here $\tilde{X}_{n,l} = X_{n,l}/\sqrt{E_h}$; $X_{n,l}$ is the root of the l -th order Bessel function.

The effective Hamiltonian H_{int} of interaction with the field of a light wave can be written as

$$H_{int} = -i \hbar \lambda_0 \sqrt{\frac{2\pi \hbar^2 \alpha^*}{m^* \omega}} I_0 \exp(i \vec{q}_s \vec{r}) (\vec{\epsilon}_{\lambda s} \nabla_{\vec{r}}), \quad (29)$$

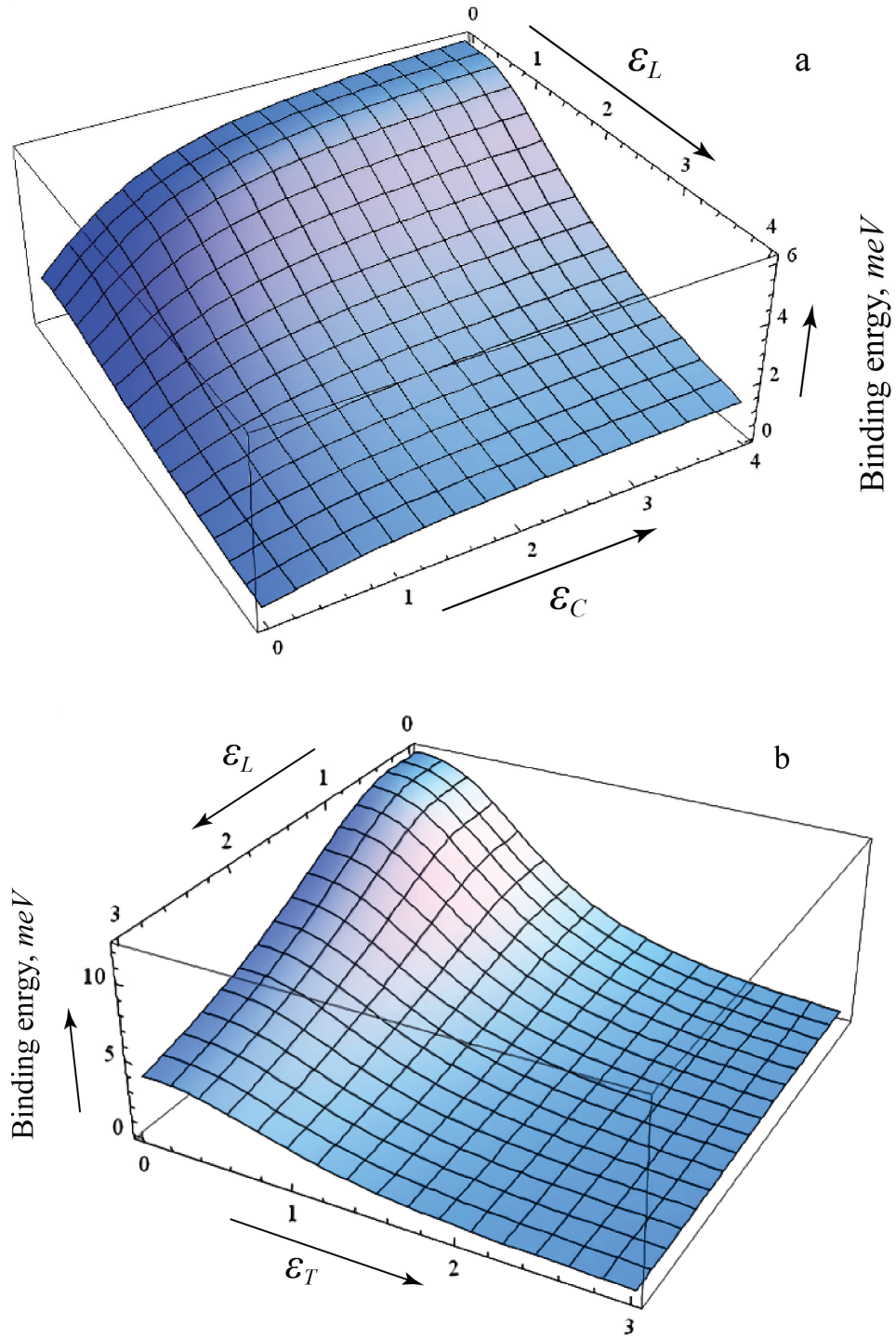


FIG. 3. Dependence of the binding energy of the quasi-stationary A^+ - state in the impurity complex $A^+ + e$ in InSb-based QDs on the parameters of dissipative tunneling at $R_0 = 70$ nm and $\eta_i = 3.5$: a – on the parameters ϵ_L and ϵ_C , at $\epsilon_T = 1$; b – from parameters ϵ_L and ϵ_T , at $\epsilon_c = 3$.

where λ_0 is the local field coefficient; α^* is the fine structure constant, taking into account the static relative permittivity ε ; I_0 is the intensity of light with frequency ω , wave vector \vec{q}_s and the unit polarization vector \vec{e}_{λ_s} ; $\nabla_{\vec{r}}$ is the Hamilton operator.

The matrix element $M_{f,\lambda}$, which determines the magnitude of the oscillator strength of the dipole optical transitions of an electron and a hole from the ground state $\Phi_{in}(\vec{r}_e, \vec{r}_h)$ to the excited states $\Phi_{out}(\vec{r}_e, \vec{r}_h)$ of the QD, is written as

$$M_{f,\lambda} = i \sqrt{\frac{2\pi\alpha^* I_0}{\omega}} \left(E_{n,1,\{0\}} - E_{1,1,0} - \left(E_{\lambda_n}^p \left\{ \begin{smallmatrix} 0 \\ \pm 1 \end{smallmatrix} \right\} - E_{\lambda_1}^s \right) \right) \times \\ \times \langle \Psi_{\lambda_n}^p(\vec{r}_h) | \Psi_{\lambda_1}^s(\vec{r}_h) \rangle \langle \psi_{n,1}(\vec{r}_e) | (\vec{e}_{\lambda_s}, \vec{r}_e) | \psi_{1,0}(\vec{r}_e) \rangle. \quad (30)$$

Taking into account the explicit form of the wave functions (18) and (25), the matrix element (30) can be written in the form

$$M_{f,\lambda} = i \sqrt{\frac{2\pi\alpha^* I_0}{\omega}} (E_{n,1,0} - E_{1,1,0} - (E_{\lambda_n}^p - E_{\lambda_1}^s)) \times \frac{8\pi}{R_0^2 J_{3/2}(X_{0,1}) J_{5/2}(X_{n,1})} \times \\ \times \sqrt{\frac{\Gamma\left(\frac{3}{4} + \frac{\varepsilon_{\lambda_1}^s}{2}\right) \Gamma\left(\frac{1}{4} + \frac{\varepsilon_{\lambda_1}^s}{2}\right)}{2\pi^2 a_{s,n}^3 \left(\psi\left(\frac{3}{4} + \frac{\varepsilon_{\lambda_1}^s}{2}\right) - \psi\left(\frac{3}{4} + \frac{\varepsilon_{\lambda_1}^s}{2}\right)\right)}} \sqrt{\frac{\Gamma\left(\frac{3}{4} + \frac{\varepsilon_{\lambda_n}^p}{2}\right) \Gamma\left(\frac{1}{4} + \frac{\varepsilon_{\lambda_n}^p}{2}\right)}{2\pi^2 a_{p,n}^3 \left(\psi\left(\frac{3}{4} + \frac{\varepsilon_{\lambda_n}^p}{2}\right) - \psi\left(\frac{3}{4} + \frac{\varepsilon_{\lambda_n}^p}{2}\right)\right)}} \times \\ \times \int_0^{R_0} W_{-\frac{\varepsilon_{\lambda_1}^s}{2}; \frac{1}{4}}\left(\frac{r_h^2}{a_{s,n}^2}\right) W_{-\frac{\varepsilon_{\lambda_n}^p}{2}; \frac{1}{4}}\left(\frac{r_h^2}{a_{p,n}^2}\right) \frac{dr_h}{r_h} \times \int_0^{R_0} dr_e r_e^2 J_{1/2}\left(\frac{r_e}{R_0} X_{1,0}\right) J_{\frac{3}{2}}\left(\frac{r_e}{R_0} X_{n,1}\right) \times \\ \times \int_0^\pi d\theta \sin\theta \int_0^{2\pi} d\varphi Y_{0,0}(\theta, \varphi) Y_{1,m}(\theta, \varphi) \cos\theta. \quad (31)$$

After calculating the integrals in (31) and taking into account the expression for the energy spectrum of charge carriers (23) for the matrix element (31), we obtain (in the Bohr units)

$$M_{f,\lambda}(X) = i a_h E_h (2^5 \pi)^{1/2} X_{n,1} \sqrt{\frac{\alpha^* I_0 E_h}{X}} \frac{\left(\frac{k(X_{n,1}^2 - \pi^2)}{R_0^{*2}} - (\eta_n^{p2} - \eta_1^{s2}) \right)}{(3X_{n,1} \cos X_{n,1} + \sin X_{n,1} (X_{n,1}^2 - 3))} \times \\ \times \sqrt{\frac{\Gamma\left(\frac{3}{4} + \frac{\varepsilon_{\lambda_1}^s}{2}\right) \Gamma\left(\frac{1}{4} + \frac{\varepsilon_{\lambda_1}^s}{2}\right)}{\left(\psi\left(\frac{3}{4} + \frac{\varepsilon_{\lambda_1}^s}{2}\right) - \psi\left(\frac{3}{4} + \frac{\varepsilon_{\lambda_1}^s}{2}\right)\right)}} \frac{\Gamma\left(\frac{3}{4} + \frac{\varepsilon_{\lambda_n}^p}{2}\right) \Gamma\left(\frac{1}{4} + \frac{\varepsilon_{\lambda_n}^p}{2}\right)}{\left(\psi\left(\frac{3}{4} + \frac{\varepsilon_{\lambda_n}^p}{2}\right) - \psi\left(\frac{3}{4} + \frac{\varepsilon_{\lambda_n}^p}{2}\right)\right)} \times \\ \times \frac{(X_{n,1}^2 \cos X_{n,1} (X_{n,1}^2 - \pi^2) + \sin X_{n,1} (3X_{n,1}^2 - \pi^2)) (\gamma_n^s \gamma_n^p)^{-3/4}}{R_0^* J_{3/2}(X_{0,1}) J_{5/2}(X_{n,1}) (\pi^2 - X_{n,1}^2)^2} \times \\ \times \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} \frac{(-1)^j (\gamma_n^{s-1} + \gamma_n^{p-1})^j (\gamma_n^s)^{-i}}{2^j j! i!} \left(\frac{\Gamma\left(-\frac{1}{2}\right) \left(\frac{1}{4} + \frac{\varepsilon_{\lambda_n}^s}{2}\right)_i}{\Gamma\left(\frac{1}{4} + \frac{\varepsilon_{\lambda_n}^s}{2}\right) \left(\frac{3}{2}\right)_i} \left[\frac{R_0^{*2} \left(\frac{3}{2} + i + j\right) \Gamma\left(-\frac{1}{2}\right)}{\left(\frac{3}{2} + i + j\right) \Gamma\left(\frac{1}{4} + \frac{\varepsilon_{\lambda_n}^p}{2}\right)} \times \right. \right. \\ \times {}_2F_2\left(\frac{3}{2} + i + j, \frac{3}{4} + \frac{\varepsilon_{\lambda_n}^p}{2}; \frac{5}{2} + i + j, \frac{3}{2}; \frac{R_0^{*2}}{\gamma_n^p}\right) + \frac{\sqrt{\gamma_n^p} R_0^{*2(1+i+j)} \Gamma\left(\frac{1}{2}\right)}{\left(\frac{3}{2} + i + j\right) \Gamma\left(\frac{3}{4} + \frac{\varepsilon_{\lambda_n}^p}{2}\right)} \times \\ \times {}_2F_2\left(1 + i + j, \frac{1}{4} + \frac{\varepsilon_{\lambda_n}^p}{2}; \frac{1}{2}, 2 + i + j; \frac{R_0^{*2}}{\gamma_n^p}\right) \left. \right] + \frac{\sqrt{\gamma_n^s} \left(\frac{1}{4} + \frac{\varepsilon_{\lambda_n}^s}{2}\right)_i}{\Gamma\left(\frac{3}{4} + \frac{\varepsilon_{\lambda_n}^s}{2}\right) \left(\frac{1}{2}\right)_i} \times \\ \times \left[\frac{R_0^{*2(1+i+j)} \Gamma\left(-\frac{1}{2}\right)}{(1+i+j) \Gamma\left(\frac{1}{4} + \frac{\varepsilon_{\lambda_n}^p}{2}\right)} {}_2F_2\left(1 + i + j, \frac{3}{4} + \frac{\varepsilon_{\lambda_n}^p}{2}; 2 + i + j, \frac{3}{2}; \frac{R_0^{*2}}{\gamma_n^p}\right) + \right. \\ \left. + \frac{\sqrt{\gamma_n^p} R_0^{*2\left(\frac{1}{2} + i + j\right)} \Gamma\left(\frac{1}{2}\right)}{\left(\frac{1}{2} + i + j\right) \Gamma\left(\frac{3}{4} + \frac{\varepsilon_{\lambda_n}^p}{2}\right)} {}_2F_2\left(\frac{1}{2} + i + j, \frac{1}{4} + \frac{\varepsilon_{\lambda_n}^p}{2}; \frac{1}{2}, \frac{3}{2} + i + j; \frac{R_0^{*2}}{\gamma_n^p}\right) \right] \right), \quad (32)$$

where $R_0^* = R_0/a_h$; $k = m_h/m_e$; $\eta_1^s = \sqrt{E_{\lambda_1}^s/E_h}$; $\eta_n^p = \sqrt{E_{\lambda_n}^p/E_h}$; ${}_pF_q(a_1, \dots, a_p; b_1, \dots, b_q; z)$ is the generalized hypergeometric function.

Let us further assume that the dispersion u of QD sizes arises during the phase decomposition of a supersaturated solid solution and is satisfactorily described by the Lifshitz–Slezov formula [14]

$$P(u) = \begin{cases} \frac{3^4 e u^2 \exp[-1/(1-2u/3)]}{2^{\frac{5}{3}} (u+3)^{\frac{7}{3}} (3/2-u)^{\frac{11}{3}}}, & u < \frac{3}{2}; \\ 0, & u > \frac{3}{2}, \end{cases} \quad (33)$$

where $u = R_0/\bar{R}_0$, R_0 and \bar{R}_0 – the QD radius and its average value, respectively; e is the base of the natural logarithm.

With relatively small changes in the permittivity ε ($\Delta\varepsilon \leq 3\varepsilon$), change in the value ε during photoexcitation in the dipole approximation, taking into account the dispersion of the QD radius, is determined from the following formula (see [15–17]):

$$\Delta\varepsilon(\omega) = \frac{4\pi N_0 I_0}{\hbar\omega} \sum_n \int_0^{3/2} \sigma_n(\omega) \tau_n \alpha_n P(u) du, \quad (34)$$

where I_0 is the radiation intensity; ω is the incident light frequency; α_n is the polarizability of excited states of an electron in QD; τ_n is the lifetime of excited electronic states; $\sigma_n(\omega)$ is the partial photon absorption cross section for QD with the complex $A^+ + e$.

Since the value $1/\tau_n$ determines the total probability, per unit time, for spontaneous emission of photons during quantum transitions of an electron from the p-state (with different quantum numbers n) to the ground state, then, assuming that the system is characterized only by the radiative lifetime, the expression for τ_n will be written as

$$\frac{1}{\tau_n} = \varpi, \quad (35)$$

where the total transition probability is given by an expression of the next form

$$\varpi = \frac{2\pi}{\hbar} \int \sum_{\sigma} |\langle \Phi_{out}(\vec{r}_e, \vec{r}_h) | \mathbf{V} | \Phi_{in}(\vec{r}_e, \vec{r}_h) \rangle|^2 d\rho(E_{out}), \quad (36)$$

the quantity $d\rho(E_{out})$ characterizes density of the final states number for the system and is defined as

$$d\rho(E_{out}) = \frac{V(\tilde{\omega})^2 d\Omega}{(2\pi)^3 \hbar c^3}, \quad (37)$$

where $\tilde{\omega} = (E_{in} - E_{out})/\hbar = (\hbar^2(X_{n,1}^2 - \pi^2)/2m_e^* a_h^2 R_0^{*2} - (E_{\lambda_n}^p - E_{\lambda_1}^s))/\hbar$; $d\Omega$ is an infinitesimal element of the solid angle, and the interaction operator of a particle with an electromagnetic field (in the one-photon approximation) has the next form

$$\mathbf{V} = -\frac{e}{m_e c} \mathbf{A}(\vec{r}) \mathbf{p}. \quad (38)$$

Here $\mathbf{A}(\vec{r})$ is the vector potential operator defined by the formula (in the Coulomb gauge)

$$\mathbf{A}(\vec{r}) = \sum_{k,\sigma} \left(\frac{2\pi\hbar c^2}{V\omega_k} \right)^{1/2} \vec{e}_{k,\sigma} \left(\mathbf{a}_{k,\sigma} e^{i\vec{k}\vec{r}} + \mathbf{a}_{k,\sigma}^\dagger e^{-i\vec{k}\vec{r}} \right). \quad (39)$$

Respectively, $\mathbf{a}_{k,\sigma}$ and $\mathbf{a}_{k,\sigma}^\dagger$ are the operators of annihilation and creation of a photon with a wave vector \vec{k} and polarization σ , respectively. In the case of the dipole approximation, the expression for the total probability, in the Bohr units, taking into account the QD size dispersion, will be determined as follows

$$\varpi = \frac{4e^2 E_h^3}{3\hbar^4 c^3} \left(\frac{k(X_{n,1}^2 - \pi^2)}{(\bar{R}_0^* u)^2} - (\eta_{\lambda_n}^p - \eta_{\lambda_h}^s) \right)^3 |P_{n,1,0}|^2, \quad (40)$$

where the value $P_{n,1,0}$ is determined by the next expression

$$P_{n,1,0}^p = \langle \Psi_{\lambda_h}^p(\vec{r}_h) | \Psi_{\lambda_h}(\vec{r}_h) \rangle \langle \Psi_{n,1,0}(\vec{r}_e) | (\vec{e}_{\lambda}^s, \vec{r}_e) | \Psi_{1,0}(\vec{r}_e) \rangle. \quad (41)$$

Taking into account the previously obtained expression for the matrix element (32), the expression for (41) can be written as

$$\begin{aligned}
P_n^p(\bar{R}_0^*u) &= \frac{4X_{n,1}(X_{n,1}^2 \cos X_{n,1}(X_{n,1}^2 - \pi^2) + \sin X_{n,1}(3X_{n,1}^2 - \pi^2))}{\bar{R}_0^*u J_{3/2}(X_{0,1}) J_{5/2}(X_{n,1})(\pi^2 - X_{n,1}^2)^2 (a_{s,n} a_{p,n})^{3/2}} \times \\
&\times \sqrt{\frac{\Gamma\left(\frac{3}{4} + \frac{\varepsilon_{\lambda 1}^s}{2}\right) \Gamma\left(\frac{1}{4} + \frac{\varepsilon_{\lambda 1}^s}{2}\right)}{\left(\psi\left(\frac{3}{4} + \frac{\varepsilon_{\lambda 1}^s}{2}\right) - \psi\left(\frac{3}{4} + \frac{\varepsilon_{\lambda 1}^s}{2}\right)\right)}} \left[\frac{\Gamma\left(\frac{3}{4} + \frac{\varepsilon_{\lambda n}^p}{2}\right) \Gamma\left(\frac{1}{4} + \frac{\varepsilon_{\lambda n}^p}{2}\right)}{\left(\psi\left(\frac{3}{4} + \frac{\varepsilon_{\lambda n}^p}{2}\right) - \psi\left(\frac{3}{4} + \frac{\varepsilon_{\lambda n}^p}{2}\right)\right)} \right]^{\frac{3}{2}} \times \\
&\times \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} \frac{(-1)^j (a_{s,n}^{-2} + a_{p,n}^{-2})^j}{2^j a_{s,n}^{2i} j! i! (3X_{n,1} \cos X_{n,1} + \sin X_{n,1}(X_{n,1}^2 - 3))} \left(\frac{\Gamma(-\frac{1}{2}) \left(\frac{1}{4} + \frac{\varepsilon_{\lambda n}^s}{2}\right)_i}{\Gamma\left(\frac{1}{4} + \frac{\varepsilon_{\lambda n}^s}{2}\right) \left(\frac{3}{2}\right)_i} \right) \times \\
&\times \left[\frac{(\bar{R}_0^*u)^{2\left(\frac{3}{2}+i+j\right)} \Gamma\left(-\frac{1}{2}\right)}{\left(\frac{3}{2}+i+j\right) \Gamma\left(\frac{1}{4} + \frac{\varepsilon_{\lambda n}^p}{2}\right)} {}_2F_2\left(\frac{3}{2}+i+j, \frac{3}{4} + \frac{\varepsilon_{\lambda n}^p}{2}; \frac{5}{2}+i+j, \frac{3}{2}; \frac{(\bar{R}_0^*u)^2}{a_{p,n}^2}\right) + \right. \\
&+ \left. \frac{a_{p,n}(\bar{R}_0^*u)^{2(1+i+j)} \Gamma\left(\frac{1}{2}\right)}{\left(\frac{3}{2}+i+j\right) \Gamma\left(\frac{3}{4} + \frac{\varepsilon_{\lambda n}^p}{2}\right)} {}_2F_2\left(1+i+j, \frac{1}{4} + \frac{\varepsilon_{\lambda n}^p}{2}; \frac{1}{2}, 2+i+j; \frac{(\bar{R}_0^*u)^2}{a_{p,n}^2}\right) \right] + \\
&+ \frac{a_{s,n} \left(\frac{1}{4} + \frac{\varepsilon_{\lambda n}^s}{2}\right)_i}{\Gamma\left(\frac{3}{4} + \frac{\varepsilon_{\lambda n}^p}{2}\right) \left(\frac{1}{2}\right)_i} \left[\frac{(\bar{R}_0^*u)^{2(1+i+j)} \Gamma\left(-\frac{1}{2}\right)}{(1+i+j) \Gamma\left(\frac{1}{4} + \frac{\varepsilon_{\lambda n}^p}{2}\right)} {}_2F_2\left(1+i+j, \frac{3}{4} + \frac{\varepsilon_{\lambda n}^p}{2}; 2+i+j, \frac{3}{2}; \frac{(\bar{R}_0^*u)^2}{a_{p,n}^2}\right) + \right. \\
&+ \left. \frac{a_{p,n}(\bar{R}_0^*u)^{2\left(\frac{1}{2}+i+j\right)} \Gamma\left(\frac{1}{2}\right)}{\left(\frac{1}{2}+i+j\right) \Gamma\left(\frac{3}{4} + \frac{\varepsilon_{\lambda n}^p}{2}\right)} {}_2F_2\left(\frac{1}{2}+i+j, \frac{1}{4} + \frac{\varepsilon_{\lambda n}^p}{2}; \frac{1}{2}, \frac{3}{2}+i+j; \frac{(\bar{R}_0^*u)^2}{a_{p,n}^2}\right) \right] \Bigg]. \quad (42)
\end{aligned}$$

Accordingly, for the lifetime of an electron at an excited level, taking into account the dispersion of the QD sizes, we obtain the following expression

$$\tau_n^p(\bar{R}_0^*u) = \frac{3\hbar^4 c^3}{4e^2 E_h^3 \left(\frac{k(X_{n,1}^2 - \pi^2)}{(\bar{R}_0^*u)^2} - (\eta_{\lambda n}^p)^2 - \eta_{\lambda 1}^s \right)^3 |P_n^p(\bar{R}_0^*u)|^2}. \quad (43)$$

The polarizability $\alpha_n^p(\bar{R}_0^*u)$ of electronic states in a QD with a complex $A^+ + e$ in the Bohr units is determined by an expression of the next form

$$\alpha_n^p(\bar{R}_0^*u) = \frac{4a_h^3}{E_h} \frac{|P_n^p|^2}{\frac{k(X_{n,1}^2 - \pi^2)}{(\bar{R}_0^*u)^2} - (\eta_{\lambda n}^p)^2 - \eta_{\lambda 1}^s}. \quad (44)$$

The partial photons absorption cross sections for QDs with the complex $A^+ + e$ are determined by the following formula

$$\sigma_n^p(\omega, R_0) = \frac{2\pi}{\hbar I_0} |M_{f,\lambda}(\omega, R_0)|^2 \frac{\Gamma_0}{\frac{\hbar^2 \Gamma_0^2}{4} + \left(\frac{\hbar^2 (X_{n,1}^2 - \pi^2)}{2m_e^* R_0^2} - (E_{\lambda n}^p - E_{\lambda 1}^s) - \hbar\omega \right)^2}, \quad (45)$$

Where $M_{f,\lambda}(\omega)$ is determined by formula (32). Rewriting (45) in the Bohr units, we obtain

$$\sigma_n^p(X, \bar{R}_0^*u) = \frac{2\pi}{\hbar^2 I_0 E_h} \frac{|M_{f,\lambda}(X)|^2 \Gamma_0^*}{\frac{\Gamma_0^{*2}}{4} + \left(\frac{k(X_{n,1}^2 - \pi^2)}{(\bar{R}_0^*u)^2} - (\eta_{\lambda n}^p)^2 - \eta_{\lambda 1}^s - X \right)^2}. \quad (46)$$

Thus, expression (34) takes the form

$$\begin{aligned}
\Delta\varepsilon(X) &= \frac{N_0 I_0 12\pi^2 \hbar^4 c^3 a_h^5 \alpha^*}{e^2 E_h^5 X^2} \sum_{n=1}^N \int_0^{3/2} P(u) du \frac{|P_n^p(\bar{R}_0^*u)|^2}{\left(\frac{k(X_{n,1}^2 - \pi^2)}{(\bar{R}_0^*u)^2} - (\eta_{\lambda n}^p)^2 - \eta_{\lambda 1}^s \right)^2} \times \\
&\times \frac{\Gamma_0^*}{\frac{\Gamma_0^{*2}}{4} + \left(\frac{k(X_{n,1}^2 - \pi^2)}{(\bar{R}_0^*u)^2} - (\eta_{\lambda n}^p)^2 - \eta_{\lambda 1}^s - X \right)^2}, \quad (47)
\end{aligned}$$

or, taking into account (32), for (47) we finally obtain

$$\begin{aligned}
\Delta\varepsilon(X) = & \frac{N_0 I_0 12 \pi^2 \hbar^4 c^3 a_h^5 \alpha^*}{e^2 E_h^5 X^2} \sum_{n=1}^N \int_0^{3/2} du \frac{P(u)}{\left(\frac{k(X_{n,1}^2 - \pi^2)}{(\bar{R}_0^* u)^2} - (\eta_{\lambda n}^p)^2 - \eta_{\lambda 1}^s)^2 \right)^2} \times \\
& \times \left| \frac{4X_{n,1} (X_{n,1}^2 \cos X_{n,1} (X_{n,1}^2 - \pi^2) + \sin X_{n,1} (3X_{n,1}^2 - \pi^2))}{\bar{R}_0^* u J_{3/2}(X_{0,1}) J_{5/2}(X_{n,1}) (\pi^2 - X_{n,1}^2)^2 (a_{s,n} a_{p,n})^{3/2}} \right| \times \\
& \times \sqrt{\frac{\Gamma\left(\frac{3}{4} + \frac{\varepsilon_{\lambda 1}^s}{2}\right) \Gamma\left(\frac{1}{4} + \frac{\varepsilon_{\lambda 1}^s}{2}\right)}{\left(\psi\left(\frac{3}{4} + \frac{\varepsilon_{\lambda 1}^s}{2}\right) - \psi\left(\frac{3}{4} + \frac{\varepsilon_{\lambda 1}^s}{2}\right)\right)}}{\left[\frac{\Gamma\left(\frac{3}{4} + \frac{\varepsilon_{\lambda n}^p}{2}\right) \Gamma\left(\frac{1}{4} + \frac{\varepsilon_{\lambda n}^p}{2}\right)}{\left(\psi\left(\frac{3}{4} + \frac{\varepsilon_{\lambda n}^p}{2}\right) - \psi\left(\frac{3}{4} + \frac{\varepsilon_{\lambda n}^p}{2}\right)\right)}\right]^{\frac{3}{2}}} \times \\
& \times \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} \frac{(-1)^j (a_{s,n}^{-2} + a_{p,n}^{-2})^j}{2^j a_{s,n}^{2i} j! (3X_{n,1} \cos X_{n,1} + \sin X_{n,1} (X_{n,1}^2 - 3))} \left(\frac{\Gamma(-\frac{1}{2}) \left(\frac{1}{4} + \frac{\varepsilon_{\lambda n}^s}{2}\right)_i}{\Gamma\left(\frac{1}{4} + \frac{\varepsilon_{\lambda n}^s}{2}\right) \left(\frac{3}{2}\right)_i} \right) \times \\
& \times \left[\frac{(\bar{R}_0^* u)^{2(\frac{3}{2}+i+j)} \Gamma(-\frac{1}{2})}{\left(\frac{3}{2} + i + j\right) \Gamma\left(\frac{1}{4} + \frac{\varepsilon_{\lambda n}^p}{2}\right)} {}_2F_2\left(\frac{3}{2} + i + j, \frac{3}{4} + \frac{\varepsilon_{\lambda n}^p}{2}; \frac{5}{2} + i + j, \frac{3}{2}; \frac{(\bar{R}_0^* u)^2}{a_{p,n}^2}\right) + \right. \\
& \left. + \frac{a_{p,n} (\bar{R}_0^* u)^{2(1+i+j)} \Gamma\left(\frac{1}{2}\right)}{\left(\frac{3}{2} + i + j\right) \Gamma\left(\frac{3}{4} + \frac{\varepsilon_{\lambda n}^p}{2}\right)} {}_2F_2\left(1 + i + j, \frac{1}{4} + \frac{\varepsilon_{\lambda n}^p}{2}; \frac{1}{2}, 2 + i + j; \frac{(\bar{R}_0^* u)^2}{a_{p,n}^2}\right) \right] + \\
& + \frac{a_{s,n} \left(\frac{1}{4} + \frac{\varepsilon_{\lambda n}^s}{2}\right)_i}{\Gamma\left(\frac{3}{4} + \frac{\varepsilon_{\lambda n}^p}{2}\right) \left(\frac{1}{2}\right)_i} \left[\frac{(\bar{R}_0^* u)^{2(1+i+j)} \Gamma(-\frac{1}{2})}{(1 + i + j) \Gamma\left(\frac{1}{4} + \frac{\varepsilon_{\lambda n}^p}{2}\right)} {}_2F_2\left(1 + i + j, \frac{3}{4} + \frac{\varepsilon_{\lambda n}^p}{2}; 2 + i + j, \frac{3}{2}; \frac{(\bar{R}_0^* u)^2}{a_{p,n}^2}\right) \right. \\
& \left. + \frac{a_{p,n} (\bar{R}_0^* u)^{2(\frac{1}{2}+i+j)} \Gamma\left(\frac{1}{2}\right)}{\left(\frac{1}{2} + i + j\right) \Gamma\left(\frac{3}{4} + \frac{\varepsilon_{\lambda n}^p}{2}\right)} {}_2F_2\left(\frac{1}{2} + i + j, \frac{1}{4} + \frac{\varepsilon_{\lambda n}^p}{2}; \frac{1}{2}, \frac{3}{2} + i + j; \frac{(\bar{R}_0^* u)^2}{a_{p,n}^2}\right) \right] \Bigg|^2 \times \\
& \times \frac{\Gamma_0^*}{\frac{\Gamma_0^{*2}}{4} + \left(\frac{k(X_{n,1}^2 - \pi^2)}{(\bar{R}_0^* u)^2} - (\eta_{\lambda n}^p)^2 - \eta_{\lambda 1}^s - X \right)^2}. \quad (48)
\end{aligned}$$

Figure 4(a,b,c) shows the spectral dependence of the relative change in permittivity $\Delta\varepsilon/\varepsilon$ for the quasi-zero-dimensional structure with InSb QDs for various dissipative tunneling parameters ε_L^* , ε_C^* and ε_T^* , calculated by formula (48). As can be seen from Fig. 4(a,b), an increase in the phonon mode value and temperature (respectively, parameters ε_L^* and ε_T^*) also leads to a significant increase in $\Delta\varepsilon/\varepsilon$, which is associated with an increase in the effective radius of the excited states of the impurity complex $A^+ + e$ under conditions of tunneling decay of the quasistationary A^+ state.

An increase in the degree of "viscosity" of the contact medium or of the heat-bath (parameter ε_C^*) leads to the suppression of the PDE due to an increase in the localization of the wave function of the quasi-stationary A^+ -state (see Fig. 4(c)). From Fig. 4(a,b,c) it is also seen that the PDE spectral dependence curves contain a peak that appears when the photon energy becomes comparable with the average energy of the optical transition. In this case, position of the peak depends on the parameters of dissipative tunneling, which, as shown above, have a significant influence on the effective localization radius of the quasi-stationary A^+ state.

4. Conclusion

Influence of dissipative tunneling on the PDE, associated with the excitation of impurity complexes $A^+ + e$ during intraband optical transitions of an electron in a QD, has been theoretically studied, taking into account change in the profile of the adiabatic potential of an electron.

In contrast to [15], where the case of a localized A^+ -state is considered, in this paper we consider the case of a quasi-stationary A^+ -state (the impurity level is located between the bottom of the adiabatic potential and the energy level of the ground state of the hole, see Fig. 1). In this case, the width of the potential barrier is finite, and the influence of the tunnel decay of the quasi-stationary-state should be taken into account. As a result, new possibilities appear for controlling the PDE in a quasi-zero-dimensional structure by varying such parameters of dissipative tunneling as temperature, the frequency of the phonon mode, and the constant of interaction with the contact medium, which differs significantly from the case of a localized A^+ -state [15], when the PDE is controlled only by changing the average quantum dot radius.

In the dipole approximation, an analytical formula is obtained for the spectral dependence of the change in the permittivity of a quasi-zero-dimensional structure upon photoexcitation of impurity complexes $A^+ + e$ under conditions

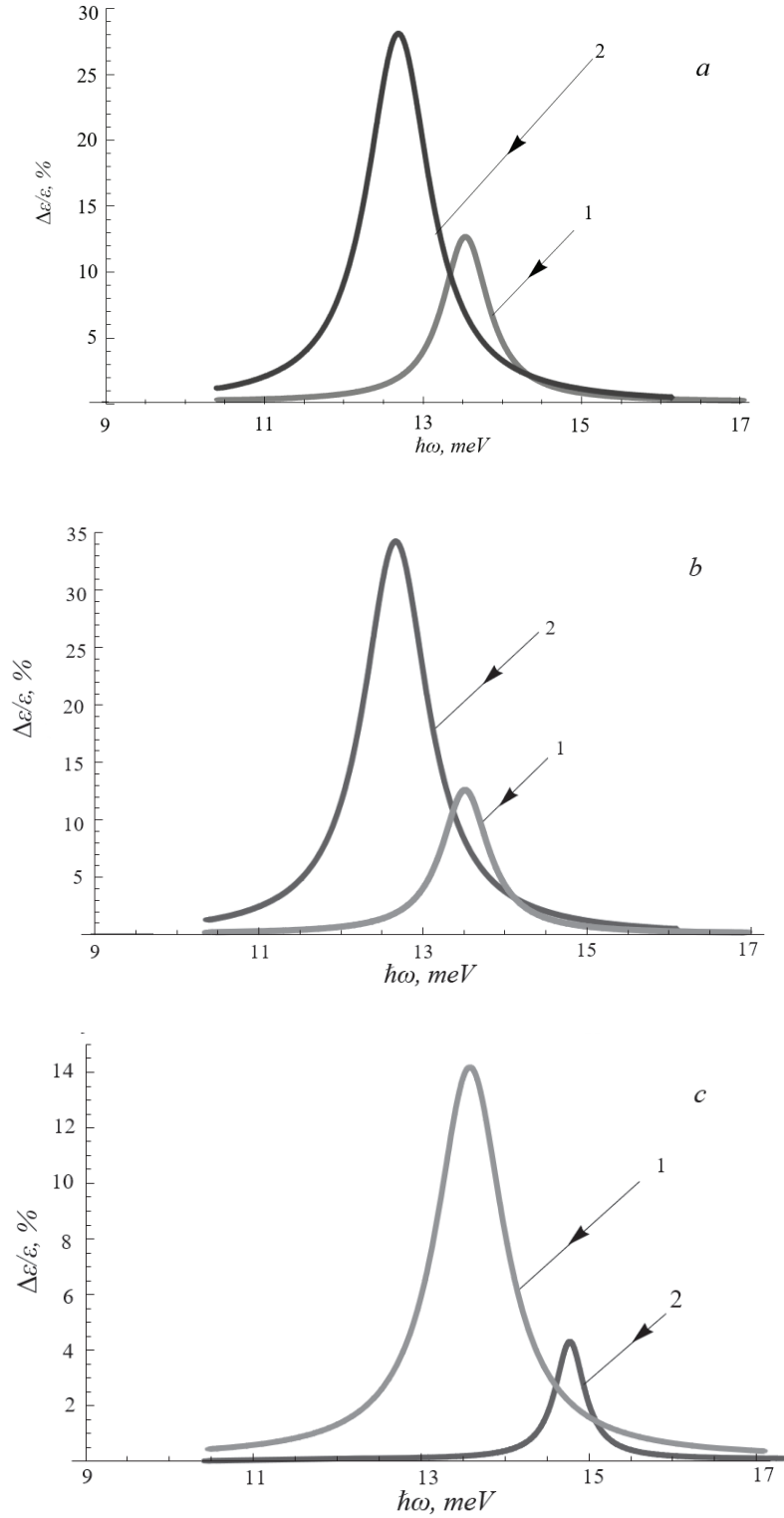


FIG. 4. Spectral dependence of the relative change in the permittivity of a quasi-zero-dimensional structure with InSb QDs at $E_i = 3.5$ meV; $\bar{R}_0 = 70$ nm, for different values of the dissipative tunneling parameters:

- a) 1 – $\varepsilon_L^* = 1$, $\varepsilon_T^* = 1$, $\varepsilon_C^* = 3$; 2 – $\varepsilon_L = 2$, $\varepsilon_T^* = 1$, $\varepsilon_C^* = 3$;
 b) 1 – $\varepsilon_L^* = 1$, $\varepsilon_T^* = 1$, $\varepsilon_C^* = 3$; 2 – $\varepsilon_L = 2$, $\varepsilon_T^* = 3$, $\varepsilon_C^* = 3$;
 c) 1 – $\varepsilon_L^* = 1$, $\varepsilon_T^* = 1$, $\varepsilon_C^* = 3$; 2 – $\varepsilon_L = 1$, $\varepsilon_T^* = 1$, $\varepsilon_C^* = 4$.

of tunnel decay of a quasi-stationary A^+ -state, taking into account the dispersion of the QD radius. It is found that varying the dissipative tunneling parameters leads to a change in the effective localization radius of the quasi-stationary A^+ -state in the QD and, as a consequence, to a noticeable change in the permittivity of the quasi-zero-dimensional structure. It is shown that the spectral dependence of the PDE is characterized by a peak that appears when the photon energy becomes comparable with the average energy of the optical transition. The position of the peak depends on the parameters of dissipative tunneling. It is revealed that an increase in the degree of “viscosity” of the contact medium is accompanied by the suppression of the PDE due to the strengthening of the localization of the wave function of the quasi-stationary A^+ -state. Thus, the controllability of the PDE in a quasi-zero-dimensional structure with impurity complexes $A^+ + e$ in the presence of dissipative tunneling has been demonstrated.

References

- [1] Kumari I. D., Jayasumana S., Attygalle D. Development of a photosensor based on photo dielectric effect of cadmium sulphide. *Moratuwa Engineering Research Conference (MERCCon)*, 2019, P. 349–353.
- [2] Nagai T., Takahashi H., et al. Optical control of dielectric permittivity in $LaAl_{0.99}Zn_{0.01}O_{3-\delta}$. *Appl. Phys. Lett.*, 2017, **110**, 172901.
- [3] Nagai T., Tanabe K., Terasaki I., Taniguchi H. Systematic tuning of the photo-dielectric effect in $Ba(Al_{1-x}Zn_x)_2O_{4-\delta}$. *Appl. Phys. Lett.*, 2018, **113**, 212902.
- [4] Avanesyan V.T., Puchkov M.Y. Photodielectric effect in thin-film structures of the poly[NiSalen] metallopolymer. *Phys. Solid State*, 2009, **51**, P. 2178–2181.
- [5] Shashin D.E., Sushentsov N.I. Development of manufacturing technology of photo-dielectric sensitive element of ultraviolet range on the basis of thin films of zinc oxide. *Herald of the Bauman Moscow State Technical University, Series Instrument Engineering*, 2019, **6**, P. 99–109.
- [6] Qasrawi A.F., Abu-Zaid S.F., Ghanameh S.A., Gasanly N.M. Dielectric and photo-dielectric properties of TlGaSeS crystals. *Bull. Mater. Sci.*, 2014, **37** (3), P. 505–509.
- [7] Nishikawa H., Sano K., Araoka F. Anisotropic fluid with phototunable dielectric permittivity. *Nature Communications*, 2022, **1142**.
- [8] Avanesyan V.T., Piskovatskova I.V. Photodielectric effect in $Bi_{12}SiO_{20}$ sillenite crystals. *Semiconductors*, 2020, **54**, P. 19–21.
- [9] Pillai P.K.C., Nath R. Photodielectric effect in photoconductors. *Phys. Stat. Sol. (a)*, 1976, **37** (2), P. 491–498.
- [10] Zhukovsky V.Ch., Dakhnovskii Yu.I., et al. Study of dissipative tunneling controllability in systems of interacting quantum molecules *Moscow University Physics Bulletin*, 2006, **62**, P. 73–77.
- [11] Krevchik V.D., Levashov A.V. Energy spectrum and optical properties of the quantum dot-impurity center complex *Semiconductors*, 2002, **36** (2), P. 208–212.
- [12] Demkov Yu.N., Ostrovskii V.N. *Zero-Range Potentials and Their Applications in Atomic Physics (Physics of Atoms and Molecules)*. Springer, New York, 1988, 296 p.
- [13] Bateman H. *Higher Transcendental Functions*. McGraw-Hill Book Company, 1953.
- [14] Lifshitz I.M., Slezov V.V. Kinetics of diffusion decomposition of supersaturated solid solutions. *Soviet Physics JETP*, 1959, **35** (2(8)), P. 331–339.
- [15] Krevchik V.D., Levashov A.V. Photodielectric effect associated with the excitation of the complex $A^+ + e$ in quasi-zero-dimensional structures. *Proceedings of the Volga region state universities. Physical and mathematical sciences*, 2007, **3**, P. 77–92.
- [16] Godik E.E., Kuznetsov A.I. Photodielectric effect associated with the excitation of impurities in semiconductors. *Proceedings of the Academy of Sciences of the USSR. (Physical series)*, 1978, **42** (6), 1206.
- [17] Krevchik V.D., Razumov A.V., Budyansky P.S. Features of the photodielectric effect associated with the excitation of impurity complexes $A^+ + e$ in quasi-zero-dimensional structures in an external magnetic field. *Proceedings of the Volga region state universities. Physical and mathematical sciences*, 2015, **4**, P. 114–144.

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