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Estimating the flux of the 14.4 keV solar axions

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In this paper we present a calculation of the expected flux of the mono-energetic 14.4 keV solar axions emitted by the M1 type nuclear transition of ^{57}Fe in the Sun. These axions can be detected, e.g., by inverse coherent Bragg-Primakoff conversion in single-crystal TeO_2 bolometers. The ingredients of this calculation are i) the axion nucleon coupling, estimated in several popular axion models and ii)the nuclear spin matrix elements involving realistic shell model calculations with both proton and neutron excitations. For the benefit of the experiments we have also calculated the branching ratio involving axion and photon emission. We find the solar axion flux on Earth to be $\Phi_a = 0.703 \times 10^9 \mathrm{cm}^{-2}\mathrm{s}^{-1} \left(\frac{10^7 \mathrm{GeV}}{f_a}\right)^2$ and the branching ratio of axion to photon for the same model to be: $\frac{\omega_a}{\omega_\gamma} = 0.229 \times 10^{-15} \approx 2 \times 10^{-16}$

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I. INTRODUCTION

The strong CP problem in Quantum Chromodynamics (QCD), predicts the electric dipole moment of the neutron to be much larger than the observed upper limit [1, 2]. Peccei and Quinn [3] devised an elegent solution by introducing a new U(1)PQ global symmetry that is spontaneously broken at an energy scale f_a. A consequence of this U(1)PQ symmetry breaking is that a new neutral spin-zero pseudoscalar particle (Nambu-Goldstone boson), the axion, is generated [4, 5]. The axion acquires a mass through non-perturbative QCD effects, which is inversely proportional to the symmetry-breaking parameter f_a. The standard axion with f_a at the electroweak (EW) scale of 250 GeV had a convenient mass, which made it easily detectable, but was quickly excluded by early searches. This led to models with much higher value of f_a, which made the axions long-lived and very weakly coupled to photons, nucleons, electrons and quarks, which makes them difficult to detect directly. This leads to the notion of "'invisible axions"'. The two most widely cited models of invisible axions are the KSVZ (Kim, Shifman, Vainshtein and Zakharov) or hadronic axion models [6, 7] and the DFSZ (Dine, Fischler, Srednicki and Zhitnitskij) or GUT axion model [8, 9]. This also led to the interesting scenario of the axion being a candidate for dark matter in the universe [10–13] and it can be searched for by real experiments [14–17]. The relevant phenomenology has recently been reviewed [18]. Since axions, or more generally, axion-like particles (ALPs), can couple with electromagnetic fields or directly with leptons or quarks, the Sun could be an excellent axion source. Solar axions could be generated by Primakoff conversion of photons, by Bremsstrahlung processes, by Compton scattering, by electron atomic recombination, by atomic de-excitation, and by nuclear M1 transitions. Axions produced in nuclear processes are mono-energetic because their energies correspond to the energy difference of a specific nuclear transition. These axions can be emitted and escape from the solar

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core due to the very weak interaction between the axion and matter. Searches for solar axions have been carried out with various experimental techniques: magnetic helioscopes [19, 20], low temperature bolometers [21] and thin foil nuclear targets [22]. CUORE(Cryogenic Underground Observatory for Rare Events) [23–26] is designed to search for neutrinoless double beta decay ($0\nu\beta\beta$) using a very low background low temperature bolometric detector. CUORE can also be used to search for dark matter weakly interacting massive particles (WIMPs) and solar axions [27].

In this paper we calculate the expected rate of 14.4 keV solar axions produced in the M1 nuclear transition of ^{57}Fe. We also calculate the axion to photon branching ratio and the axion flux on Earth. To this end in section II we discuss a set of parameters g_{aN} given in various axion models and in section III we present a realistic shell model calculation containing both proton and neutron excitations. In section IV we combine these results to get the effective matrix element to be used in section V to yield the rate for axion production. In section VI we discuss the axion to photon branching ratio and in section VII we calculate the axion flux on Earth, using appropriate density and temperature profiles for the Sun. From this flux, the axions can be detected via the coherent inverse Primakoff process in TeO_2 single crystals. A brief summary of our results is given in section VIII.

II. THE PARTICLE MODEL

The axion a is a pseudoscalar particle. Its coupling to the quarks is given by:

$$\mathcal{L} = \frac{g_q}{f_a} i\partial_\mu a \bar{\psi}(\mathbf{p}', s)\gamma^\mu\gamma_5\psi(\mathbf{p}, s), \tag{1}$$

where g_q is a coupling constant and f_a a scale parameter with the dimension of energy. The space component, $\mu \neq 0$, in the non relativistic limit is given by

$$\mathcal{L} = \langle\phi|\Omega|\phi\rangle,\ \Omega = \frac{g_{aq}}{2f_a}\boldsymbol{\sigma}.\mathbf{k}, \tag{2}$$

with $\boldsymbol{\sigma}$ the Pauli matrices and ϕ the quark wave function and $\mathbf{k}$ is the axion momentum.

We will concentrate on the last term involving the operator $g_{aq}\sigma$. The quantities g_{aq} can be evaluated at various axion models.

$$\mathrm{me}_q = \langle q| \left(g^0_{aq} + g_{as} + g^3_{aq}\tau_3\right)\sigma|q\rangle, \tag{3}$$

where we have ignored the contribution of heavier quarks and

$$g^3_{aq} = \frac{1}{2}(g_{au} - g_{ad}),\ g^0_{aq} = \frac{1}{2}(g_{au} + g_{ad}). \tag{4}$$

Some authors use the notation c_q instead of g_{aq}.
Then, following a procedure analogous for the determination of the nucleon spin from that of the quarks [28, 29], the matrix element at the nucleon level can be written as:

$$\mathrm{me}_N = \langle N| \left(g^0_{aq}(\delta_0 - \Delta s) + g_{as}\Delta s + g^3_{aq}\tau_3\delta_1\right)\sigma|N\rangle, \tag{5}$$

where

$$\begin{aligned}\delta_0 &= (\Delta u + \Delta d + \Delta s),\\ \delta_1 &= (\Delta u - \Delta d),\end{aligned} \tag{6}$$

$\delta_0 - \Delta s = \Delta u + \Delta d$. The quantities Δu, Δd and Δs will be given below.
Alternatively these quantities can be expressed in terms of the quantities D and F defined by Ellis [30]

$$\delta_1 = F + D,\ \delta_0 = 3F - D + 3\Delta s.$$

The quantity δ_1 is essentially fixed by the axial current to be approximately 1.24. No such constraint exists for the isoscalar part, so for that we have to rely on models. For the quantities D and F one can use experimental information[30]. Thus, e.g., from hyperon beta decays and flavor SU(3) symmetry one gets

$$\frac{3F - D}{\sqrt{3}} = 0.34 \pm 0.02.$$

On the other hand measurements of νp and $\nu \bar{p}$ elastic scattering of the recent MicroBooNE experiment [31] indicate that $\Delta s = \pm 0.036 \pm 0.003$.
The following model parameters are going to be considered:

i) The quantities Δq recently obtained in [32], which are consistent with lattice gauge calculations [33]

$$\Delta u = 0.897(27),\ \Delta d = -0.376(27),\ \Delta s = -0.026(4), \tag{7}$$

which yield

$$\delta_0 = 0.495,\ \delta_1 = 1.273,\ \Delta s = -0.026 \Rightarrow D = 0.812,\ F = 0.462. \tag{8}$$

These are also consistent with the recent results [34] for both connected and disconnected contributions, which yield

$$\Delta u = 0.826, \quad \Delta d = -0.386, \quad \Delta s = -0.042.$$

From these we obtain:

$$\delta_0 = 0.398, \quad \delta_1 = 1.212,\ \Delta s = -0.042 \Rightarrow D = 0.778, \quad F = 0.434. \tag{9}$$

ii) The quantities Δq prescribed by Ellis [30], namely

$$\Delta u = 0.78 \pm 0.02, \quad \Delta d = -0.48 \pm 0.02, \quad \Delta s = -0.15. \pm 0.02,$$

From these we find [35]

$$\delta_0 = 0.15, \quad \delta_1 = 1.26,\ \Delta s = -0.15 \Rightarrow D = 0.795, \quad F = 0.461. \tag{10}$$

The small isoscalar part is consistent with the so-called proton spin crisis, i.e. the observation that the spin of the nucleon comes mainly from the gluon spins not the quark spin (EMC effect)[36–38] . The isovector, as we have already mentioned, is well known from weak interaction theory.

iii) The quantities Δq of a recent analysis [28, 29], found also appropriate for pseudoscalar couplings to quarks [39], namely

$$\Delta u = 0.84, \Delta d = -0.43 \Delta s = -0.02 \Leftrightarrow D = 0.86,\ F = 0.41.$$

Thus

$$\delta_0 = 0.43,\ \delta_1 = 1.27,\ \Delta s = -0.02. \tag{11}$$

Thus the effective nucleon coupling becomes:

$$C_p = g^0_{aq}(\delta_0 - \Delta s) + g_{as}\Delta s + g^3_{aq}\delta_1,\ C_n = g^0_{aq}(\delta_0 - \Delta s) + g_{as}\Delta s - g^3_{aq}\delta_1 \tag{12}$$

Since Δs is small, it seems that the largest uncertainty comes from the determination of the parameters g^0_{aq} and g^3_{aq}. If these happen to be comparable, the smallness of δ_0 makes the isoscalar contribution negligible, i.e.

$$C^{\rm eff}_p = -C^{\rm eff}_n = g^3_{aq}\delta_1. \tag{13}$$

As a result one has in this special case essentially one unknown parameter. This, however, is not realized in the axion models considered below. As a result a relevant effective quark couplings cannot be extracted from experiment in the presence of both proton and neutron components in the nuclear wave functions.

III. SHELL-MODEL CALCULATION

In the present work we have performed a shell model calculation for ^{57}Fe in fp model space using a truncation with the KB3G [40–42] effective interaction. For protons and neutrons we put no restriction for the $f_{7/2}$, $p_{3/2}$, and $p_{1/2}$ orbitals, but we only allow a maximum of two particles in the $f_{5/2}$ orbital. The shell model code NuShellX@MSU [43] was used for diagonalization of matrices. The interaction KB3G [40] is a monopole-corrected version of the previous KB3 [41, 42] interaction in order to treat properly the $N = 28$ and $Z = 28$ shell closures and their surroundings. The single-particle energies for the KB3G effective interaction are taken to be -8.6000, -6.6000, -4.6000 and -2.1000 MeV for the $f_{7/2}$, $p_{3/2}$, $p_{1/2}$ and $f_{5/2}$ orbits, respectively.

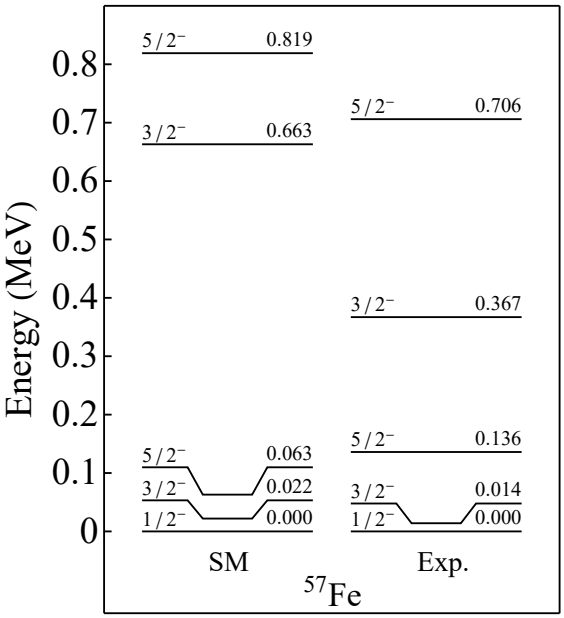

FIG. 1: Computed and experimental energy spectra for ^{57}Fe up to 1 MeV.

The computed and experimental energy spectra are given in Fig. 1. The present calculation correctly produces the $1/2^-$ as a ground state and the $3/2^-$ as the first excited state, with the

predicted excited state energy at 22 keV. The difference, however, between the experimental result of 14.4 keV and shell model result is only 7.6 keV. Also the order of the five lowest energy states is reproduced by our calculation, although there is a considerably wider gap between the $5/2_1^-$ and $3/2_2^-$ states in the shell-model spectrum than in the experimental one. In Table I, we report the comparison of experimental and shell model results of magnetic and quadrupole moments. In these calculations we have used bare g-factors and effective charges $e_p = 1.5e$ and $e_n = 0.5e$. The obtained results show good agreement with available experimental data. Furthermore, we have computed the transition strength $B(\mathrm{M1}: 3/2_1^- \to 1/2_{\mathrm{gs}}^-)$ to be 0.0356 W.u., while the experimental value is 0.0078(3) W.u. For the E2 transition we computed $B(\mathrm{E2}: 3/2_1^- \to 1/2_{\mathrm{gs}}^-) = 0.487$ W.u., while the experimental value is 0.37(7) W.u.. We find transition probabilities of $T(\mathrm{M1}: 3/2_1^- \to 1/2_{\mathrm{gs}}^-) = 3.4\,\mu\mathrm{s}^{-1}$, and $T(\mathrm{E2}: 3/2_1^- \to 1/2_{\mathrm{gs}}^-) = 4.8\,\mathrm{s}^{-1}$, for the M1 and E2 transitions respectively. We note that the lowest transition is dominantly M1. Based on the above comparisons with experimental data, we are confident that the magnetic properties and transitions between the lowest lying states are described within reasonable accuracy by the present calculations.

TABLE I: Comparison of experimental [44] and calculated shell model magnetic dipole moments in units of (μ_N), using $g_s^{\mathrm{eff}} = g_s^{\mathrm{free}}$ (columns 3 and 4), and electric quadrupole moments in units of (eb), using $e_p = 1.5e$ and $e_n = 0.5e$ (columns 5 and 6).

State	E_x (keV)	μ_{expt}	μ_{SM}	Q_{expt}	Q_{SM}
$1/2^-$	0	+0.09044(7)	+0.130	-	-
$3/2^-$	22	−0.1549(2)	−0.388	+0.082(8)	+0.17

IV. THE NUCLEAR MATRIX ELEMENT (ME)

With the above information one can proceed to calculate the rate for axion production in the spin induced de-excitation of a nuclear level J_i to a final level J_f. Then, the nuclear matrix element, obtained by summing over the final and averaging over all initial m-substates, becomes

$$\begin{aligned}\mathcal{M}^2 &= \frac{1}{2J_i+1}\sum_{M_f,M_i}\left|\langle J_f M_f|\sum_n \left(g_{aq}^0(\delta_0-\Delta s)+g_{as}\Delta s+g_{aq}^3\tau_3(n)\delta_1\right)\boldsymbol{\sigma}(n).\mathbf{k}|J_i M_i\rangle\right|^2 \\ &= \frac{1}{2J_i+1}\frac{1}{3}|\mathrm{ME}|^2, \end{aligned} \tag{14}$$

where

$$\mathrm{ME} = \langle J_f||\sum_n \left(g_{aq}^0(\delta_0-\Delta s)+g_{as}\Delta s+g_{aq}^3\tau_3(n)\delta_1\right)\boldsymbol{\sigma}(n)||J_i\rangle. \tag{15}$$

The last expression is the usual reduced matrix of the one body operator in the isospin basis. The sum over n involves all active nucleons.
Alternatively, the reduced matrix element can be computed in the proton-neutron representation

$$\Omega_p = \langle J_f||\sum_n\left(\frac{1}{2}(1+\tau_3(n))\right)\boldsymbol{\sigma}(n)||J_i\rangle,\ \Omega_n = \langle J_f||\sum_n\left(\frac{1}{2}(1-\tau_3(n))\right)\boldsymbol{\sigma}(n)||J_i\rangle. \tag{16}$$

Thus the reduced ME can be written as:

$$\mathrm{ME} = C_p\Omega_p + C_n\Omega_n. \tag{17}$$

Our shell model calculation for the ^{57}Fe transition predicts:

$$\Omega_p = 0.1054, \quad \Omega_n = 0.7932, \tag{18}$$

As expected the neutron component dominates, but the proton may make an important contribution for some coupling choices.
Our shell model calculation also predicts the matrix element of the isovector orbital magnetic moment operator, namely:

$$\langle J_f || \sum_k \ell(k)\tau_3(k) || J_i \rangle = 0.8291, \tag{19}$$

which enters the de-excitation of the 14.4 keV state via photon emission, which is needed to estimate the axion to photon branching ratio discussed below.
To proceed further in the evaluation of the MEs, we need some elementary particle input, e.g. those recently obtained for the hadronic model of the recent calculation[32]:

$$g_{au} = -0.47, g_{ad} = 0.02 \Rightarrow g^0_{aq} = -0.2250, g^3_{aq} = -0.2465.$$

Combining this with the δ_0, Δ_s and δ_1 given by Eq.(8) we get

$$C_p = (g^0_{aq}(\delta_0 - \Delta_s) + g^3_{aq}\delta_1) = -0.429,\ C_n = (g^0_{aq}\delta_0 - \Delta_s) - g^3_{aq}\delta_1) = 0.195 \text{ (MODEL A)}.$$

On the other hand using Eqs (10) and (11) we get

$$C_p = -0.378, C_n = 0.241 \text{ (MODEL B)};\ C_p = -0.404, C_n = 0.219 \text{ (MODEL C)}.$$

Finally if one considers renormalization effects in the KSVZ model these authors conclude [32]

$$C_p^{\text{KSVZ}} = -0.47,\ C_n^{\text{KSVZ}} = -0.02 \text{ (MODEL D)}.$$

Models A, B, C lead to large values for the neutron coupling, but in the presence of renormalization the neutron contribution is greatly suppressed and even its sign changes. So the first three models are included for orientation purposes, but they should not be taken seriously.

The coupling of axion to matter has been investigated [32, 45], in particular in the context of the DFSZ axion models [9, 11]. At the quark level they find :

$$g_{au} = g_{ac} = g_{at} = \frac{1}{3}\sin^2\beta,\ g_{ad} = g_{as} = g_{ab} = \frac{1}{3} - \frac{1}{3}\sin^2\beta$$

Thus restricting ourselves to quarks u, d and s, which are relevant for nucleons, we get:

$$g^0_{aq} = \frac{1}{6},\ g_{as} = \frac{1}{3} - \frac{1}{3}\sin^2\beta,\ g^1_{aq} = -\frac{1}{6} + \frac{1}{3}\sin^2(\beta),$$

where $\tan\beta$ is the ratio of the vacuum expectation values of the two doublets of the model, which, like the well known case in supersymmetry, is not constrained by the SM physics.
In this case we will explore the dependence of the results on the quark model and $\tan\beta$. In a fashion analogous to the constrained parameter space of supersymmetry, we will consider values of $\tan\beta$, e.g. $\tan\beta = 10$ as well as a small value, e.g. $\tan\beta = 1$. The obtained results are presented in table II.

From table II we make a reasonable selection for our calculations

$$\begin{aligned} C_p &= 0.0663, C_n = 0.0663 \text{ for } \tan\beta = 1 \text{ MODEL E}, \\ C_p &= 0.2712, C_n = -0.1248 \text{ for } \tan\beta = 10 \text{ MODEL F}. \end{aligned} \tag{20}$$

TABLE II: The results obtained in the DSFZ models for various isoscalar and isovector contributions and two choices of $\tan\beta$

	$\tan\beta = 1$				$\tan\beta = 10$			
	$Eq.(8)$	$Eq.(9)$	$Eq.(10)$	$Eq.(11)$	$Eq.(8)$	$Eq.(9)$	$Eq.(10)$	$Eq.(11)$
C_p	0.0825	0.0663	0.025	0.065	0.2947	0.2712	0.2553	0.2757
C_n	0.0825	0.0663	0.025	0.065	-0.1212	-0.1248	-0.1563	-0.1392
ME	0.074	0.0596	0.0225	0.0584	-0.0651	-0.0704	-0.0971	-0.0814
$\frac{\mathcal{M}^2}{10^{-3}}$	0.458	0.2961	0.0421	0.2843	0.353	0.4131	0.7856	0.5516

Combining these with the reduced nuclear matrix elements for protons and neutrons we obtain:

$$\text{ME} = 0.0596 \text{ for } \tan\beta = 1 \text{ MODEL E, ME} = -0.0704 \text{ for } \tan\beta = 10 \text{ MODEL F}$$

With supersymmetry as a guide we expect the larger value of β as the most likely.

Before ending this exposition we should mention that there exist a fairly old model which depends on essentially only one family parameter developed long time ago by Kaplan [46], based on the DFSZ [9, 11] axion. In this case there exists one value c_q indicated by $(1/2)X_u$ for the charge 2/3 quarks and one indicated by $(1/2)X_d$ for the charge -1/3 quarks, with the condition $X_u + X_d = 1$, $X_u > 0$ and $X_d > 0$. At the nucleon level the isovector contribution depends on c_q, but the isoscalar contribution becomes independent of c_q, in other words:

$$g^0_{aq} = \frac{1}{4},\ g_{as} = \frac{1}{2}x_d,\ g^3_{aq} = \frac{1}{4}(2X_u - 1), g^3_{aq} \leq g^0_{aq}.$$

We will consider the isoscalar and isovector component found in cases i) ii) and iii) above (see section II) and the following 3 quark couplings X_u and X_d:

a) $X_u = X_d = 1/2$ Then we obtain:

	$Eq.(8)$	$Eq.(9)$	$Eq.(10)$	$Eq.(11)$
Cp	0.2540	0.2055	0.1125	0.2000
C_n	0.2540	0.2095	0.1125	0.2000

b) $X_u = 3/4, X_d = 1/4$ Then

	$Eq.(8)$	$Eq.(9)$	$Eq.(10)$	$Eq.(11)$
C_p	0.4165	0.3663	0.2888	0.3613
C_n	0.0981	0.0633	0.0213	0.0438

c) $X_u = 1/4, X_d = 3/4$ Then

	$Eq.(8)$	$Eq.(9)$	$Eq.(10)$	$Eq.(11)$
C_p	0.09163	0.0523	-0.0638	0.0388
C_n	0.4099	0.3557	0.2513	0.3565

.

TABLE III: The various couplings and matrix elements entering axion production

$MODEL$	C_p	C_n	ME	$\mathcal{M}^2$	$\lvert\mathrm{rME}\rvert^2$
A	-0.4291	0.1947	0.1092	0.000993	0.1008
B	-0.3762	0.2412	0.1517	0.001917	0.1945
C	-0.4034	0.2189	0.1311	0.001433	0.1454
D	-0.4700	-0.0200	-0.0654	0.000356	0.0362
E	0.0663	0.0663	0.0596	0.000296	0.0300
F	0.2712	-0.1248	-0.0704	0.000413	0.0419
G	0.1125	0.1125	0.1011	0.000852	0.0864
H	0.3663	0.0633	0.0888	0.000657	0.0667
I	0.0388	0.3563	0.2867	0.006848	0.6948

From the above set we select three typical cases:

$$\begin{aligned} C_p &= C_n = 0.1125 \text{ (MODEL G) } ; C_p = 0.3663, C_n = 0.0633 \text{ (MODEL H) } ; \\ C_p &= 0.0388, C_n = 0.3565 \text{ (MODEL I) } . \end{aligned} \tag{21}$$

Since the suppression of the neutron coupling looms on the horizon, we had to consider an elaborate shell model calculation for ^{57}Fe in which the proton components of the wave function may be involved and contribute significantly to the width for axion production. The obtained results are given in table III. The quantity $|\mathrm{rME}|^2$ enters in the axion to photon production branching ratio to be discussed below.

V. THE RATE OF AXION PRODUCTION

Once the Matrix element $\mathcal{M}^2$, see Eq. (14), is known the axion production width is given by

$$\Gamma(J_i \to J_f a) = \frac{1}{2\pi} \frac{\left(\sqrt{\Delta^2 - m_a^2}\right)^3}{4 f_a^2} \mathcal{M}^2, \tag{22}$$

where as we have seen in the previous section the expression for $\mathcal{M}^2$ involves the needed particle and nuclear physics inputs.
The mass of the axion is expected to be much less than the transition energy Δ [32], e.g. $m_a = 5.7\mu\mathrm{eV} \times \frac{1}{(f_a/10^{12}\mathrm{GeV})}$. Even for the unrealistically small scale of $f_a = 10^6$ GeV we find $m_a = 5.7$eV. Thus

$$\begin{aligned} \Gamma(J_i \to J_f a) &= \Lambda \left(\frac{\Delta}{14.4 \text{ keV}}\right)^3 , \\ \Lambda &= 1.2 \times 10^{-21}\mathrm{eV} \left(\frac{10^7\mathrm{GeV}}{f_a}\right)^2 \mathcal{M}^2 = 1.2 \times 10^{-7}\mathrm{eV} \left(\frac{f_a}{1\mathrm{GeV}}\right)^{-2} \mathcal{M}^2 \end{aligned} \tag{23}$$

or

$$\Lambda = 1.83 \times 10^{-6}\mathrm{s}^{-1} \left(\frac{10^7\mathrm{GeV}}{f_a}\right)^2 \mathcal{M}^2 = 1.83 \times 10^{8}\mathrm{s}^{-1} \left(\frac{f_a}{1\mathrm{GeV}}\right)^{-2} \mathcal{M}^2. \tag{24}$$

Using the $\mathcal{M}^2$ obtained above and the excitation energy of $\Delta = 14.4$ keV we find the width and transition probability as follows:

MODEL	A	B	C	D	E	F	G	H	I
$\frac{\Gamma}{(10^{-10})}\mathrm{eV}\left(\frac{f_a}{1\mathrm{GeV}}\right)^{-2}$	1.19	2.30	1.72	0.4282	0.355	0.496	1.022	0.788	8.22
$\frac{\omega_a}{(10^5)}\mathrm{s}^{-1}\left(\frac{f_a}{1\mathrm{GeV}}\right)^{-2}$	1.82	3.51	2.62	0.652	0.542	0.756	1.56	1.20	12.5

VI. THE AXION TO PHOTON PRODUCTION BRANCHING RATIO

Experimentally it is of interest to estimate the axion to photon branching ratio. Following Haxton and Lee [47] and [48] we write

$$\begin{aligned} \frac{\omega_a}{\omega_\gamma} &= \frac{1}{2\pi\alpha}\left(\frac{m_N}{2f_a}\right)^2 \frac{|\mathrm{rME}|^2}{\left((\mu_0 - \frac{1}{2}\beta + \mu_1 - \eta\right)^2}, \\ \mathrm{rME} &= \beta(C_p + C_n) + C_p - C_n, \end{aligned} \tag{25}$$

where m_N is the nucleon mass, $\mu_0 = 0.88$, $\mu_1 = 4.77$,

$$\beta = \frac{(\Omega_p + \Omega_n)}{(\Omega_p - \Omega_n)}, \quad \eta = -\langle J_f || \sum_k \ell(k)\tau_3(k) || J_i\rangle / \langle J_f || \sum_k \sigma(k)\tau_3(k) || J_i\rangle.$$

The quadrupole transition contribution δ is negligible in this case, but it may have to be included in other experimentally interesting nuclei, like ^{65}Cu [49]. Our nuclear calculation yields $\beta = -1.3065$ and $\eta = 1.2054$. i.e. $\left(\mu_0 - \frac{1}{2}\right)\beta + \mu_1 - \eta = 3.068$. This must be compared with the value 3.458 of Haxton and Lee [47]. We thus get

$$\frac{\omega_a}{\omega_\gamma} = 0.58\left(\frac{M_N}{f_a}\right)^2 (rME)^2 . \tag{26}$$

This equation can be used to extract a limit on the parameter $(rME)^2$ from the branching ratio data. The values of $|\mathrm{rME}^2|$ for the models considered in this work are given in table III. Using these parameters and for the value $f_a = 10^7$GeV, see section VII, we obtain:

	MODEL								
	A	B	C	D	E	F	G	H	I
$\left(\frac{\omega_a}{\omega_\gamma}\right) / \left(10^{-15}\right)$	0.548	1.060	0.794	0.197	0.164	0.229	0.472	0.363	3.786

(27)

VII. AXION FLUX IN THE EARTH

One must first estimate the number of ^{57}Fe in the excited state with $J_1 = 3/2$. This is given by the Boltzmann factor

$$\frac{N^*}{N} = \frac{2J_1 + 1}{2J_0 + 1} e^{-\frac{\Delta}{kT}},$$

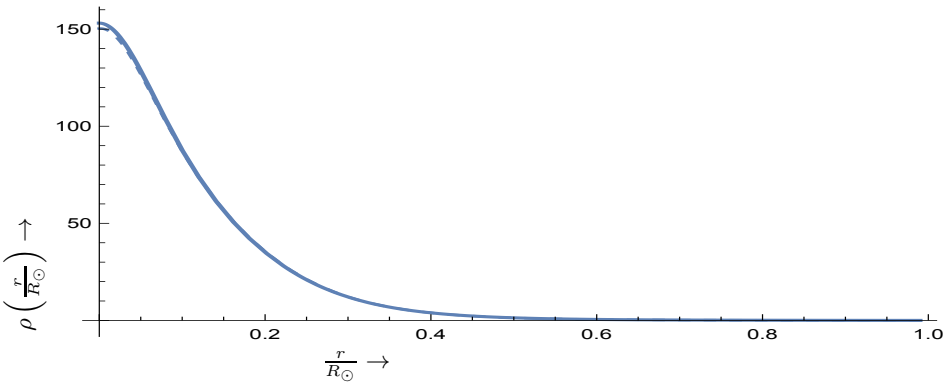

FIG. 2: The solar density profiles in two models described in Ref. [52].

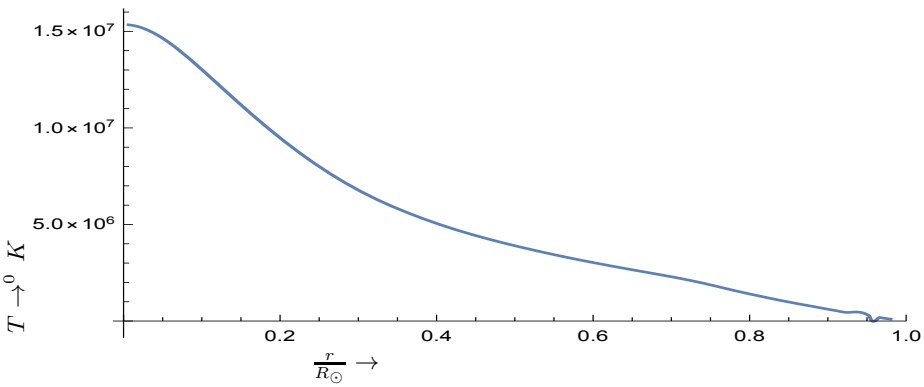

FIG. 3: The solar temperature profile obtained from the data of Ref. [53].

where $J_0 = 1/2$ is the ground state angular momentum, N is the number of ^{57}Fe present and T the temperature of the Sun. This fraction depends on the position in the Sun. Thus we can write [50, 51]

$$\frac{N^*}{N} = \frac{2J_1+1}{2J_0+1} fr = 2fr,\ fr = \frac{\int_0^{R_\odot} e^{-\frac{\Delta}{kT(r)}} 4\pi r^2 \rho(r) dr}{\int_0^{R_\odot} 4\pi r^2 \rho(r) dr}, \tag{28}$$

where $\rho(r)$ and $T(r)$ are the density and temperature profiles of the Sun respectively. We found it convenient to express the above expression in units of $x = \frac{r}{R_\odot}$ and write:

$$fr = \frac{\int_0^1 e^{-\frac{\Delta}{kT(x)}} 4\pi x^2 \rho(x) dx}{\int_0^1 4\pi x^2 \rho(x) dx}. \tag{29}$$

To proceed further we need the temperature and the density profile of the Sun. The density profile [52] is presented in Fig. 2. A solar temperature profile can be found in Ref. [53] and is exhibited in Fig. 3. For the solar density profiles in two models described in Ref. [52] we obtain almost the same result $fr = 6.75 \times 10^{-7}$ and $fr = 6.71 \times 10^{-7}$. On the other hand using the density and temperature files found in ref. [54] we find $fr = 7.75 \times 10^{-7}$. Adopting the first

value and taking as the number of ^{57}Fe nuclei in the Sun per gram to be $3 \times 10^{17}\mathrm{g}^{-1}$ [50, 51], we find the total number N^* of ^{57}Fe nuclei decaying in the Sun to be

$$N^* = 2 \times 6.75 \times 10^{-7} \times 3 \times 10^{17} \times 2.0 \times 10^{33} = 0.81 \times 10^{45}.$$

Adopting the value of 1.0×10^{45} the rate of the emitted axions is:

$$N_a = N^* \Gamma(J_i \to J_f a)$$

and the flux of axions at the Earth is:

$$\Phi_a = \frac{N*}{4\pi d_{SE}^2} \mathcal{M}^2 \times 1.83 \times 10^8 \mathrm{s}^{-1} \left(\frac{f_a}{1\mathrm{GeV}}\right)^{-2} = 6.86 \times 10^{25} \mathrm{cm}^{-2}\mathrm{s}^{-1} \mathcal{M}^2 \left(\frac{f_a}{1\mathrm{GeV}}\right)^{-2}, \quad (30)$$

where d_{SE} is the average distance between the Earth and the Sun. Thus we find:

	MODEL										
	A	B	C	D	E	F	G	H	I	units	f_a factor
Φ_a	0.681	1.32	0.983	0.245	0.203	0.283	0.584	0.451	4.70	$\times 10^{23}\mathrm{cm}^{-2}\mathrm{s}^{-1}$	$\left(\frac{f_a}{1\mathrm{GeV}}\right)^{-2}$

(31)

in the above order of the ME.
The numerical value depends, of course, on the value of f_a. It may be useful to relate this to the usual axion-photon coupling $g_{a\gamma\gamma}$ given by:

$$g_{a\gamma\gamma} = \frac{\alpha C_\gamma}{2\pi f_a}, \quad (32)$$

where C_γ is a model dependent parameter, which in the KSVZ model [6, 7] takes the value 1.95 ± 0.08. Taking now the rather optimistic value $g_{a\gamma\gamma} = 0.66 \times 10^{-10}\mathrm{GeV}^{-1}$, extracted from axion searches, e.g. the CAST [55] limit $g_{a\gamma\gamma} \leq 1.16 \times 10^{-10}\mathrm{GeV}^{-1}$, and the more recent [56] CAST limit (0.66×10^{10} GeV^1 at 95% confidence level) as well as astrophysical limits $g_{a\gamma\gamma} \leq 10^{-10}\mathrm{GeV}^{-1}$ [57], one obtain $f_a \geq 3.4 \times 10^7$GeV. We will adopt the value of 10^7 GeV in our work. Thus the obtained flux is

	MODEL									
	A	B	C	D	E	F	G	H	I	units
Φ_a	1.691	3.262	2.437	0.607	0.504	0.703	1.449	1.119	11.669	$\times 10^9\mathrm{cm}^{-2}\mathrm{s}^{-1}$

(33)

VIII. DISCUSSION

In this paper we have performed a calculation of the expected flux of the mono-energetic 14.4 keV solar axions emitted by the M1 type nuclear transition of ^{57}Fe in the Sun. To this end we have included the following ingredients:

- We have employed the spin induced axion quark couplings obtained in various axion models.
- We appropriately transformed these isovector and isoscalar couplings at the nucleon model.
- We performed a realistic shell model calculation of the 14.4 keV state of ^{57}Fe containing both protons and neutrons
- With the previous input we were able to obtain the nuclear matrix elements for axion production as well as the branching ratio of axion to photon production.

- We also obtained the number of excited nuclei found in the Sun employing a Boltzmann factor obtained with appropriate density and temperature profiles in the Sun.
- With the above ingredients we have obtained the flux of solar axions in the surface of the Earth.

We give a formula which allows the experimentalists to extract the effective matrix element $\mathcal{M}^2$ from the data, Eq. (30) and then compare it to the predictions of the various axion models, Eqs. (31) and (33). We also give analogous formulas for the branching ratio of axions to photons involving the matrix element $|\mathrm{rME}|^2$. A value can be extrcted from the data via Eq. (26) and an estimate for the expected branching ratio is given by Eq. (27). Since, however, our nuclear model involves both protons and neutrons, in extracting the above values from the data, one cannot disentangle the couplings from the nuclear matrix elements.

As a conclusion we can say that, within the popular DFSZ axion model for large $\tan\beta$ considered here and a realistic calculation of the nuclear matrix elements employed, involving both proton and neutron configurations, a reasonably high flux of axions is expected on earth, coming from the decay 14.4 keV state of ^{57}Fe in the Sun, namely:

$$\Phi_a = 0.703 \times 10^9 \mathrm{cm}^{-2}\mathrm{s}^{-1} \left(\frac{10^7 \mathrm{GeV}}{f_a}\right)^2 . \tag{34}$$

Furthermore the branching ratio of axion to photon for the same model is is predicted to be:

$$\frac{\omega_a}{\omega_\gamma} = 0.229 \times 10^{-15} \approx 2 \times 10^{-16} \tag{35}$$

Finally it is worth mentioning that the width of the 14.4 keV state of ^{57}Fe involved here is of the order of few eV. while that of the axion distribution due to the Primakoff effect is of some keV.

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