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# DECAY AND IN-BEAM STUDIES OF NEUTRON-DEFICIENT Po AND Ra ISOTOPES AT JYFL\*

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An extensive program to study the production, decay properties, and nuclear structure of very neutron-deficient polonium and radium nuclei is underway at the Department of Physics, University of Jyväskylä, Finland (JYFL). The main tools used in these studies are the gas-filled recoil separator RITU and various germanium gamma-ray arrays. In the course of these studies, among others the following new isotopes have been produced:  $^{204}\mathrm{Ra}$ ,  $^{203}\mathrm{Ra}$ , and  $^{202}\mathrm{Ra}$ . Isomeric alpha decaying states have been discovered in  $^{203}\mathrm{Ra}$  and  $^{191}\mathrm{Po}$ . Fine structure in the decay of  $^{192}\mathrm{Po}$  to the oblate and prolate band heads in  $^{188}\mathrm{Pb}$  has been observed. In-beam gamma-ray spectra have been, for the first time, measured for  $^{192}\mathrm{Po}$ ,  $^{206}\mathrm{Ra}$ ,  $^{208}\mathrm{Ra}$ , and  $^{210}\mathrm{Ra}$ . Development of collectivity in nuclei in the Po-Ra region and the systematics of reduced alpha widths will be discussed.

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#### 1. Introduction

The availability of large escape suppressed Ge detector arrays [1] has during the past few years made it possible to study the development of collective phenomena in nuclei far from stability. The established method of producing these nuclei is heavy ion induced fusion. In the region of heavy elements where fission of the compound system strongly dominates over particle evaporation, efficient filtering is needed to extract in-beam gamma-rays originating from weak evaporation channels. This can be achieved by combining the Ge array with a recoil separator. Only those gamma-rays will be accepted which are in coincidence with separated evaporation residues. Due to the small production cross sections of these neutron-deficient nuclei, a high-transmission separator is desirable. Such devices do not always provide a unique identification of the separated nuclei. This drawback can be overcome by using the Recoil Decay Tagging (RDT) method [2]. The separated nucleus is identified through its characteristic decay in a position sensitive focal plane silicon detector. Most often, alpha decay is used for identification although proton radioactivity can also be used. An example of the power of the RDT method will be shown in Sect. 6 in connection with the discussion of the <sup>206</sup>Ra in-beam study.

Due to the selectivity of the process through which nuclear levels are populated by the emission of gamma-rays following the production of evaporation residues, important non-yrast low lying levels are not always accessible to study using in-beam methods. A complementary method is then provided by radioactive alpha or beta decay. An example is the observation of deformed 2p-2h intruder  $0^+$  states in the closed shell nuclei  $^{192-198}$ Pb from beta decay of Bi isotopes [3]. The determination of hindrance factors in alpha decay may also provide information on nuclear structure [4].

The region of neutron-deficient nuclei around Pb with the closed Z=82 proton shell is particularly interesting for the study of shape coexistence phenomena and the related intruder states. The closed proton shell favours spherical shape but around midshell (N=104) the interaction of 2p-2h and 4p-4h and higher excitations across the shell with the large number of active valence neutrons leads to a delicate competition between spherical and deformed states at low spin and excitation energy [5,6]. Another way of looking at the deformation is based on deformation driving high-j neutron orbitals [7]. Here we will base the discussion on the np-nh excitation concept.

In this paper, we will present results from studies of neutron-deficient Po and Ra nuclei performed at the Department of Physics, University of Jyväskylä, or JYFL. In all the experiments, the gas-filled heavy ion recoil separator RITU [8] was used to separate and identify the nuclei. Heavy ions were accelerated using the  $K=130~{\rm MeV}$  JYFL cyclotron. Both in-beam gamma-ray studies and alpha decay studies will be discussed.

## 2. Observation of excited states in <sup>192</sup>Po

The Po isotopes present an illuminating example of the development of collective phenomena far from stability [3,7,9]. A wide variety of states can be studied through in-beam methods as well as through alpha and beta decay. On the other hand, alpha decay of Po isotopes provides an efficient complementary method to study the structure of Pb isotopes. This will be discussed in Sect. 3.

Prior to our work on  $^{192}$ Po, excited states had been known down to  $^{194}$ Po which was studied using the Fragment Mass Analyzer (FMA) at Argonne [10]. In that work, a continuation of the steep decline of level energies of positive parity yrast levels up to  $10^+$  as compared with heavier Po isotopes was observed. It was of interest to extend these studies and to approach the neutron midshell at N=104 ( $^{188}$ Po).

This experiment was performed using 178 MeV  $^{36}$ Ar ions to bombard a 70% enriched 500  $\mu g/cm^2$  thick  $^{160}$ Dy target. The results have been published in Ref. [11] and repeated in Ref. [12]. Prompt gamma-rays from the target were detected by the DORIS array consisting of nine TESSA type [13] escape suppressed Ge detectors and having an efficiency of 0.6% at 1.3 MeV.

The ground state band from  $^{192}$ Po was observed up to the 8<sup>+</sup> level. The tentative spin and parity assignments are based on transition intensities, level systematics and on  $\gamma$ - $\gamma$  coincidence measurements. Unambiguous assignment to  $^{192}$ Po was based on the use of the RDT method. The low-energy systematics for even-even Po isotopes is shown in Fig. 1. The data are from Refs. [7,9–11,14].

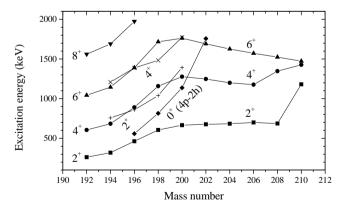


Fig. 1. Systematics of low-lying level energies of even-even Po isotopes.

# 3. Fine structure in the $\alpha$ decay of <sup>192</sup>Po

Prolate structures in the neutron-deficient  $^{184}$ Pb,  $^{186}$ Pb and  $^{188}$ Pb nuclei were discovered recently in in-beam experiments [15–17]. It was concluded [16] that the structure of the observed bands is different from the 2p-2h proton intruder states in  $^{190-208}$ Pb mentioned above. A comparison with the well established prolate bands observed in the  $^{184,186}$ Hg isotones indicated that the observed structures are indeed associated with the prolate minimum predicted by the potential energy surface calculations (see for example [6]). Due to decay out of the bands, the  $0^+$  band heads were not observed in these experiments.

In another recent work [18], on fine structure in the  $\alpha$  decay of <sup>192</sup>Po, an excited  $0^+$  state was observed in <sup>188</sup>Pb. It was concluded that this was the oblately deformed 2p-2h proton intruder state observed in heavier Pb isotopes.

In the work described here [19], the standard position-sensitive focal plane detector was combined with an auxiliary detector system for observing alpha particles and conversion electrons escaping from the stop detector at backward angles. The reaction used to study fine structure in the alpha decay of  $^{192}$ Po was  $^{36}$ Ar +  $^{160}$ Dy. The bombarding energy was varied between 172 and 184 MeV and the target thickness was 500  $\mu$ g/cm<sup>2</sup>.

When alpha decay takes place from the ground state of  $^{192}$ Po to low-lying excited  $0^+$  states in  $^{188}$ Pb, the excitation energy in the daughter nucleus is expected to be removed primarily through conversion electrons from E0 transitions. Thus, by demanding a coincidence between alpha decays in the stop detector and conversion electrons in the auxiliary detector, it was possible to observe alpha decay fine structure to two low-lying  $0^+$  states in  $^{188}$ Pb. One of these, at an excitation energy of  $(568\pm4)$  keV, was identified as the oblate band head thus confirming the result of Ref. [18]. The other state has an excitation energy of  $(767\pm12)$  keV and was identified as the prolate  $0^+$  band head.

It is of interest to deduce the mixing amplitude of the oblate 4p-2h state in the ground state of  $^{192}$ Po. This can be achieved through experimental alpha decay hindrance factors and mixing amplitudes for the excited  $0^+$  states in  $^{188}$ Pb [4]. The hindrance factors as determined relative to alpha decay to the ground state of  $^{188}$ Pb were deduced to be  $0.67\pm0.10$  (decay to the oblate  $0^+$  state) and  $0.22\pm0.08$  (decay to the prolate  $0^+$  state). The mixing amplitudes were determined following the procedure of Dracoulis [20] by fitting to the experimental levels from this work and from Ref. [16]. Then, by representing the ground state of  $^{192}$ Po as a combination of 2p-0h and 4p-2h states, it was possible to determine the admixture of  $\sim 63\%$  for the intruder 4p-2h configuration in the ground state of  $^{192}$ Po. The unmixed

prolate  $0^+$  level energy of  $\sim 730$  keV in <sup>188</sup>Pb agrees well with the value of  $\sim 710$  keV estimated from the energies of higher members of the band [16].

## 4. Alpha decay studies of neutron-deficient Po isotopes

The main interest behind  $\alpha$  decay studies of neutron-deficient Pb and Po nuclei lies in the determination of reduced alpha decay widths [21] and in the nuclear structure information to be deduced from these. An early measurement using a gas-filled separator at Lawrence Berkeley Laboratory of the half-life of <sup>192</sup>Po [22] gave the surprising result that the reduced width of <sup>192</sup>Po is lower than that of its heavier Po neighbours. This behaviour differs from that of heavier elements for which the reduced alpha width increases as the neutron number decreases. This result was confirmed by the findings in the fine structure studies [18,19]. This work was extended to the isotope <sup>190</sup>Po for which a half-life of  $(1.9^{+0.6}_{-0.4})$  ms was measured at JYFL [23]. It was found that the saturation trend i. e. the near-constancy of reduced widths persisted for <sup>190</sup>Po. In later work using RITU at JYFL, the half-life value for this nucleus was improved to  $(2.53\pm0.33)$  ms [24].

One should note that the saturation of the Po  $\alpha$  decay widths can be removed by summing up the widths of the decay to the even-even Pb ground state and to the oblate excited 2p-2h  $0^+$  state [23]. In Fig. 2 we show the systematics of the reduced  $\alpha$  decay widths for even-even nuclei from Po to Th. The data are from Refs. [19,24–29]. Concerning new data on the alpha decay of the odd isotope <sup>191</sup>Po, we refer to Ref. [30].

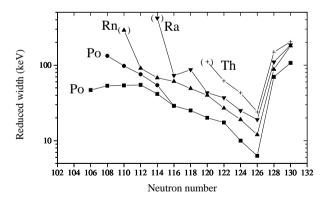


Fig. 2. Reduced alpha decay widths of even-even Po, Rn, Ra, and Th isotopes. The round symbols for Po represent sums of widths for decays to the Pb ground state and to the oblate 0<sup>+</sup> excited state. The values for <sup>196</sup>Rn, <sup>202</sup>Ra, and <sup>210</sup>Th (in brackets) are based on the observation of only three, one and two decay chains, respectively and have rather large asymmetric error bars.

## 5. Alpha decay studies of neutron-deficient Ra isotopes

During the last few years, the three new radium isotopes <sup>202,203,204</sup>Ra were discovered using RITU [25]. The results concerning <sup>202</sup>Ra are tentative since they are based on the observation of only one decay chain corresponding to a cross section of 2 nb in the reaction <sup>36</sup>Ar + <sup>170</sup>Yb. These data give information on the nuclear mass surface around the proton drip line as well as on reduced widths. They are also of interest due to the predicted appearance of nuclear ground state deformation in the vicinity of <sup>200</sup>Rn [31]. The predicted ground state deformation for <sup>200</sup>Rn, about 0.2, would correspond to an excitation energy of  $\sim 140$  keV for the first  $2^+$  state. Such a low energy was excluded in our work [25] since no decays to the  $2^+$  state were observed. This result has later been confirmed by in-beam RDT data [32] where the  $2^+$  state was found to be at 433 keV excitation energy. In <sup>203</sup>Ra, two alpha decaying isomeric states were found. They were observed to decay to corresponding states with  $I^{\pi} = 3/2^{-}$  and  $13/2^{+}$ , respectively, in <sup>201</sup>Rn. The hindrance factors determined for the two transitions were  $1.6^{+1.2}_{-0.4}~(13/2^+ \rightarrow 13/2^+)$  and  $\sim 0.04~(3/2^- \rightarrow 3/2^-)$ . The first of these is based on the observation of seven decay chains and is the basis for the assumption that the alpha decay is unhindered and connects states with identical spin and parity. The latter value is quite uncertain since it is based on the observation of only one decay chain. It would be desirable to collect more data on the decay of the corresponding  $3/2^-$  isomer. The reduced  $\alpha$ decay widths of even-even neutron-deficient Ra isotopes are shown in Fig. 2.

## 6. In-beam studies of $^{206,208,210}$ Ra

Radium nuclei have six protons outside the closed Z=82 shell. When also the number of neutron holes increases as one moves away from the closed N=126 shell towards the proton drip line it is expected that a new region of ground state deformation will be encountered. As discussed above, such a region has been predicted to emerge rather suddenly around <sup>200</sup>Rn and <sup>204</sup>Ra [31]. The rapid drop of production cross sections as one moves towards more neutron-deficient nuclei makes it difficult to study these nuclei. In addition, the large number of valence particles does not allow for a straightforward interpretation of the deduced level schemes. Nevertheless, an attempt was made to study these nuclei in-beam using the JUROSPHERE array. JU-ROSPHERE consisted of 11 TESSA-type [13] and 14 Eurogam phase I type [33] Compton suppressed Ge detectors in this particular experiment. It had a total photopeak efficiency of 1.6% at 1.3 MeV. The Eurogam Ge detectors were provided by the French/UK Loan Pool. Prior to our studies, the lightest Ra isotope for which transitions were known was <sup>212</sup>Ra [34]. Due to the expected presence of isomers in these nuclei, additional Ge detectors were placed at the focal plane of RITU. For a more detailed description of the experimental setup see Ref. [35].

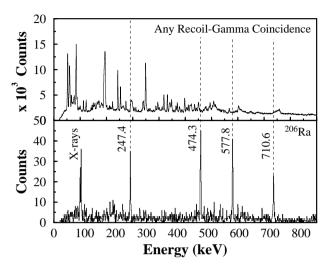


Fig. 3. Gamma-ray spectra measured in-beam in the reaction  $^{40}{\rm Ar}$  +  $^{170}{\rm Yb}$ . Upper panel: gamma-rays in coincidence with any fusion product observed at the focal plane of the separator. Lower panel: Gamma-rays in coincidence with  $^{206}{\rm Ra}$  nuclei, identified on the basis of their characteristic alpha decay. The dashed lines have been drawn to guide the eye.

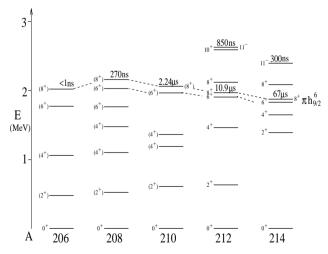


Fig. 4. Systematics of low-lying level energies for neutron-deficient even-even Ra isotopes.

To demonstrate the power of the RDT method, we show in Fig. 3 a comparison of in-beam gamma-ray spectra measured for  $^{206}\mathrm{Ra}$  with and without the RDT-based identification of the separated nuclei. The production cross section of  $^{206}\mathrm{Ra}$  in the reaction employed, 180 MeV  $^{40}\mathrm{Ar} + ^{170}\mathrm{Yb}$ , was only  $\sim 5~\mu\mathrm{b}$  and the recoil-gated spectrum is completely dominated by the much more abundant Fr and Rn nuclei produced in charged particle evaporation channels. The RDT spectrum, on the other hand, shows no peaks from contamination activities.

In Fig. 4, a compilation of the level schemes deduced for  $^{206,208,210}$ Ra from the present work and for  $^{212,214}$ Ra from earlier work [34,36,37] is shown.

#### 7. Discussion

Our new data on the level structure of  $^{192}$ Po show that the trend of decreasing level energies persists as one proceeds still farther towards the proton drip line. However, the steepness of the drop is significantly reduced. Similarly to the even-mass Pt nuclei, this flattening of energy systematics can be interpreted as evidence for a ground state intruder configuration. An interesting example of the kind of complementary information one can deduce from decay experiments is our value for the admixture of the oblate 4p-2h component in the ground state of  $^{192}$ Po as deduced on the basis of alpha decay hindrance factors,  $\sim 63\%$  [18,19]. Moreover, on the basis of Nilsson–Strutinsky type of calculations it was suggested by May et~al. [5] that in  $^{192}$ Po an oblate deformed minimum becomes the ground state.

The most obvious feature of the systematics of level energies of Ra isotopes shown in Fig. 4 is the smooth and gradual decrease of the  $2^+$  energies. There is no evidence for an abrupt increase in ground state deformation. The excitation energies of the  $6^+$  and  $8^+$  members of the  $\pi h_{9/2}^6$  multiplet increase with decreasing neutron number until at  $^{206}$ Ra where a drop in the excitation energy of the  $8^+$  state is observed. Furthermore, the  $8^+$  state decays to the (lowest)  $6^+$  state through a fast transition in  $^{206}$ Ra while in  $^{208}$ Ra this transition is significantly hindered. Assuming that the lowest  $6^+$  states in  $^{206}$ Ra and  $^{208}$ Ra at very nearly identical excitation energy have the same configuration, perhaps  $\pi h_{9/2}{}^5 f_{7/2}$ , this would imply that the  $8^+$  states in  $^{206}$ Ra and  $^{208}$ Ra have different origin.

Concerning reduced widths for alpha decay, the summing up of widths of both ground state to ground state and ground state to excited 2p-2h 0<sup>+</sup> state transitions for Po nuclei removes the anomalous saturation behaviour of Po nuclei. To study the width systematics in more detail, it would be of interest to perform precision measurements on the half-life of <sup>196,198</sup>Rn and of <sup>202,204</sup>Ra. The values for <sup>196</sup>Rn and <sup>202</sup>Ra are based on the observation of only three decay chains [26] and one decay chain [25], respectively. Further-

more, it would be of interest to confirm the slightly anomalous behaviour of the reduced width between <sup>206</sup>Ra and <sup>204</sup>Ra.

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