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Finite $N_{\rm c}$ corrections in the Balitsky–Kovchegov equation at next-to-leading order

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We study the finite-$N_{\rm c}$ corrections to the next-to-leading order (NLO) Balitsky-Kovchegov (BK) equation. This contains correlators of six Wilson lines, which we express in terms of the two-point function using the Gaussian approximation. Numerically, the effects of these finite-$N_{\rm c}$ corrections on the NLO BK equation are found to be smaller than the expected $1/N_{\rm c}^2 \sim 10\%$. Corrections may be large for individual correlators, but have less of an influence on the shape of the amplitude as a function of the dipole size. There is an even smaller effect on the evolution speed as a function of rapidity.

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1. Introduction

The Color Glass Condensate effective field theory is used to describe QCD in the region where parton densities grow to the order of the inverse of the QCD coupling. Cross sections for scattering processes can be expressed in this framework in terms of n-point correlators of Wilson lines - these describe the eikonal propagation of a parton in the target field. The energy dependence of the target is obtained by solving the Balitsky–Kovchegov (BK) equation for the 2-point correlator, which was derived to next-to-leading order (NLO) accuracy in Ref. [1]. The NLO BK equation is known to be unstable [2] and that its finite-N_c corrections are smaller than the naive expectation of $O(1/N_c^2)$ [3, 4]. It contains six-point correlators, for which we derive analytical parametric equations using the Gaussian approximation [5–7]. We study numerically the effect of finite-N_c corrections from the six-point correlators on NLO BK evolution. For the full details, see [8].

2. NLO BK correlators using the Gaussian approximation

For any product of $n/2$ pairs of fundamental Wilson lines $UU^\dagger$, we use the notation $S_{x_1,x_2,\ldots,x_{n-1},x_n}^{(n)} := \text{tr}\left(U_{x_1} U_{x_2}^\dagger \ldots U_{x_{n-1}} U_{x_n}^\dagger\right)/N_c$. The NLO BK equation (with $n_f = 0$) reads [1]

$$\partial_Y \left\langle S_{x,y}^{(2)} \right\rangle = \frac{\alpha_s N_c}{2\pi^2} \int_z K_1^{\text{BC}} \langle D_1 \rangle + \frac{\alpha_s^2 N_c^2}{16\pi^4} \int_{z,z'} (K_{2,1} \langle D_{2,1} \rangle + K_{2,2} \langle D_{2,2} \rangle) + O(n_f), \qquad (1)$$

where the brackets $\langle\rangle$ refer to the expectation value over target color field configurations. The kernels are simple functions of $r^2 = (x-y)^2$, $X^2 = (x-z)^2$, $X'^2 = (x-z')^2$, $Y^2 = (y-z)^2$, $Y'^2 = (y-z')^2$ and $Z^2 = (z-z')^2$. The Wilson line operators are

$$\langle D_1 \rangle = \left\langle S_{x,z}^{(2)} S_{z,y}^{(2)} \right\rangle - \left\langle S_{x,y}^{(2)} \right\rangle, \qquad \langle D_{2,1} \rangle = \left\langle S_{x,z}^{(2)} S_{z,z'}^{(2)} S_{z',y}^{(2)} \right\rangle - \frac{1}{N_c^2} \left\langle S_{x,z,z',y,z,z'}^{(6)} \right\rangle - (z' \to z), \quad (2)$$

$$\langle D_{2,2} \rangle = \left\langle S_{x,z}^{(2)} S_{z,z'}^{(2)} S_{z',y}^{(2)} \right\rangle - (z' \to z). \qquad (3)$$

We refer to two pieces of the right side of Eq. (1) as the "LO-like" contribution $\frac{\alpha_s N_c}{2\pi^2} K_1^{\text{BC}} \otimes \langle D_1 \rangle$ and the "NLO-like" contribution $\frac{\alpha_s^2 N_c^2}{16\pi^4} K_{2,1} \otimes \langle D_{2,1} \rangle + \frac{\alpha_s^2 N_c^2}{16\pi^4} K_{2,2} \otimes \langle D_{2,2} \rangle$. In the large-$N_c$ limit, one can drop terms suppressed by $1/N_c$. Then averages of products factorize into products of averages.

In the Gaussian approximation, all correlators are parametrized by a two-point function as

$$\langle O[U] \rangle_\eta = \exp\left\{ -\frac{1}{2} \int^\eta d\tilde\eta \int_{u_1,u_2} G_{u_1,u_2}(\tilde\eta) L_{u_1}^a L_{u_2}^a \right\} O[U], \qquad (4)$$

where $\int_u = \int d^2 u$, η is a parametrization rapidity and L_u^a is a Lie derivative that acts on Wilson lines according to $L_u^a U_x = -ig\delta^{(2)}(x-u) t^a U_x$. In the case of n-point correlators larger than the dipole, $\langle O[U] \rangle$ is an $n \times n$ matrix of correlators, denoted $\mathcal{A}(\eta)$ and the differential form of Eq. (4) becomes an $n \times n$ matrix differential equation $\partial_\eta \mathcal{A}(\eta) = -\mathcal{M}(\eta)\mathcal{A}(\eta)$.

The procedure to find parametric equations for the six-point correlators is as follows. First, we choose a multiplet basis B for the space of all six-point correlators. Then, we construct the correlator matrix $\mathcal{A}$ by taking $B \left(U_z \otimes U_{z'}^\dagger \otimes U_v \otimes U_y^\dagger \otimes U_x \otimes U_w^\dagger \right) B^T$. Next, we construct the transition matrix $\mathcal{M}$ by summing (for each element in $\mathcal{A}$) all possible one-gluon diagrams obtained with the double Lie derivative operator and rewriting the result in terms of elements of $\mathcal{A}$.

The most efficient way to construct a basis is to exploit the structure of the six-point correlators that are actually needed for the NLO BK equation, which only have four distinct coordinates. It is then easy to see that there is one way to join the endpoints of six Wilson lines with these coordinates such that four Wilson lines cancel (due to unitarity) and only a dipole remains. Similarly, the choice of two more particular basis elements results in two correlators that reduce to four-point correlators. The remaining three basis elements are chosen such that they are orthonormal to the first three.

The differential equation, which has an exponential solution, now decouples into three independent equations. Instead of exponentiating a six-by-six matrix; at most we will need to exponentiate a three-by-three matrix, which can be done analytically. The first transition sub-matrix gives the parametrization for the dipole correlator. The second transition sub-matrix gives the parametrization for the four-point correlator with one repeated coordinate [6]. Taking linear combinations of the elements of the last transition sub-matrix gives the six-point correlators required.

To verify the validity of our parametric equations, we perform two checks. Firstly, the Gaussian approximation has the built-in property that it should be consistent in color algebra. Secondly, when the equation for $\langle S^{(2)} S^{(2)} S^{(2)} \rangle$ is taken in the dilute limit, it should be the same up to second order as the parametric equation for the large-N_c counterpart operator.

3. Numerical results

In Fig. 1, we plot $\langle D_{2,1} \rangle$ and $\langle D_{2,2} \rangle$. Since both quantities reduce to the same expression in the large-N_c limit, only one curve is shown for the large-N_c case. The dashed curves show the differences between large and finite N_c for each quantity. The finite-N_c corrections are negligible at $aQ_s \ll 1$, but become numerically important when $aQ_s \gtrsim 1$. At $aQ_s = 1$, the finite-N_c corrections to $\langle D_{2,1} \rangle$ and $\langle D_{2,2} \rangle$ are approximately 20% and 16%, respectively. In comparison, the LO-like operator $\langle D_1 \rangle$ has a finite-N_c correction of approximately 8%. Since the evolution is driven by the dipole sizes $r \lesssim 1/Q_s$, the effect of the $1/N_c^2$ suppressed contributions to the evolution can be smaller.

In Fig. 2, we plot the difference between the large-N_c and finite-N_c cases, separately for the LO-like and NLO-like terms. The difference has the opposite sign in the LO-like and the NLO-like terms and so part of the difference cancels in the total evolution speed. At $rQ_s = 1$, the relative finite-N_c correction is approximately 8% in the LO-like contribution and 13% in the NLO-like contribution. The relative magnitude of the total $1/N_c^2$ suppressed contribution is 5%.

In Fig. 3, we show the ratio of N obtained by solving the full NLO BK equation at finite N_c to that at large N_c. At $r \gtrsim 1/Q_s$, when the details of the initial condition are lost and one enters the geometric scaling region, the difference between the large-N_c and finite-N_c cases evolves only very slowly. At small dipoles, the ratio grows approximately linearly in Y. The fact that the total finite-N_c correction is positive at small dipole sizes and negative at large dipoles, as seen in Fig. 2, is found to hold also asymptotically after many units of rapidity evolution.
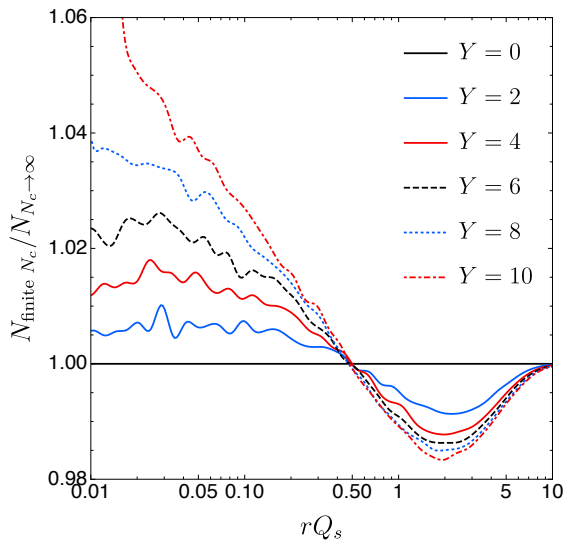
The evolution speed of the saturation scale, $\partial_Y \ln Q_s^2$, is shown in Fig. 4. The finite-N_c corrections are seen to be more important at the initial condition, slowing down the evolution of Q_s^2 by approximately 5%. Later in the evolution, the difference becomes smaller - of the order of 1%.

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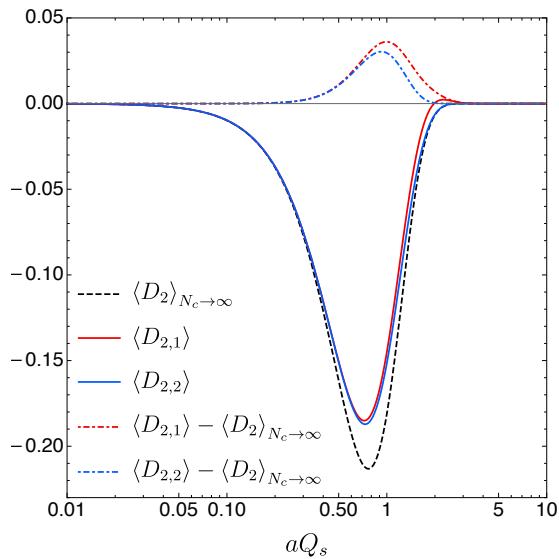
Figure 1: Correlator factors $\langle D_{2,1}\rangle$ and $\langle D_{2,2}\rangle$ in the NLO-like part of the BK equation (1), in the line configuration of coordinates. Both factors reduce to the same expression $\langle D_2\rangle_{\text{Large }N_c}$ in the large-N_c limit, as also shown in the figure.
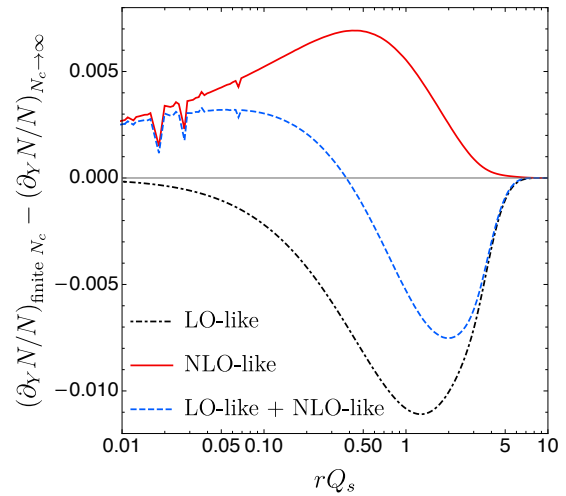
Figure 2: Difference of the evolution speeds at finite N_c and at large N_c, shown separately for the LO-like, NLO-like and total (LO-like + NLO-like) contributions.

Figure 3: Evolution for the ratio of the dipole amplitudes obtained by performing the finite-N_c and large-N_c evolutions with the same initial condition.
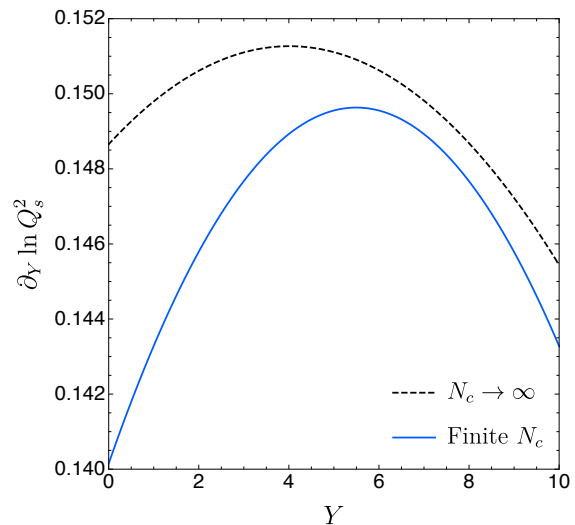
Figure 4: Evolution speed of the saturation scale Q_s^2 as a function of rapidity at large N_c and at finite N_c.

4. Conclusions

The overall finite-N_c corrections to the NLO-like part of the BK equation are somewhat smaller than what is naively expected. However, they need to be considered carefully in the NLO BK equation, since they may have a non-negligible effect at the required accuracy. When

correlators are considered between Wilson lines separated by large distances relative to $1/Q_\mathrm{s}$, $1/N_\mathrm{c}^2$ suppressed corrections may be considerable. Despite these potentially large corrections to individual correlators, these configurations do not contribute much to the right side of the BK equation. There is a small but significant effect on the shape of the dipole amplitude as a function of r. The finite-N_c corrections are watered down further when one considers the evolution speed of Q_s as a function of rapidity.

Acknowledgements

References

[1] I. Balitsky and G. A. Chirilli, *Next-to-leading order evolution of color dipoles*, *Phys. Rev.* **D77** (2008) 014019 [`arXiv:0710.4330 [hep-ph]`].

[2] T. Lappi and H. Mäntysaari, *Direct numerical solution of the coordinate space Balitsky-Kovchegov equation at next to leading order*, *Phys. Rev.* **D91** (2015) no. 7 074016 [`arXiv:1502.02400 [hep-ph]`].

[3] K. Rummukainen and H. Weigert, *Universal features of JIMWLK and BK evolution at small x*, *Nucl. Phys.* **A739** (2004) 183 [`arXiv:hep-ph/0309306 [hep-ph]`].

[4] Y. V. Kovchegov, J. Kuokkanen, K. Rummukainen and H. Weigert, *Subleading-N(c) corrections in non-linear small-x evolution*, *Nucl. Phys.* **A823** (2009) 47 [`arXiv:0812.3238 [hep-ph]`].

[5] H. Fujii, F. Gelis and R. Venugopalan, *Quark pair production in high energy pA collisions: General features*, *Nucl. Phys.* **A780** (2006) 146 [`arXiv:hep-ph/0603099 [hep-ph]`].

[6] C. Marquet and H. Weigert, *New observables to test the Color Glass Condensate beyond the large-N_c limit*, *Nucl. Phys.* **A843** (2010) 68 [`arXiv:1003.0813 [hep-ph]`].

[7] F. Dominguez, C. Marquet, B.-W. Xiao and F. Yuan, *Universality of Unintegrated Gluon Distributions at small x*, *Phys. Rev.* **D83** (2011) 105005 [`arXiv:1101.0715 [hep-ph]`].

[8] T. Lappi, H. Mäntysaari and A. Ramnath, *Next-to-leading order balitsky-kovchegov equation beyond large n_c*, `arXiv:2007.00751 [hep-ph]`.