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Title: Nuclear matrix elements for Majoron-emitting double- β decay

Year: 2021

Version: Published version

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Please cite the original version:

Kotila, J., & Iachello, F. (2021). Nuclear matrix elements for Majoron-emitting double- β decay. *Physical Review C*, 103(4), Article 044302. <https://doi.org/10.1103/PhysRevC.103.044302>

Nuclear matrix elements for Majoron-emitting double- β decay

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(Received 3 November 2020; accepted 18 March 2021; published 1 April 2021)

A complete calculation of the nuclear matrix elements (NMEs) for Majoron emitting neutrinoless double beta decay within the framework of IBM-2 for spectral indices $n = 1, 3, 7$ is presented. By combining the results of this calculation with previously calculated phase-space factors (PSFs) we give predictions for expected half-lives. By comparing with experimental limits on the half-lives we set limits on the coupling constants $\langle g_{ee}^M \rangle$ of all proposed Majoron-emitting models.

DOI: [10.1103/PhysRevC.103.044302](https://doi.org/10.1103/PhysRevC.103.044302)

I. INTRODUCTION

In recent years, increased accuracy has been achieved in the measurement of double- β decay (DBD) with the emission of two neutrinos, $2\nu\beta\beta$ decay, especially in the measurement of the summed electron spectra. High-statistics experiments have been reported by GERDA (^{76}Ge) [1], NEMO3 (^{100}Mo) [2], CUORE (^{130}Te) [3], EXO (^{136}Xe) [4], and KamLAND-Zen (^{136}Xe) [5]. High-statistics experiments have provided information on the mechanism of DBD in $2\nu\beta\beta$ decay, in particular on the question of single-state dominance (SSD) versus high-state dominance (HSD), CUPID-0 (^{82}Se) [6] and CUPID-Mo (^{100}Mo) [7]. With the degree of accuracy reached in the latest experiments, one can also test nonstandard mechanisms of DBD and set stringent limits on them [8].

One of the nonstandard mechanisms is that occurring with the emission of additional bosons called Majorons. Majorons were introduced years ago [10,11] as massless Nambu-Goldstone bosons arising from global $B - L$ (baryon number minus lepton number symmetry) broken spontaneously in the low-energy regime. These bosons couple to the Majorana neutrinos and give rise to neutrinoless double- β decay, accompanied by Majoron emission $0\nu\beta\beta M$ [12], as schematically shown in Fig. 1(a). Although these older models are disfavored by precise measurements of the width of the Z boson decay to invisible channels [13], several other models of $0\nu\beta\beta M$ decay have been proposed in which one or two Majorons, denoted by χ_0 , are emitted (see Fig. 1):

$$\begin{aligned} (A, Z) &\rightarrow (A, Z + 2) + 2e^- + \chi_0, \\ (A, Z) &\rightarrow (A, Z + 2) + 2e^- + 2\chi_0. \end{aligned} \quad (1)$$

Table I lists some of the models proposed to describe these decays [14–17]. The different models are distinguished by the nature of the emitted Majoron(s), i.e., whether it is a Nambu-

Goldstone boson (NG), the leptonic charge of the emitted Majoron (L), and the spectral index of the model, n .

The half-life for all these models can be written as

$$[\tau_{1/2}^{0\nu M}]^{-1} = G_{m\chi_0 n}^{(0)} |\langle g_{\chi_{ee}^M} \rangle|^{2m} |M_{0\nu M}^{(m,n)}|^2, \quad (2)$$

where $G_{m\chi_0 n}^{(0)}$ is a phase-space factor (PSF), $\langle g_{\chi_{ee}^M} \rangle$ is the effective coupling constant of the Majoron to the neutrino, $m = 1, 2$ for the emission of one or two Majorons, respectively, and $M_{0\nu M}^{(m,n)}$ the nuclear matrix element (NME).

II. PHASE-SPACE FACTORS

In a previous article [9] we calculated the PSF and from these the single-electron spectrum, the summed electron spectrum, and the angular correlation between the two electrons. Particularly interesting are the summed electron spectra whose shape depends crucially on the spectral index n . In Fig. 2, the summed electron spectra for $n = 1$, $n = 3$ and $n = 7$, obtained from Ref. [9] by normalizing the spectra so that the area covered by each of them is the same, are plotted as a function of $\varepsilon_1 + \varepsilon_2 - 2m_e c^2$. In this figure, also the summed electron spectrum for $2\nu\beta\beta$ decay [18] is shown again with area normalized to 1. This spectrum has a spectral index $n = 5$. The summed electron spectrum of the “bulk” model $n = 2$ is also shown in Fig. 2. Exact Dirac wave functions, nuclear finite size, and electron screening are included in this calculation, as discussed in Ref. [18]. Previous calculations [19–22] make use of Fermi functions, which are an approximation to the relativistic Dirac wave functions. For comparison between the values reported in Ref. [22] and our values [9] we note that our PSFs are divided by a factor of $g_A^4 = 2.593$ since we include this factor in the NME. We estimated the error in using the old calculation of the PSFs [19–22] instead of the new [9], $(G_{m\chi_0 n}^{(0)\text{old}} - G_{m\chi_0 n}^{(0)\text{new}})/G_{m\chi_0 n}^{(0)\text{new}}$, to be 6% in ^{76}Ge and 28% in ^{136}Xe . The reason why the error is larger in ^{136}Xe ($Z = 54$) than in ^{76}Ge ($Z = 32$) is the neglect in the old calculation of relativistic effects and electron screening which increase as a large power of Z . While in ^{76}Ge and ^{82}Se the use of the old

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FIG. 1. Schematic representation of neutrinoless double- β decay accompanied by the emission of one or two Majorons. Adapted from Ref. [9].

calculation may still be reasonable, it is definitely not so in the other nuclei of current interest: ^{100}Mo , ^{130}Te , ^{136}Xe , and ^{150}Nd . Although experimentally not easily accessible, we also plot in Fig. 3 the single electron spectra with area normalized to 1, and in Fig. 4 the angular correlation between the two electrons, for $n = 1$, $n = 2$, $n = 3$, $n = 7$, and $2\nu\beta\beta$ ($n = 5$) as a function of $\epsilon_1 - m_e c^2$, both of which have been measured by the NEMO3 collaboration in ^{130}Te [23]. In this article we present a calculation of the nuclear matrix elements $M_{0\nu M}^{(m,n)}$.

III. NUCLEAR MATRIX ELEMENTS

Nuclear matrix elements for Majoron-emitting DBD were derived in a seminal paper by Hirsch *et al.* [22]. These authors derived an explicit form for the nuclear matrix elements of all the models of Table I, except for the “bulk” model. We have converted the form of Ref. [22] to our notation, added some higher-order terms not included in the original form and calculated the corresponding matrix elements within the framework of the microscopic interacting boson model IBM-2 [24,25] with isospin restoration [26]. Explicitly, we introduce the matrix elements

$$\begin{aligned}\mathcal{M}_F &= \langle f \| v_m \| i \rangle, \\ \mathcal{M}_{GT} &= \langle f \| v_m \sigma_1 \cdot \sigma_2 \| i \rangle, \\ \mathcal{M}_T &= \langle f \| v_m S_{12} \| i \rangle,\end{aligned}$$

TABLE I. Different Majoron-emitting models [14–17]. The third, fourth, and fifth columns indicate whether the Majoron is a Nambu-Goldstone boson, its leptonic charge L , and the model’s spectral index, n . The sixth column indicates the nuclear matrix elements of Sec. II appropriate for each model.

Model	Decay mode	NG boson	L	n	NME
IB	$0\nu\beta\beta\chi_0$	No	0	1	M_1
IC	$0\nu\beta\beta\chi_0$	Yes	0	1	M_1
ID	$0\nu\beta\beta\chi_0\chi_0$	No	0	3	M_3
IE	$0\nu\beta\beta\chi_0\chi_0$	Yes	0	3	M_3
IIB	$0\nu\beta\beta\chi_0$	No	-2	1	M_1
IIC	$0\nu\beta\beta\chi_0$	Yes	-2	3	M_2
IID	$0\nu\beta\beta\chi_0\chi_0$	No	-1	3	M_3
IIE	$0\nu\beta\beta\chi_0\chi_0$	Yes	-1	7	M_3
IIF	$0\nu\beta\beta\chi_0$	Gauge boson	-2	3	M_2
“Bulk”	$0\nu\beta\beta\chi_0$	Bulk field	0	2

FIG. 2. Summed electron spectra for the $n = 1, 2, 3$, and 7 , as well as for the $2\nu\beta\beta$ ($n = 5$) decays of ^{136}Xe .

$$\begin{aligned}\mathcal{M}_{GTR} &= \langle f \| v_R \sigma_1 \cdot \sigma_2 \| i \rangle, \\ \mathcal{M}_{TR} &= \langle f \| v_R S_{12} \| i \rangle, \\ \mathcal{M}_{F\omega^2} &= \langle f \| v_{\omega^2} \| i \rangle, \\ \mathcal{M}_{GT\omega^2} &= \langle f \| v_{\omega^2} \sigma_1 \cdot \sigma_2 \| i \rangle, \\ \mathcal{M}_{T\omega^2} &= \langle f \| v_{\omega^2} S_{12} \| i \rangle,\end{aligned}\quad (3)$$

FIG. 3. Single electron spectra for the $n = 1, 2, 3$, and 7 , as well as for the $2\nu\beta\beta$ ($n = 5$) decays of ^{136}Xe .

FIG. 4. Angular correlation between the two emitted electrons for the $n = 1, 2, 3$, and 7 , as well as for the $2\nu\beta\beta$ ($n = 5$) decays of ^{136}Xe . The calculation for $n = 1, 2, 3, 7$ stops at the point where the single electron spectrum goes to zero. Beyond that point it becomes unstable as $G^{(0)}$ goes to zero faster than $G^{(1)}$. For $2\nu\beta\beta$ ($n = 5$) this is avoided by taking into account the individual energies of the neutrinos, ω_1 and ω_2 , as in Eq. (22) of Ref. [18], instead of the Majoron energy q as in Eq. (5) of Ref. [9].

where the isospin operators $\tau_1^+\tau_2^+$ have been dropped for simplicity. These matrix elements are the same as in Ref. [22] with the addition of the tensor matrix elements.

The neutrino potentials needed for the calculation of these matrix elements, when converted to the notation used in IBM-2 [24–26] are

$$\begin{aligned} v_m &= \frac{2}{\pi} \frac{1}{q(q+\tilde{A})}, & v_R &= \frac{2}{\pi} \frac{1}{Rm_p} \frac{q+\frac{\tilde{A}}{2}}{q(q+\tilde{A})^2}, \\ v_{\omega^2} &= \frac{2}{\pi} m_e^2 \frac{q^2 + \frac{9}{8}q\tilde{A} + \frac{3}{8}\tilde{A}^2}{q^3(q+\tilde{A})^3}, \end{aligned} \quad (4)$$

with $R = 1.2A^{1/3}$ fm, $m_p = 938$ MeV = 4.76 fm $^{-1}$, $m_e = 0.511$ MeV = 0.00259 fm $^{-1}$. $\tilde{A}$ is the closure energy that we take as in Refs. [24–26], $\tilde{A} = 1.12A^{1/2}$ MeV, where A denotes the mass number. We note that the last term in v_{ω^2} diverges at the origin as q^{-3} . We regularize this term by multiplying it by $q/(q+\tilde{A})$, that is

$$v_{\omega^2} = \frac{2}{\pi} m_e^2 \frac{q^2 + \frac{9}{8}q\tilde{A} + \frac{3}{8}\tilde{A}^2 \frac{q}{q+\tilde{A}}}{q^3(q+\tilde{A})^3}. \quad (5)$$

From the neutrino potentials we construct the quantities

$$h(q) = v(q)\tilde{h}(q), \quad (6)$$

where $\tilde{h}_{F,GT,T}(q) = \tilde{h}_{F\omega^2,GT\omega^2,T\omega^2}$ are given in Table II of Ref. [25] which includes the form factors and higher-order corrections and

$$\tilde{h}_R(q) = \frac{1}{(1+q^2/m_V^2)^2} \frac{1}{(1+q^2/m_A^2)^2}, \quad (7)$$

which includes the form factors with $m_V = 0.84$ GeV and $m_A = 1.09$ GeV, as in Refs. [25,26].

The matrix elements for the three classes of Majoron models are

$$\begin{aligned} M_1 &= g_A^2 \mathcal{M}_1 = g_A^2 \left[-\left(\frac{g_V^2}{g_A^2}\right) \mathcal{M}_F + \mathcal{M}_{GT} - \mathcal{M}_T \right], \\ M_2 &= g_A^2 \mathcal{M}_2 = g_A^2 \left[\left(\frac{g_V}{g_A}\right) \frac{f_W}{3} \mathcal{M}_{GTR} - \left(\frac{g_V}{g_A}\right) \frac{f_W}{6} \mathcal{M}_{TR} \right], \\ M_3 &= g_A^2 \mathcal{M}_3 = g_A^2 \left[-\left(\frac{g_V^2}{g_A^2}\right) \mathcal{M}_{F\omega^2} + \mathcal{M}_{GT\omega^2} - \mathcal{M}_{T\omega^2} \right], \end{aligned} \quad (8)$$

where we have used the overall sign convention as in Ref. [27] and in our previous papers [24–26]. In Eq. (8), $f_W = 1 + \kappa_\beta = 4.70$, where κ_β is the isovector magnetic moment of the nucleon. In the calculation of the matrix elements in Eq. (8) also short-range correlations are included as in Refs. [24–26]. Our results are shown in Table II. The nuclear matrix elements M_1, M_2, M_3 are associated with Majoron-emitting models of $0\nu\beta\beta M$ decays as in the last column of Table I.

Sensitivity to parameter changes, model assumptions, and operator assumptions

The matrix element $\mathcal{M}_1$ for index $n = 1$ is identical to the matrix element of ordinary $0\nu\beta\beta$ decay without Majoron emission. The sensitivity of IBM-2 calculations to parameter changes, model assumptions, and operator assumption for this NME was discussed in great detail in Refs. [25,26]. Our error estimate for M_1 is therefore 16% for all nuclei, as in Ref. [26].

For the matrix element $\mathcal{M}_2$ we have an additional error coming from the neglect of higher-order terms of the type

$$\begin{aligned} \frac{(\mathbf{Q} \cdot \boldsymbol{\sigma}_1)(\mathbf{Q} \cdot \boldsymbol{\sigma}_2)}{4m_p^2} &\simeq \frac{(\mathbf{q} \cdot \boldsymbol{\sigma}_1)(\mathbf{q} \cdot \boldsymbol{\sigma}_2)}{4m_p^2} \\ &= \frac{q^2}{4m_p^2} \left[\frac{1}{3} \boldsymbol{\sigma}_1 \cdot \boldsymbol{\sigma}_2 + \frac{1}{3} S_{12} \right], \end{aligned} \quad (9)$$

where $\mathbf{Q}$ is the total momentum and $\mathbf{q}$ the relative momentum of the nucleons and we have assumed $\mathbf{Q} \simeq \mathbf{q}$ [20]. We estimate the neglected contribution of these higher-order terms to be about 4%, giving a total estimated error of 20% for $\mathcal{M}_2$.

The matrix element $\mathcal{M}_3$ depends strongly on the closure energy $\tilde{A}$ as given in Eq. (4). In the present calculation we have assumed the standard choice $\tilde{A} = 1.12A^{1/2}$ MeV. We have investigated variations of $\tilde{A}$ around the standard values and estimate an additional error in the calculation of $\mathcal{M}_3$ of $\approx 10\%$, bringing the total estimated error to 30%. An estimate of the sensitivity of $\mathcal{M}_3$ to the closure energy was also given in Ref. [22]. In this reference also a discussion of the sensitivity to model assumptions of Majoron-emitting DBD was given.

TABLE II. Majoron-emitting DBD NMEs $\mathcal{M}_i$ ($i = 1, 2, 3$) calculated in this work using the quenched value $g_A = 1.0$ and the convention $\mathcal{M}_i > 0$.

Isotope	$\mathcal{M}_F$	$\mathcal{M}_{GT}$	$\mathcal{M}_T$	$\mathcal{M}_1$	$\mathcal{M}_{GTR}$	$\mathcal{M}_{TR}$	$\mathcal{M}_2$	$\mathcal{M}_{F\omega^2}$ $\times 10^3$	$\mathcal{M}_{GT\omega^2}$ $\times 10^3$	$\mathcal{M}_{T\omega^2}$ $\times 10^3$	$\mathcal{M}_3$ $\times 10^3$
⁷⁶ Ge	-0.780	5.582	-0.281	6.642	0.225	-0.037	0.381	-0.017	2.530	-0.009	2.556
⁸² Se	-0.667	4.521	-0.270	5.458	0.178	-0.034	0.305	-0.014	1.967	-0.009	1.993
⁹⁶ Zr	-0.361	3.954	0.250	4.065	0.147	0.031	0.205	-0.006	1.672	0.009	1.668
¹⁰⁰ Mo	-0.511	5.075	0.318	5.268	0.187	0.038	0.263	-0.008	1.904	0.011	1.901
¹¹⁰ Pd	-0.425	4.024	0.243	4.206	0.144	0.030	0.203	-0.006	1.411	0.009	1.409
¹¹⁶ Cd	-0.335	2.888	0.118	3.105	0.102	0.019	0.144	-0.005	0.945	0.006	0.945
¹²⁴ Sn	-0.572	3.099	-0.118	3.789	0.104	-0.017	0.177	-0.013	1.161	-0.005	1.179
¹²⁸ Te	-0.718	3.965	-0.115	4.798	0.132	-0.020	0.223	-0.016	1.505	-0.006	1.527
¹³⁰ Te	-0.651	3.586	-0.159	4.396	0.118	-0.018	0.199	-0.014	1.291	-0.005	1.311
¹³⁴ Xe	-0.686	3.862	-0.121	4.669	0.126	-0.018	0.212	-0.015	1.456	-0.006	1.477
¹³⁶ Xe	-0.522	2.958	-0.123	3.603	0.096	-0.013	0.160	-0.012	1.161	-0.004	1.112
¹⁴⁸ Nd	-0.362	2.283	0.125	2.521	0.074	0.012	0.107	-0.006	0.648	0.004	0.650
¹⁵⁰ Nd	-0.507	3.371	0.119	3.759	0.110	0.017	0.159	-0.008	0.836	0.005	0.839
¹⁵⁴ Sm	-0.340	2.710	0.122	2.928	0.086	0.015	0.122	-0.006	0.858	0.005	0.859
¹⁶⁰ Gd	-0.415	3.838	0.250	4.002	0.120	0.023	0.170	-0.006	1.261	0.008	1.260
¹⁹⁸ Pt	-0.329	2.021	0.119	2.230	0.061	0.009	0.089	-0.005	0.393	0.003	0.395
²³² Th	-0.444	3.757	0.251	3.950	0.104	0.019	0.148	-0.006	0.930	0.007	0.930
²³⁸ U	-0.525	4.470	0.244	4.751	0.122	0.022	0.174	-0.007	1.118	0.008	1.118

IV. LIMITS ON THE COUPLING CONSTANTS

From the PSF of Ref. [9], the NME of this article, and experimental limits on half-lives for each type of Majoron

model, one can derive limits on the coupling constants $g_{\chi_{ee}^M}$. These limits depend on the value of the coupling constant g_A . This coupling constant is renormalized in nuclei by many-body effects. Three possible values are [28] (i) the free

TABLE III. Limits on the Majoron-neutrino coupling constants $\langle g_{\chi_{ee}^M} \rangle$ for $g_A = 1$. PSF from Ref. [9]. NME from this paper.

Decay mode	Spectral index	Model type	$\mathcal{M}$	$G_{m\chi_0 n}^{(0)} [10^{-18} \text{ yr}]$	$\tau_{1/2} [\text{yr}]$	$ \langle g_{\chi_{ee}^M} \rangle $
⁷⁶ Ge [32]						
$0\nu\beta\beta\chi_0$	1	IB,IC,IIB	6.64	44.2	$>4.2 \times 10^{23}$	$<3.5 \times 10^{-5}$
$0\nu\beta\beta\chi_0\chi_0$	3	ID,IE,IID	0.0026	0.22	$>0.8 \times 10^{23}$	<1.7
$0\nu\beta\beta\chi_0$	3	IIC,IIF	0.381	0.073	$>0.8 \times 10^{23}$	$<0.34 \times 10^{-1}$
$0\nu\beta\beta\chi_0\chi_0$	7	IIE	0.0026	0.420	$>0.3 \times 10^{23}$	<1.9
$0\nu\beta\beta\chi_0$	2	Bulk			$>1.8 \times 10^{23}$	
¹³⁰ Te [29]						
$0\nu\beta\beta\chi_0$	1	IB,IC,IIB	4.40	413	$>2.2 \times 10^{21}$	$<2.4 \times 10^{-4}$
$0\nu\beta\beta\chi_0\chi_0$	3	ID,IE,IID	0.0013	3.21	$>0.9 \times 10^{21}$	<3.8
$0\nu\beta\beta\chi_0$	3	IIC,IIF	0.199	1.51	$>2.2 \times 10^{21}$	$<0.87 \times 10^{-1}$
$0\nu\beta\beta\chi_0\chi_0$	7	IIE	0.0013	14.4	$>0.9 \times 10^{21}$	<2.6
$0\nu\beta\beta\chi_0$	2	Bulk			$>2.2 \times 10^{21}$	
¹³⁰ Te [23]						
$0\nu\beta\beta\chi_0$	1	IB,IC,IIB	4.40	413	$>1.6 \times 10^{22}$	$<8.8 \times 10^{-5}$
¹³⁶ Xe [31]						
$0\nu\beta\beta\chi_0$	1	IB,IC,IIB	3.60	409	$>1.2 \times 10^{24}$	$<1.3 \times 10^{-5}$
$0\nu\beta\chi_0\chi_0$	3	ID,IE,IID	0.0011	3.05	$>2.7 \times 10^{22}$	<1.8
$0\nu\beta\beta\chi_0$	3	IIC,IIF	0.160	1.47	$>2.7 \times 10^{22}$	$<0.31 \times 10^{-1}$
$0\nu\beta\beta\chi_0\chi_0$	7	IIE	0.0011	12.5	$>6.1 \times 10^{21}$	<1.8
$0\nu\beta\beta\chi_0$	2	Bulk			$>2.5 \times 10^{23}$	
¹³⁶ Xe [30]						
$0\nu\beta\beta\chi_0$	1	IB,IC,IIB	3.60	409	$>2.6 \times 10^{24}$	$<8.5 \times 10^{-6}$
$0\nu\beta\beta\chi_0\chi_0$	3	ID,IE,IID	0.0011	3.05	$>4.5 \times 10^{24}$	<0.49
$0\nu\beta\beta\chi_0$	3	IIC,IIF	0.160	1.47	$>4.5 \times 10^{24}$	$<0.24 \times 10^{-2}$
$0\nu\beta\beta\chi_0\chi_0$	7	IIE	0.0011	12.5	$>1.1 \times 10^{22}$	<1.6
$0\nu\beta\beta\chi_0$	2	Bulk			$>1.0 \times 10^{24}$	

value, $g_A = 1.269$, (ii) the quark value, $g_A = 1.0$, and (iii) the value extracted from $2\nu\beta\beta$ decay, which, in IBM-2 can be parametrized as $g_{A,eff}^{IBM-2} = 1.269A^{-0.18}$. To allow for different values of g_A , we rewrite Eq. (2) as

$$[\tau_{1/2}^{0\nu M}]^{-1} = G_{m\chi_0 n}^{(0)} |g_{\chi_{ee}^M}|^{2m} g_A^4 |\mathcal{M}_{0\nu n}^{(m,n)}|^2, \quad (10)$$

where $\mathcal{M}_{0\nu n}^{(m,n)}$ are the NME given in Table II. In extracting limits on $g_{\chi_{ee}^M}$ we use in this article $g_A = 1$. From Eq. (10) it is straightforward to obtain limits for other values of g_A .

Limits on half-lives for Majoron-emitting models have been reported by several groups [23,29–32]. In Table III we provide our limits on the coupling constants $g_{\chi_{ee}^M}$.

The most stringent limits come from the KamLAND-Zen collaboration [30] and from the EXO collaboration [31]. The reason why one obtains such small limits for Majoron-emitting models with index $n = 1$ was discussed in Ref. [22]. The larger limits of $g_{\chi_{ee}^M}$ for Majoron-emitting models with

index $n = 3$ and $n = 7$ are due to the smaller values of the PSF for these indices.

V. CONCLUSIONS

We have presented here a complete calculation of NME for Majoron-emitting neutrinoless double- β decay within the framework of the Interacting Boson Model IBM-2. Our results when combined with the phase space factors of Ref. [9] provide up-to-date predictions for lifetimes, single-electron spectra, summed electron spectra, and angular distributions for Majoron-emitting neutrinoless double- β decay which can be used in the analysis of recent high-statistics experiments [1–7].

ACKNOWLEDGMENTS

This work was supported in part by the Academy of Finland Grants No. 314733 and No. 320062.

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