

**This is a self-archived version of an original article. This version may differ from the original in pagination and typographic details.**

**Author(s):** Becchi, C.; Ferretti, Jacopo; Giachino, A.; Maiani, L.; Santopinto, E.

**Title:** A study of  $ccc\bar{c}\bar{c}$  tetraquark decays in 4 muons and in  $D(D^-)$  at LHC

**Year:** 2020

**Version:** Published version

**Copyright:** © 2020 The Author(s). Published by Elsevier B.V.

**Rights:** CC BY 4.0

**Rights url:** <https://creativecommons.org/licenses/by/4.0/>

**Please cite the original version:**

Becchi, C., Ferretti, J., Giachino, A., Maiani, L., & Santopinto, E. (2020). A study of  $ccc\bar{c}\bar{c}$  tetraquark decays in 4 muons and in  $D(D^-)$  at LHC. *Physics Letters B*, 811, Article 135952. <https://doi.org/10.1016/j.physletb.2020.135952>



# A study of $cc\bar{c}\bar{c}$ tetraquark decays in 4 muons and in $D^{(*)}\bar{D}^{(*)}$ at LHC

C. Becchi<sup>a</sup>, J. Ferretti<sup>b</sup>, A. Giachino<sup>a</sup>, L. Maiani<sup>c,\*</sup>, E. Santopinto<sup>a</sup>

<sup>a</sup> Istituzione Nazionale di Fisica Nucleare (INFN), Sezione di Genova, via Dodecaneso 33, 16146 Genova, Italy

<sup>b</sup> Department of Physics, University of Jyväskylä, P.O. Box 35, 40014 Jyväskylä, Finland

<sup>c</sup> Dipartimento di Fisica and INFN, Sapienza Università di Roma, Piazzale Aldo Moro 2, I-00185 Roma, Italy



## ARTICLE INFO

### Article history:

Received 28 June 2020

Received in revised form 19 October 2020

Accepted 10 November 2020

Available online 13 November 2020

Editor: B. Grinstein

## ABSTRACT

We perform a quantitative analysis of the decays of  $cc\bar{c}\bar{c}$  tetraquarks with  $J^{PC} = 0^{++}, 2^{++}$  into 4 muons and into hidden- and open-charm mesons and estimate, for the first time, the fully charmed tetraquark decay width. The calculated cross section upper limit is  $\sim 39(780)$  fb for the 4 muons channel, and  $\sim 42(616)$  pb for the  $D^{(*)}\bar{D}^{(*)} \rightarrow e\mu$  channel, in the  $0^{++}(2^{++})$  case. Decay widths depend upon the additional parameter  $\xi = |\Psi_{\mathcal{T}}(0)|^2/|\Psi_{J/\psi}(0)|^2$ , which can be computed with a considerable error. We find  $\Gamma(0^{++}) \sim \Gamma(2^{++}) = 97 \pm 30$ . On the basis of our results and with the present sensitivity, LHCb should detect both  $0^{++}$  and  $2^{++}$  fully-charmed tetraquarks.

© 2020 The Author(s). Published by Elsevier B.V. This is an open access article under the CC BY license (<http://creativecommons.org/licenses/by/4.0/>). Funded by SCOAP<sup>3</sup>.

## 1. Introduction

In this note we consider production and decay at proton colliders of the fully charmed tetraquarks,  $\mathcal{T} = cc\bar{c}\bar{c}$ . In particular, we consider the  $4\mu$  and meson-meson decays, the latter revealed through the  $e\mu$  signature of meson pair weak decays. We focus on the ground states with  $J^{PC} = 0^{++}, 2^{++}$ . We shall use the method recently applied to production and decay of fully bottom tetraquarks,  $bb\bar{b}\bar{b}$  in [1], briefly described in the following.

Evidence for a  $4\mu$  resonance has been announced in a recent paper of the LHCb Collaboration [2], which is in line with our estimates and indicates that  $4\mu$  and meson-meson channels may be the key to the study of these truly exotic hadrons.

The hypothetical existence of hadronic states with more than minimal quark content ( $q\bar{q}$  or  $qqq$ ) was proposed by Gell-Mann in 1964 [3] and Zweig [4], followed by a quantitative model by Jaffe [5] for the lightest scalar mesons described as diquark anti-diquark pairs. Recent years have seen considerable growth in the observation of four valence quark states that cannot be included in the well-known systematics of  $q\bar{q}$  mesons, like  $Z(4430)$  [6,7],  $Z(3900)$  and  $Z(4020)$  [8]. Similar particles have also been found in the bottom sector,  $Z_b(10610)$  and  $Z_b(10650)$ , observed by the Belle collaboration [9] (see [10–15] for recent reviews).

Earlier predictions of a fully-charmed  $cc\bar{c}\bar{c}$  tetraquark were made in Refs. [16–22], followed by more recent studies in [23–33].

Refs. [29,34] have estimated the  $J^{PC} = 0^{++}$ , fully-bottom tetraquark decay width.

Theoretically,  $J^{PC} = 0^{++}$  is expected for the  $cc\bar{c}\bar{c}$  ground-state. Following Ref. [1] we present a calculation of decay widths and branching ratios of the main, hidden- and open-charm channels of  $cc\bar{c}\bar{c}$  tetraquarks.

To be explicit, we assume that such states do indeed exist, as in [33, Table III].<sup>1</sup>

We restrict, for definiteness, to diquarks in colour  $\bar{\mathbf{3}}$ . The case of colour 6 diquarks can be worked out as a simple extension.

We extend to fully charm tetraquarks the recent work done on doubly heavy tetraquarks in the quark model [36–38] and in Lattice QCD [39]. It is worth noticing that in the last reference no evidence was found of bound doubly heavy diquarks in colour  $\bar{\mathbf{6}}$ . The presence of colour  $\bar{\mathbf{6}}$  diquarks component in the doubly heavy tetraquark has been noted in [40] and found to vanish for increasing heavy to light quark mass ratio.

In a recent paper tetraquarks with  $\bar{\mathbf{3}}$  diquarks and  $\bar{\mathbf{6}}$  anti-diquarks have also been considered, in the presence of explicit gluon fields [41]. This situation is definitely beyond reach of our method.

## 2. Results

Decay rates are proportional to the ratio of overlap probabilities of the annihilating  $c\bar{c}$  pairs in  $\mathcal{T}$  and  $J/\psi$ :

$$\xi = \frac{|\Psi_{\mathcal{T}}(0)|^2}{|\Psi_{J/\psi}(0)|^2} \quad (1)$$

<sup>1</sup> The spectrum given in [33] has to be shifted by a constant  $\Delta E$ , determined so as to reproduce the experimental mass of the tetraquark ground-state, see [35].

\* Corresponding author.

E-mail address: [luciano.maiani@roma1.infn.it](mailto:luciano.maiani@roma1.infn.it) (L. Maiani).

**Table 1**  
Branching fractions of fully-charmed tetraquarks, assuming S-wave decay.

$[\bar{c}c\bar{c}c]$	$\eta_c + \text{any}$	$D_q \bar{D}_q (m_q < m_c)$	$D_q^* \bar{D}_q^*$	$J/\psi + \text{any}$	$J/\psi + \mu^+ \mu^-$	$4\mu$
$J^{PC} = 0^{++}$	0.75	0.021	0.061	$7.3 \cdot 10^{-4}$	$4.3 \cdot 10^{-5}$	$2.6 \cdot 10^{-6}$
$J^{PC} = 2^{++}$	0	0	0.25	$2.9 \cdot 10^{-3}$	$1.7 \cdot 10^{-4}$	$1.0 \cdot 10^{-5}$

Branching ratios do not depend upon  $\xi$ , our predictions are reported in Table 1. In particular, we find

$$\begin{aligned} B(\mathcal{T} \rightarrow 4\mu) &= 2.6 \cdot 10^{-6} (J^{PC} = 0^{++}); \\ B(\mathcal{T} \rightarrow 4\mu) &= 10 \cdot 10^{-6} (J^{PC} = 2^{++}). \end{aligned} \quad (2)$$

The total width is expressed as:

$$\Gamma(\mathcal{T}(J = 0^{++})) = 20 \cdot \xi \text{ MeV}$$

The overlap functions for tetraquark and  $J/\psi$  can be computed by making use of a variational method with harmonic oscillator trial wave functions. To get the overlap function of the  $J/\psi$  one can also use the leptonic width. In Sect. 4 we find:

$$\xi = 4.6 \pm 1.4 \quad (3)$$

Our best estimate is then

$$\Gamma(\mathcal{T}(J = 0^{++})) = 97 \pm 30 \text{ MeV} \quad (4)$$

We extend the calculation to the  $J^{PC} = 2^{++}$ , fully-charmed tetraquark.  $J = 2$  tetraquarks are produced in  $p + p$  collisions with a statistical factor of 5 with respect to the spin 0 state. The decay  $\mathcal{T} \rightarrow \eta_c + \text{light hadrons}$  is suppressed but annihilations into meson pairs take place at a greater rate.

We find:

$$\begin{aligned} \Gamma(\mathcal{T}(J = 2^{++})) &\sim \Gamma(\mathcal{T}(J = 0^{++})) = 21 \cdot \xi \text{ MeV} = \\ &= 98 \pm 30 \text{ MeV} \end{aligned} \quad (5)$$

Branching fractions and upper limits to the cross sections of final states in  $pp$  collisions are summarised in Tables 1 and 2.

The results of Table 1 combined with the recent determination by LHCb of the cross section for  $2J/\psi$  production at 13 TeV [42], give encouraging upper bounds to the production of  $\mathcal{T} \rightarrow 4\mu$  at LHC

$$\begin{aligned} \sigma(p + p \rightarrow \mathcal{T}(0^{++}) + \dots \rightarrow 4\mu + \dots) &< 39 \text{ fb} \\ \sigma(p + p \rightarrow \mathcal{T}(2^{++}) + \dots \rightarrow 4\mu + \dots) &< 790 \text{ fb} \end{aligned} \quad (6)$$

### 3. Details of the calculation

We give here a brief description of our method. The reader may consult Ref. [1] for more details. The starting point is the Fierz transformation, which brings  $c\bar{c}$  together [14]:

$$\begin{aligned} \mathcal{T}(J = 0^{++}) &= \left| (cc)_{\frac{3}{1}}^1 (\bar{c}\bar{c})_{\frac{3}{1}}^1 \right|_1^0 \\ &= -\frac{1}{2} \left( \sqrt{\frac{1}{3}} \left| (c\bar{c})_1^1 (c\bar{c})_1^1 \right|_1^0 - \sqrt{\frac{2}{3}} \left| (c\bar{c})_8^1 (c\bar{c})_8^1 \right|_1^0 \right) + \\ &\quad + \frac{\sqrt{3}}{2} \left( \sqrt{\frac{1}{3}} \left| (c\bar{c})_1^0 (c\bar{c})_1^0 \right|_1^0 - \sqrt{\frac{2}{3}} \left| (c\bar{c})_8^0 (c\bar{c})_8^0 \right|_1^0 \right), \end{aligned} \quad (7)$$

quark bilinears are normalised to unity, subscripts denote the dimension of colour representations, and superscripts the total spin. For the  $J = 2$  tetraquark, one finds:

$$\begin{aligned} \mathcal{T}(J = 2^{++}) &= \left| (cc)_{\frac{3}{1}}^1 (\bar{c}\bar{c})_{\frac{3}{1}}^1 \right|_1^2 \\ &= \left( \sqrt{\frac{1}{3}} \left| (c\bar{c})_1^1 (c\bar{c})_1^1 \right|_1^2 - \sqrt{\frac{2}{3}} \left| (c\bar{c})_8^1 (c\bar{c})_8^1 \right|_1^2 \right). \end{aligned} \quad (8)$$

We describe  $\mathcal{T}$  decay rate as the incoherent sum of the annihilation rates of one charm quark, call it  $c_1$ , with either antiquark  $\bar{c}_1$  or  $\bar{c}_2$  (see [1]). The  $c_1\text{-}\bar{c}_1$  annihilation rate, with  $c_2$  and  $\bar{c}_2$  spectators, depends on total colour and spin of the incoming particles, which are given by (7) or (8). Rates are categorised in the following items 1 to 4. Annihilation of  $c_1$  with  $\bar{c}_2$  gives the same result and brings in a factor 2. This approximation is valid in the limit of very massive quarks (mass  $\gg \Lambda_{QCD} \sim 0.35$  GeV), which behave as classical particles that can be localised independently.

1. The colour singlet, spin 0 pair decays into 2 gluons, which are converted into confined, light hadrons with a rate of order  $\alpha_s^2$ ; taking the spectator  $c\bar{c}$  pair into account, this decay leads to:  $\mathcal{T} \rightarrow \eta_c + \text{light hadrons}$ .
2. The colour singlet, spin 1 pair decays into 3 gluons, which are converted into confined light hadrons leading to:  $\mathcal{T} \rightarrow J/\psi + \text{light hadrons}$ . The rate is of order  $\alpha_s^3$ . In addition, annihilation into one photon produces the final state  $J/\psi + \mu^+ \mu^-$  and, eventually,  $4\mu$ , with rates of order  $\alpha^2$  and  $\alpha^4$ .
3. The colour octet, spin 1 pairs annihilate into one gluon, which materialises into a pair of light quark flavours,  $q = u, d, s$ ; the latter recombine with the spectator pair to produce a pair of open-charm particles, in particular meson pairs,  $D_q \bar{D}_q$  and  $D_q^* \bar{D}_q^*$ , with a rate of order  $\alpha_s^2$ .
4. The colour octet, spin 0 pairs annihilate into two gluons, which have to produce a pair of light quarks to neutralise the colour of the spectator  $c\bar{c}$  pair, with amplitude of order  $\alpha_s^2$  and rate of the order of  $\alpha_s^4$ , which we neglect.

The total  $\mathcal{T}$  decay rate is the sum of individual decay rates, obtained from the simple formula [44]

$$\Gamma((c\bar{c})_c^s) = |\Psi(0)_{\mathcal{T}}|^2 v \sigma((c\bar{c})_c^s \rightarrow f) \quad (9)$$

$|\Psi(0)_{\mathcal{T}}|^2$  is the overlap probability of the annihilating pair,  $v$  the relative velocity,  $\sigma$  the spin-averaged annihilation cross section in the final state  $f$  and suffixes  $s$  and  $c$  denote spin and colour.<sup>2</sup>

For tetraquarks near the  $2J/\psi$  threshold, the spectator  $c\bar{c}$  pair appears as  $\eta_c$  or  $J/\psi$  on the mass shell, or combines with the outgoing  $q\bar{q}$  pair into a pair of open-charm particles.

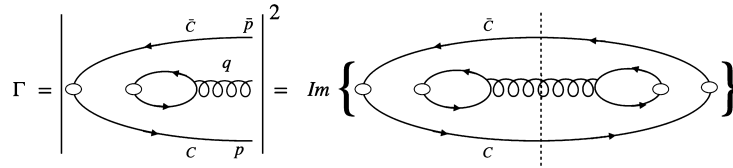
We normalise the overlap probabilities to  $|\Psi_{J/\psi}(0)|^2$ , derived from the  $J/\psi$  decay rate into lepton pairs. Eq. (9) applied to this case gives:

$$\Gamma(J/\psi \rightarrow \mu^+ \mu^-) = Q_c^2 \frac{4\pi\alpha^2}{3} \frac{4}{m_{J/\psi}^2} |\Psi_{J/\psi}(0)|^2. \quad (10)$$

In terms of the Vector Meson Dominance parameter [45] defined by

<sup>2</sup> Our method of calculation is borrowed from the theory of  $K$  electron capture, where an atomic electron reacts with a proton in the nucleus to give a final nucleus and a neutrino, see [1].





**Fig. 2.** To order  $g_s$ , the virtual gluon in  $c\bar{c}$  annihilation is far off-shell ( $q^2 = 4m_c^2$ ) and the amplitude in the r.h.s. of the figure has no imaginary part, implying a vanishing rate to order  $\alpha_s$ .

**Table 2**

Upper limits of two- and four-lepton cross sections via  $\mathcal{T}$  production, estimated from the production cross sections of  $2\psi(1S)$  (LHC, 13 TeV) [42].

$[cc][\bar{c}\bar{c}]$	Decay channel	$BF$ in $\mathcal{T}$ decay	Cross section upper limit (fb)
$J = 0^{++}$	$\mathcal{T} \rightarrow D^{(*)+} \bar{D}^{(*)-} \rightarrow e + \mu + \dots$	$2.3 \cdot 10^{-3}$	$3.6 \cdot 10^4$ (36 pb)
	$\mathcal{T} \rightarrow D^{(*)0} \bar{D}^{(*)0} \rightarrow e + \mu + \dots$	$0.36 \cdot 10^{-3}$	$0.55 \cdot 10^4$ (6 pb)
	$\mathcal{T} \rightarrow 4\mu$	$2.6 \cdot 10^{-6}$	39
$J = 2^{++}$	$\mathcal{T} \rightarrow D^{*+} \bar{D}^{*-} \rightarrow e + \mu + \dots$	$7.0 \cdot 10^{-3}$	$53 \cdot 10^4$ (532 pb)
	$\mathcal{T} \rightarrow D^{*0} \bar{D}^{*0} \rightarrow e + \mu + \dots$	$1.1 \cdot 10^{-3}$	$8.3 \cdot 10^4$ (83 pb)
	$\mathcal{T} \rightarrow 4\mu$	$1.0 \cdot 10^{-5}$	780

case the numerical wave function is extracted by means of the relativized diquark model of Refs. [29] and [33]. We find<sup>5</sup>

$$|\Psi_{(J/\psi, \text{h.o.})}(0)|^2 = 0.070 \text{ GeV}^3 \quad (20)$$

$$|\Psi_{(\mathcal{T}, \text{h.o.})}(0)|^2 = 0.42 \text{ GeV}^3 \quad (21)$$

The harmonic oscillator overlap probability for  $J/\psi$  is smaller than the value obtained from  $\Gamma(J/\psi \rightarrow \mu^+ \mu^-)$ , Eq. (12). To estimate the  $\mathcal{T}$  width, we take for  $\xi$  the average of the two estimates and use their difference for the error

$$\xi_{\text{h.o.}} = \frac{|\Psi_{(\mathcal{T}, \text{h.o.})}(0)|^2}{|\Psi_{(J/\psi, \text{h.o.})}(0)|^2} = 6.0; \quad \xi_{\text{h.o.}, J/\psi} = \frac{|\Psi_{(\mathcal{T}, \text{h.o.})}(0)|^2}{|\Psi_{J/\psi}(0)|^2} = 3.2$$

$$\xi = 4.6 \pm 1.4 \quad (22)$$

## 5. Tetraquark cross sections

Combining Eqs. (15) and (17) we obtain, for  $J^{PC} = 0^{++}$ :

$$B_{4\mu} = B(\mathcal{T} \rightarrow 4\mu) = 2.7 \cdot 10^{-6} \quad (23)$$

and the cross section upper bound

$$\sigma_{\text{theo.}}(\mathcal{T} \rightarrow 4\mu) \leq \sigma(pp \rightarrow 2J/\psi) B_{4\mu} = 40 \text{ fb} \quad (24)$$

where  $\sigma(pp \rightarrow 2J/\psi) \simeq 15.2 \text{ nb}$  is the two- $J/\psi$  production cross section measured by LHCb at 13 TeV [42].

We focus on the  $e\mu$  inclusive channel and give in Table 2 the upper limits to  $\sigma_{\text{theo.}}(\mathcal{T} \rightarrow 2D_q^{(*)} \rightarrow e\mu + \dots)$ , calculated as

$$\begin{aligned} \sigma_{\text{theo.}}(\mathcal{T} \rightarrow 2D_q^{(*)} \rightarrow e\mu + \dots) \\ = \sigma(pp \rightarrow \mathcal{T} + \dots) BF(\mathcal{T} \rightarrow 2D_q^{(*)} \rightarrow e\mu + \dots) \\ \leq \sigma(pp \rightarrow 2J/\psi + \dots) BF(\mathcal{T} \rightarrow 2D_q^{(*)} \rightarrow e\mu + \dots) \end{aligned} \quad (25)$$

The largest part of the signal (the total signal for  $J^{PC} = 2^{++}$ ) arises from the decay of  $\mathcal{T}$  into a pair of vector mesons. Vector particles decay promptly into a pseudoscalar plus a soft pion or photon(s) and contribute to the signal on the same basis as the pseudoscalars.

<sup>5</sup> The numerical wave functions of the  $J/\psi$  and  $0^{++}$  ground-state tetraquark can be fitted by simple harmonic oscillator wave functions,  $\Psi(\mathbf{r}) = \frac{1}{\pi^{3/4}} \alpha_{\text{ho}}^{3/2} e^{-\frac{1}{2}\alpha_{\text{ho}}^2 r^2}$  with  $\alpha_{\text{ho}, J/\psi} = 0.73 \text{ GeV}$  and  $\alpha_{\text{ho}, \mathcal{T}} = 1.3 \text{ GeV}$ . For  $\mathcal{T}$ ,  $\mathbf{r}$  is the distance of the c.o.m. of  $cc$  and  $\bar{c}\bar{c}$ . The wave function in the origin is:  $\Psi(0) = \frac{1}{\pi^{3/4}} \alpha_{\text{ho}}^{3/2}$ .

In conclusion, production in the  $4\mu$  channel and decay rates that we estimate for the  $cc\bar{c}\bar{c}$  tetraquarks are tantalizingly similar to the preliminary results presented by the LHCb Collaboration [2]. The meson-meson channel with the  $e\mu$  signature may provide an additional, complementary tool to identify and study the spectacular, exotic  $cc\bar{c}\bar{c}$  tetraquarks.

We thank Sheldon Stone, for an enlightening discussion and advice on a preliminary, March 2020, version of these notes, Michelangelo Mangano for an interesting exchange on the hadronization of the light quark pair and Liupan An for interesting correspondence on her seminar at CERN.

## Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

## References

- [1] C. Becchi, A. Giachino, L. Maiani, E. Santopinto, Phys. Lett. B 806 (2020) 135495.
- [2] R. Aaij, et al., LHCb, arXiv:2006.16957 [hep-ex].
- [3] M. Gell-Mann, Phys. Lett. 8 (1964) 214.
- [4] G. Zweig, CERNTH-401, An SU3 Model for Strong Interaction Symmetry and Its Breaking, 1964.
- [5] R.L. Jaffe, Phys. Rev. D 15 (1977) 267, <https://doi.org/10.1103/PhysRevD.15.267>.
- [6] S. Choi, et al., Belle, Phys. Rev. Lett. 100 (2008) 142001.
- [7] R. Aaij, et al., LHCb, Phys. Rev. Lett. 112 (22) (2014) 222002.
- [8] M. Ablikim, et al., BESIII, Phys. Rev. Lett. 111 (24) (2013) 242001.
- [9] A. Bondar, et al., Belle Collaboration, Phys. Rev. Lett. 108 (2012) 122001.
- [10] A. Esposito, A. Pilloni, A.D. Polosa, Phys. Rep. 668 (2017) 1.
- [11] R.F. Lebed, R.E. Mitchell, E.S. Swanson, Prog. Part. Nucl. Phys. 93 (2017) 143.
- [12] S.L. Olsen, T. Skwarnicki, D. Zieminska, Rev. Mod. Phys. 90 (2018) 015003.
- [13] F.K. Guo, C. Hanhart, Ulf-G. Meißner, Q. Wang, Q. Zhao, B.S. Zou, Rev. Mod. Phys. 90 (2018) 015004.
- [14] A. Ali, L. Maiani, A.D. Polosa, Multiquark Hadrons, Cambridge University Press, 2019.
- [15] Y.R. Liu, H.X. Chen, W. Chen, X. Liu, S.L. Zhu, Prog. Part. Nucl. Phys. 107 (2019) 237.
- [16] Y. Iwasaki, Prog. Theor. Phys. 54 (1975) 492.
- [17] K.T. Chao, Z. Phys. C 7 (1981) 317.
- [18] L. Heller, J.A. Tjon, Phys. Rev. D 32 (1985) 755.
- [19] N. Barnea, J. Vijande, A. Valcarce, Phys. Rev. D 73 (2006) 054004.
- [20] J. Vijande, A. Valcarce, J.M. Richard, Phys. Rev. D 76 (2007) 114013.
- [21] D. Ebert, R.N. Faustov, V.O. Galkin, W. Lucha, Phys. Rev. D 76 (2007) 114015.
- [22] A.V. Berezhnoy, A.V. Luchinsky, A.A. Novoselov, Phys. Rev. D 86 (2012) 034004.
- [23] J. Wu, Y.R. Liu, K. Chen, X. Liu, S.L. Zhu, Phys. Rev. D 97 (9) (2018) 094015.
- [24] W. Chen, H.X. Chen, X. Liu, T.G. Steele, S.L. Zhu, Phys. Lett. B 773 (2017) 247.
- [25] Y. Bai, S. Lu, J. Osborne, arXiv:1612.00012 [hep-ph].
- [26] Z.G. Wang, Eur. Phys. J. C 77 (2017) 432.
- [27] V.R. Debastiani, F.S. Navarra, Tetraquark, Chin. Phys. C 43 (1) (2019) 013105.

- [28] J.M. Richard, A. Valcarce, J. Vijande, Phys. Rev. D 95 (2017) 054019.
- [29] M.N. Anwar, J. Ferretti, F.K. Guo, E. Santopinto, B.S. Zou, Eur. Phys. J. C 78 (2018) 647.
- [30] J.M. Richard, A. Valcarce, J. Vijande, Phys. Rev. C 97 (3) (2018) 035211.
- [31] A. Esposito, A.D. Polosa, Eur. Phys. J. C 78 (2018) 782.
- [32] M.S. Liu, Q.F. Lü, X.H. Zhong, Q. Zhao, Phys. Rev. D 100 (1) (2019) 016006.
- [33] M.A. Bedolla, J. Ferretti, C.D. Roberts, E. Santopinto, arXiv:1911.00960 [hep-ph].
- [34] M. Karliner, S. Nussinov, J.L. Rosner, Phys. Rev. D 95 (2017) 034011.
- [35] L. Maiani, Sci. Bull. 65 (23) (2020) 1949–1951, arXiv:2008.01637 [hep-ph].
- [36] M. Karliner, J.L. Rosner, Phys. Rev. Lett. 119 (2017) 202001.
- [37] E.J. Eichten, C. Quigg, Phys. Rev. Lett. 119 (2017) 202002.
- [38] L. Maiani, A.D. Polosa, V. Riquer, Phys. Rev. D 100 (2019) 074002.
- [39] P. Junnarkar, N. Mathur, M. Padmanath, Phys. Rev. D 99 (2019) 034507.
- [40] E. Hernández, J. Vijande, A. Valcarce, J.M. Richard, Phys. Lett. B 800 (2020) 135073, <https://doi.org/10.1016/j.physletb.2019.135073>, arXiv:1910.13394 [hep-ph].
- [41] J.F. Giron, R.F. Lebed, arXiv:2008.01631 [hep-ph].
- [42] R. Aaij, et al., LHCb Collaboration, J. High Energy Phys. 1706 (2017) 047, Erratum: J. High Energy Phys. 1710 (2017) 068.
- [43] S. Godfrey, N. Isgur, Phys. Rev. D 32 (1985) 189.
- [44] V.B. Berestetskii, E.M. Lifshitz, L.P. Pitaevskii, Quantum Electrodynamics, Pergamon Press, 1982, p. 371; A more recent application to particle physics is found in R. van Royen, V.F. Weisskopf, Nuovo Cimento A 50 (1967) 617.
- [45] see e.g. D. Schildknecht, Acta Phys. Pol. B 37 (2006) 595.
- [46] M. Tanabashi, et al., Particle Data Group, Phys. Rev. D 98 (2018) 030001 and 2019 update.
- [47] J.E. Paton, H.M. Chan, Nucl. Phys. B 10 (1969) 516.
- [48] C. Becchi, A. Giachino, L. Maiani, E. Santopinto, arXiv:2006.14388 [hep-ph].
- [49] R.E. Cutkosky, J. Math. Phys. 1 (1960) 429; see also N. Cabibbo, L. Maiani, O. Benhar, An Introduction to Gauge Theories, 2018, p. 205.
- [50] R.P. Feynman, The Behavior of Hadron Collisions at Extreme Energies. High Energy Collisions: Third International Conference at Stony Brook, Gordon & Breach, N.Y., ISBN 978-0-677-13950-0, 1969, pp. 237–249.
- [51] T. Barnes, S. Godfrey, E.S. Swanson, Phys. Rev. D 72 (2005) 054026.