# FACTORS OF LEVEL AND <br> SPEED OF INTELLIGENCE 

BY<br>TAPIO NUMMENMAA

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TAPIO NUMMENMAA

ACADEMIC DISSERTATION
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## PREFACE

At the preliminary stage this investigation had two aims: to investigate the relationship between the level and the speed of performance in a simple intellectual task, and to study some problems which arise if the performance is scored by limen methods. The main study has been concerned with the problem of the relationship between speed and level, and the results are reported in this monograph.

The plan of the investigation was discussed with Professor Martti Takala, who later has also discussed several central problems connected with this work. He and Professor J. M. v. Wright have read the manuscript and made valuable comments. I have also had some useful discussions on several problems, especially those relating to the study of intelligence, with other people, in particular with Mr. Urpo Kauran$n e$ who also helped in the construction and administration of the tests.

A group of pupils of the Central Trade School of Central Finland served as subjects. This was made possible by the kind permission of the Rector of the school, Mr. Osmo Valtonen. Several teachers, in particular Mr. Ensio Seies and Mr. Veikko Vilkko helped in the practical arrangements.

A fellowship granted me by the Finnish Government made it possible for me to concentrate on the analysis of the data. The manuscript was written in the rooms of Library of the Institute of Pedagogics. The translation has been checked by Miss Anne Holden and by Professor J. M. v. Wright.

I would like to thank all the persons and institutions mentioned above for the help they have given to me. I also thank the Institute of Pedagogics which has included my book in its publication series as well as the Scientific Foundation of the City of Tampere for granting financial support for the publication of my study.

Jyväskylä, May, 1960.
Tapio Nummenmaa

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## CHAPTER I

## SPEED AND LEVEL COMPONENTS OF INTELLIGENCE: COMMON VIEWS

The first chapter presents in general terms the problem of the present study. Some earlier conceptions of the problem will also be considered. It seems to the present writer that theories concerning the relationship between the speed and the level components of intelligence have been based on unanalyzed assumptions and opinions more frequently, perhaps, than the hypotheses concerning some other aspects of intelligence. It may therefore be of some interest to review these theories briefly and in particular to consider the factor theories of intelligence from the point of view of the problem of speed and level components.

## Introduction

The purpose of the present study is to investigate the relationships between the speed and level components of intellectual performances, and to link this problem to the factor theory of intelligence.

The question of the interrelationship between the speed and the level of performance is an old one. It has been presented in different forms, but the formulation of the problem has generally been dependent on theories of intelligence. In general the earlier studies were concerned with the relationship of these two variables as measured in one and the same performance or in a few performances of different kinds, whereas later studies have more often formulated the problem as a factor analytical one. In the former kind of study the problem has been to find out, whether speed and level are really independent components of a performance or are characteristics dependent on each other. In the latter kind of study the main concern has been to find out, whether there is more than one speed factor.

The problems of speed and level have not only been a matter of theoretical interest, but they have also been important from the point of view of practical intelligence testing. It is, in fact, quite possible that the problem has arisen as a consequence of the use of time-limited tests. In the beginning when the testing was mostly carried out in individual situations the time allowed was usually not limited; but when group intelligence tests were devised it became necessary for practical reasons to introduce a time-limit. Then the speed of the subject became a factor which affected his score, and thus the problem of the relationship between the level and the speed of performance became important. More recently there has been a tendency to use even more speeded tests, at least in some areas of practical application. Thus, the problem of the relationship between the level and the speed components of tests has some bearing on the interpretation of the test results. This is the case in particular, if speed and level were to be independent components. It has been emphasized, especially by Davidson and Carroll (7) that a time-limited score in fact is a weighted composite of the level and speed components. When tests are used to make predictions concerning some criterion, this weighting by means of time-limits is not necessarily the best and is at most as good as would be obtained if both components were to be measured separately and weighted to produce the maximum amount of correlation with the criterion.

## Views on the relationship of speed and level

It is perhaps of some interest to notice that in early discussions on the relationship between speed and level two opposite hypotheses or views were presented and that the opinions on this problem have since then been divided into two corresponding categories. According to the first view the level of intellectual performance is completely independent of the speed of performance. Thus, a subject may be able to perform remarkable intellectual achievements independent of whether he is quick or slow. According to the second view speed is a really essential part of intelligence, so that there would be a high correlation between the speed and the level of performance.

Both of the views presented ahove are quite common. In fact, even those doing research work on this problem disagree as to which view is the prevailing one, as can be seen from the following two excerpts.
». . . yet popular opinion is almost unanimous in asserting that a separate ability for speed exists» (39, p. 293). „Ordinarily one assumes
that those who are able to solve difficult problems in a test will work more quickly than those who cannot» (29, p. 352). Several remarks of both kinds can be found.

It is difficult, perhaps even useless, to try to find out how these different views have arisen. One could imagine that the hypothesis of the independence of these components has arisen from the consideration that the slower subjects might suffer from time-limited conditions of testing. The opposite hypothesis perhaps has some basis in everyday observations: a quick person tends to be more efficient than a slow one, and thus it may appear that he would also have a higher level of performance.

## On the definitions of intelligence

It is quite natural, that the speed versus level controversy is met with in attempts to define intelligence. The definitions of intelligence are considered below primarily from this point of view.

In definitions of intelligence two main lines of thought can be distinguished. The first of these lays stress on the belief that intelligence is the ability to learn and make use of previously learned material, in particular in new situations. To this group belong the definitions of Stern, Thorndike and several others. On the other hand, there is a tendency to define intelligence operationally as the ability to perform tests.

It is obvious that the aspect of level tends in most cases to be included in the definition, that of speed being more easily disregarded. An inspection of the definitions of intelligence shows that in both groups definitions can be found with or without stress on the speed. Among those who define intelligence as learning or easy adaptation to new situations Stern (38, p. 424) for example does not make use of speed. His definition is as follows. »Intelligence is a personal capacity to adapt, with the appropriate aid of thinking, to new circumstances». On the other hand Kaila (20, p. 64) lays heavy stress on the speed. »In all so called »intelligence» there is essentially the question of time, i.e. of that speed with which a certain purposive successful reaction takes place. A low intelligence does not mean that a certain reaction, which later on can be transferred without practice, could not take place if enough time is used. To an intelligence of high quality it is essential that a reaction takes place in short time, perhaps as a simultaneous insight. Thus, it seems appropriate to take into account in the definition of
intelligence the speed of the learning process, too, i.e. the speed with which the intended form of reaction is ascertained. Thus one could define animal intelligence as follows. A learned reaction is more intelligent and shows more insight, the larger the area into which it can be transferred without training and the more suddenly this transfer can take place川. Though in the preceding definition Kaila speaks of animal intelligence it is obvious that he thinks in the same way of human intelligence. Kaila has been here quoted at length to show that in addition to including speed into the definition of intelligence he also finds it difficult to disregard the level of performance. In the definition above the area of the transfer has a particular bearing on this point. It seems possible that the tendency to think that speed is the most essential feature of intelligence arises from the fact that in learning experiments the level of performance is often held constant.

Among the definitions, in which intelligence has been characterized as an ability to perform tasks, definitions with more or less emphasis on speed can be found. Drever (8, p. 139) in his dictionary gives two definitions. One of these is as follows: nthe ability to perform tests or tasks, involving the grasping of relationships, the degree of intelligence being proportional to the complexity, or the abstractness, or both, of the relationships». Eysenck (9, p. 38) after a more general examination of intelligence tests answers the question of what they measure, in the speed versus power respect, as follows. »If properly constructed along analytical lines, they measure speed of mental functioning, which appears quite basic to intellectual efficiency».

The preceding definitions are only examples, but they show, however, that the opinions are divided into two categories. The speed element either is or is not included in the concept of intelligence.

## Speed and level in theories of intelligence and in intelligence testing

Though tests and measurements of human performances had been made before Binet's work, it was Binet who laid the foundation of intelligence testing. This was mainly due perhaps to the applicability of the tests presented by him. Binet's speculative conceptions of intelligence are not considered here, but it may be noted that he obviously does not lay stress on the speed of performance. It may be noted, too, that the scales presented by him and Simon (5) consisted of items of
different kinds. The conditions of testing did not involve speed or only to a very small extent, the items being of a power type and becoming more difficult towards the end of the test. This is especially the case with the 1908-scale, which introduces the concept of mental age. The scales, which have been built on the same principles as the Binet scale, and are used mainly in clinical practice, have, it would appear, usually been given without any time-limit, or, if such a limit is involved, it is quite liberal.

Thorndike (42) makes an explicit statement concerning speed in his system of description of intellectual performances. This intuitive system includes four attributes of intelligence. The first of these, level or altitude, is concerned with the degree of difficulty of the problems the subject can solve. The second feature, range, refers to the amount of tasks at any given degree of difficulty that the subject can do. The third attribute is called area, and it means the sum of all the ranges over all levels of difficulty. All of these three attributes seem to be quite close to each other. The last factor is the speed with which the subject can do tasks. Thorndike holds the opinion that this last factor, in case it should be independent of the altitude, should not be given too much emphasis in practical intelligence testing.

The two factor theory of intelligence was presented by Spearman (36) as early as 1904, before Binet had published his first scale. This theory may be regarded as the first attempt towards an objective, nonspeculative theory of intelligence. As the present review is primarily concerned with the problem of speed and level, it is sufficient to state that according to Spearman's theory there exists a general factor of intelligence, "g», and factors specific to different tasks, "s", the general factor being described as an indicator of mental energy. According to Spearman (37) this mental energy, which is the basis of the g-factor, may be made use of as the level of performance or as the speed of performance, all depending on the situation. He says: »On the whole, then, g has shown itself to measure a factor both in goodness and in speed of cognitive process . . » (37, p. 258). To this he adds further: »The almost unanimous view, that some persons who are on the whole unable to think quickly and yet are quite able to think clearly would seem to be a most grave error» (37, p. 258). To give support to his views Spearman presents empirical evidence, e.g. investigations made by May (24), Ruch and Koerth (33) and Bernstein (4). These will be considered in Chapter III.

The prevailing theory of intelligence is based on Thurstone's factorial
investigations (45, 46, 47, 49). As the earlier investigations had dealt mainly with the problem of whether only one general factor of intelligence existed or not, Thurstone developed new methods to study how many different factors were required to explain the observed relations between the different tests. The result was what is called the multiple factor theory of intelligence.

As to the problem of level and speed, it may be noted that Thurstone has not considered it in his factorial investigations. He has, however, considered the problem at the conceptual level (44). This conceptual analysis will be discussed in Chapter II.

Thurstone described the factors obtained in his studies in terms of stimulus content, and factor analysts have since then been interested mainly in factors interpretable in terms of stimulus content, other factors being thought of as "supplementary or unintentional» (53, p. 80). Nevertheless, factors interpreted as speed factors are met with in factor analytical studies. There are about fifteen or twenty factors of speed mentioned in the researches surveyed by French (12). French accepts the following as generally confirmed: fluency of expression, ideational fluency, perceptual speed, reaction time, speed of accociation, speed of judgment, word fluency, and speed. Lord (22) points out that there are factors, which involve speed but which are not described as speed factors, e.g. the number factor. Tests which have high loadings in this factor are usually tests of very easy calculations and are consequently highly speeded. This factor therefore could also be described as the number speed factor. It may be noted that speed is almost always of some importance in tests. Relatively short tests are in practice inevitable. This is the case in particular in factor analytical studies, in which the purpose is to measure many aspects of the subjects' performance and in which the time allowed for one test cannot as a rule be long. If a test is to be a good, reliable measure several items have to be included. Consequently the items have to be relatively easy and the speed will be of importance.

Ahmavaara (2) has made comparisons between different factorial studies to ascertain clearly, which factors have been best confirmed. He used exact methods of comparison previously developed by him (1). The following scheme presents the factors of the „first certainty class» of Ahmavaara (2, p. 131).

| Number | Reasoning | quantitative domain |
| :---: | :---: | :---: |
| Word fluency | Verbal | verbal domain |
| Space | Visual | visual domain |
| fluency | comprehension |  |
| performances | performances |  |
| Classification | actors of first cer |  |

The above arrangement of the factors is obviously related to the distinction between speed and level components, though it is difficult to state the relationship in precise terms. This difficulty is mainly due to the absence of precise measures of the degree of speeding of the tests on the basis of which the factors have been obtained. It is only kown that all the tests involved are given with time limits wich may be quite stringent.

Summing up the discussion on the speed and level problem in factor theories of intelligence, it may be stated first of all, that Spearman put forward the hypothesis that these two are different aspects of the same thing, i. e. of the $g$-factor. Thurstone did not consider this problem in his multiple factor studies. He used tests with quite stringent timelimits in his studies, but the description of factors was made only in terms of stimulus content. This same policy has been followed by most research workers since then. It may be pointed out that this has led some investigators to take these studies as implying that it makes little difference whether the speed or the level of the performance is measured. This way, one could say perhaps that Spearman's hypothesis still has supporters. Several factors have, however, been described as speed factors by different investigators. Ahmavaara's division of the factors into fluency factors and comprehension factors implies some difference in the speed involved in the different performances.

There are some investigations directly attempting to analyze level factors as distinct from speed factors, but these are reviewed and discussed in Chapter III.

## CHAPTER II

## SCORING OF PERFORMANCE WITH RESPECT TO LEVEL AND SPEED

In the first chapter the concepts of level and speed of intellectual performance have been used without any close definition. In this chapter consideration will be given to the possibilities of measuring these variables. Both the theoretical and practical sides of the measurement will be considered. It seems reasonable to discuss these matters in a separate chapter, because technical details are very important in studies concerning the correlation of speed and level: the result may essentially depend on the way in which the concepts have been defined and the corresponding variables measured. In the following only such concepts are discussed in scoring, that are relevant to the speed versus power problem. The conceptual analysis presented by Thurstone will be used as starting point in the discussion on the measurement of level of performance, as this analysis can also be used in the discussion on the measurement of speed of performance.

## Scoring of level of performance

The most fundamental requirement which must be fulfilled in studies concerning the relationship between level and speed is, of course, that the level scores are determined technically independently of speed, and correspondingly the speed scores have to be technically independent of the level scores.

Thurstone (44) in an analysis of the concept of power (which here is synonymous with level) describes an individual's performance in terms of a psychometric surface, which is presented as Figure 1.

In the figure an individual's performance is presented as a function of three variables, difficulty of task, D ; the response time allowed, T ; and the probability of success, P. Thurstone defines the individual's power as follows: „The ability of an individual subject to perform a specified kind of task is the difficulty E at which the probability is


A Psychometric Surface Showing an Individual's Performance, According to
Thurstone
Figure 1
$1 / 2$ that he will do the task in infinite time», (44, p. 251). It is, of course, possible that the power could he defined as some other level of difficulty than that at which the probability to obtain a correct solution in infinite time is $1 / 2$, but it is very probable that similar results would be obtained, in the sense that the different sets of level scores would correlate highly with each other. It is sometimes necessary to make the assumption that the scores at successive levels of difficulty have a perfect $(+1.00)$ correlation with each other. This assumption supposedly holds good for all practical purposes. However, it has been shown by Mosier (28) and Lorr (23) that the psychometric functions of different individuals differ with respect to the slope; thus it is possible that there may be subjects, whose location on the difficulty axis may be the same for $P=1 / 2$, but different for other values of $P$.

In practice the infinite time presupposed by the definition cannot be given to the subject. It can be suggested that »infinite time» would be taken as the „time the subject will take». Gulliksen (17) considers a necessary condition for level tests to be that every subject attempts every item. Thurstone in his writing suggests another way. The time given must not necessarily be infinite; the power score can be determined by founding »the value of D for the section, parallel to PT, whose cumulative frequency curve has an asymptotic limit of P that is ${ }^{1} / 2^{\prime \prime}$ (44, p. 251).

We shall now turn to examine a model for speed and power of per-
formance, which has been presented by Eysenck (9) on the basis of Furneaux' investigation. This model introduces the important concept of persistence. This model is presented in Figure 2.

The figure presents the performances of three subjects as functions of time and difficulty level. The solid lines show each subject's performance. The broken lines show what are thought to be the subjects' performances if infinite time had been allowed. Thus, it is supposed that all these persons would obtain the same result if they had enough time. The willingness to make use of time, to continue to search for an answer is called persistence. In this model power is presented as a compound of speed and persistence. However, to this model an important qualification is added, a factor of carelessness (Eysenck admits that the word perhaps is not very good). This carelessness would be the source of wrong answers.

It may be noted that persistence comes into play in different ways depending on the way the score in »infinite time» is determined. When this »infinite time» is taken as meaning the time the subjects will take, the individual differences in persistence cause the time taken by the subjects to be different. And correspondingly when the score in »infinite time» is determined by means of the procedure suggested by Thurstone, all subjects will not make use of the whole time allowed.

The elimination of the factor of persistence seems to be quite difficult. It could be suggested that the use of very difficult items should be avoided. In a study of Porebski (32) an attempt was made to measure the level of performance by means of a few difficult problems. The subjects could take the problems home and work on them. In a repeti-


The Performances of Three Individuals in a Test, According to Eysenck Figure 2
tion of Porebski's study made by Vincent (54) these scores were shown to be measures of persistence, in the sense that all subjects who had seriously tried to solve the problems had succeeded in it. Of course, it is possible that some other practical arrangement with difficult problems could remove the drawbacks of Porebski's procedure. One possibility of avoiding the complicating effects of persistence would perhaps be to obtain the absolute ability scores by the interpoletion procedure as proposed by Thurstone, and to work this out from the relatively short times allowed for solving the problems, so that all of the subjects would be willing to make use of the whole time.

We have now to try to compare the scoring procedure of Thurstone, as described earlier, with the usual procedure of test scoring, i.e. the counting of the number of right answers. Two questions have to be considered. First whether there is any difference between determining the scores in the way suggested by Thurstone, i. e., by founding the asymptote, and determining them on the basis of a psychometric function found on a plane parallel to the plane PD (in Figure 1) at a certain point (of mmedium» position) on the time axis. The hypothesis could be put forward that the scores obtained by the two methods have a high intercorrelation, as the changes in the form of the psychometric functions mentioned above become very small from a certain point onwards. Secondly, there is the problem of the relationship between the scores obtained by the methods adapted from psychophysics to test theory (and the use of which is assumed in Thurstone's. model) and the scores obtained by the usual method of counting the: number of correct answers. It is known that the methods adapted from psychophysics yield results very similar to those obtained by the usual scoring method of counting the number correct answers. This is shown to be true with respect to the constant process by Mosier (28), Lorr (23), and Nummenmaa (30). A method corresponding to some extent to the method of minimal changes, proposed by Glaser $(13,14)$, has also been shown to give the same results as the usual scoring procedure, if there are no »floor» or »ceiling» effects in the test.

Thurstone also dealt with the possible effect of motivation on the performance of the subject. He came to the conclusion that motivation is essentially a rate concept. Thus, ability would be independent of motivation, but motivation could affect the speed with which the subject is able to solve a problem. The scores determined by the process suggested by Thurstone would show scores of absolute ability independent of both motivation and speed. Guilford (15) remarks,
however, that there possibly exists an optimal level of motivation at each level of difficulty. It has been shown in experiments in the psychology of learning, that a very strong motivation may make the performances worse; in the same way one could suppose that a very strong motivation could also make intelligence performances worse. Guilford supposes that Thurstone's description holds good for a certain middle range of intensities of motivation, but not when very extreme motivations are in question. What makes the situation problematic, however, is that the motivation need not remain constant, but it can change as a function of time. It seems probable that after many unsuccessful trials the motivation would turn out to be negative and cause an aversion from the task.

In summary of the above discussion it is suggested that level scores can be defined as scores obtained when an infinite response time is allowed. Thurstone has proposed a procedure for the determination of level scores. This method will presumably give results similar to those obtained when (a) »infinite time» is taken to mean the time a subject will take when allowed to make use of time freely, and (b) the scoring is carried out by the usual summation method. It appears further to be important to hold the factor of persistence constant. To some extent this may be achieved by avoiding the use of extremely difficult items; in this case the changes in the subjects' motivation during the performance may not be very great either.

## Scoring of speed of performance

As the starting point in the discussion of the scoring of the speed of performance we can use Thurstone's model. If speed is defined analogously to power, it can be defined as follows: The speed with which an individual subject performs a specified task is the time $T$, at which the probability is $1 / 2$ that he will do the task of zero difficulty. Thus, the speed of a subject would be determined by tasks different from those determining his level. These tasks could be very simple discrimination problems. This definition, however, has several weak points in practice. It is not easy in practice to prepare items with zero difficulty, especially when the subjects are instructed to react very rapidly. Secondly, and this is more serious, the score obtained from tasks of zero difficulty could mean something entirely different from a score obtained from tasks of some level of difficulty other than zero. There are two reasons for this. First of all, in tasks of zero difficulty it is possible that the
time spent in performing the task mentally may be short compared with the time that is spent in communicating the answer. And secondly, it seems unreasonable to measure the speed and the level in entirely different tasks. It is known that simple discrimination tasks do not work as intelligence measures; thus the speed score obtained might not be the speed of intellectual performance, either.

It should be noted that the scoring need not necessarily be done by using psychophysical methods, the use of which is assumed in the definition above. It may be done by the usual methods of test scoring. The latter methods most probably will prove to be more easy to handle. This way a great number of items of zero difficulty are given to the subjects, who have a limited time to use in working on them. The speed score would then be the number of items accomplished in the time allowed. Gulliksen (17) defines a speed test as a test of items of zero difficulty.

If, however, we choose to obtain the speed scores by using items of a level of difficulty other than zero, new problems are met. First of all, there will be both wrong und right answers, and the experimenter has to make a decision whether he will make use of one or the other of these or of both. It is quite plausible that the response times are dependent on the accuracy of the response, and consequently right and wrong answers cannot be treated alike. Several suggestions have been made; first of all, one may use only the correct answers. In this case one may be sure that the subject really has performed the task mentally; in the case of wrong answers the time may be spent for instance in estimating the problem as too difficult and in turning to the next one. This policy has on the whole been preferred. When only correct answers are used to determine the speed scores, care must be taken that the difficulty of the tasks performed by the different subjects is the same. This has been achieved in different ways. The time has been taken only from such items as all the subjects have solved. This was done by for example McFarland (26) and Sutherland (39). It has also been worked out using several items at all levels of difficulty, and the speed at any level of difficulty has been determined as the average speed in the correct answers, whether there is one or more correct answers. This has been done by for example Nummenmaa (30). Speed scores that are combined from correct and incorrect answers have also been used. This way the speed scores are completely independent of the subject's accuracy. This method has been used by Tate (41), who took the speed scores in correct and incorrect answers as a deviation
from regression lines and combined the scores to give a speed score independent of the accuracy.

When the speed scores are determined from the correct answers, one additional difficulty is met: all subjects do not have correct answers at all levels of difficulty. The speed may then be measured at some restricted level of difficulty. In this way one obtains a speed score that gives the subject's speed at a certain level, but which is not necessarily the same as the subject's score at some other level of difficulty. It is possible, too, to obtain speed scores at several levels of difficulty and to combine these to give a speed score independent of level of difficulty. It may be assumed that speed scores at levels of difficulty not very far from each other correlate quite highly with each other, and thus it is possible to combine the scores. Of course, if very difficult levels of difficulty are considered, one finds that not every subject has correct answers and consequently, if speed scores really independent of level of difficulty are wanted, they must thus be determined from both correct and incorrect answers.

When speed scores are determined at a level of difficulty other than zero, one further inconvenience is met. It lies in the instruction. The subjects may aim at either speed or accuracy. This problem does not appear at zero level of difficulty, where the subject may be given instructions with regard to speed only. We here meet the problem of the nature of the different speed rates. We could say that the fastest rate at which the subject may work without loss of accuracy defines his ability of speed, but he may of course prefer some other rate of work, either a more rapid or a slower one. Concequently, it is difficult to say, whether a speed score obtained as a rate of working really means ability as defined above or something else, personal tempo for instance. Attempts have sometimes been made to avoid this problem by instructing the subjects to work as fast as possible. This however may lead to speed rates too fast for the subject and thus to an accuracy smaller than usual.

In measuring the speed, the size of the unit of performance measured has also some significance. In particular there is the question, whether the measurement should be made item by item or by taking the total time used in a test. It seems that if the measurement is done item by item, as was done for example by McFarland (26) and Tate (41), there is better control over the behaviour of the subject.

The question whether time-limit or work-limit methods should be used in the measurement of the speed of performance has been an
additional source of controversy. The time-limit method implies that the subjects are instructed to work a certain time and the amount of work done is used in measuring speed, the work-limit method implying that the subjects are instructed to perform a certain amount of work and the time elapsed is measured. It has been shown by Paterson and Tinker (31) that in a test of reading speed the two methods yielded similar results, the correlations between the different methods of measuring the speed being about equal to the reliability coefficients. Thus, a coefficient of correlation corrected for attenuation between time-limit and work-limit scores was +1.00 .

The nature of speed scores depends on matters which have been dealt with above. The experimenter may make his choice between the possibilities in different ways. Only, when the results are considered must the methods used in the measurements be taken into account. Speed scores of several kinds shall be dealt with in the following. In particular, two broad types of score will be of central importance. The first of these is obtained by using items of about zero difficulty. A number of such items is given to the subject, who has a limited time to work on them. He is instrtucted to work with the maximum speed. The scoring is made by counting the number of right answers, which is the usual practice in test scoring. The second type of score is obtained by using items of medium difficulty or items that become more difficult towards the end of the test. The items are possibly presented item by item. The time spent on each item (or on the whole test) is recorded in units of time. Either only the response times of the correct ansvers or the response times of both the correct and incorrect answers are used in scoring.

## CHAPTER III

## EARLIER INVESTIGATIONS

In this chapter earlier investigations of the problem of the relationship of speed and level components of intelligence performances are considered. We have to deal with several kinds of study. First of all, there are studies in which the relationship of speed and level has been studied using one test only. Secondly, there are factor analytical studies in which an attempt has been made either to find a general speed factor independent of a general ability factor or to discover, whether there are several factors of speed.

Studies concerning the relationship of speed and level components in one test

Correlations between speed and level components in one test are very frequently reported, and it would be an overwhelming task to review all of them. Tryon (51) gives a review of 11 studies made before 1931. Himmelveit (19) gives a quite extensive review of literature. Here only some studies will be considered.

We can find several methods of measuring speed in these studies. First of all, there are studies, in which a score obtained in a timelimited test is used as a measure of speed. It is not always claimed, however, that this would be a speed score in a strict sense. The purpose may have been for example to find out what effect the time-limit will have on the nature of the test scores, either for practical purposes of testing, as in the studies of May (24) and Ruch and Koerth (33), or with the purpose of studying the factor structure of such scores, as Davidson and Carroll (7) have done.

In the following Table I some correlations are shown between the scores obtained in time-limited situations on the one hand and in free situations or ones with a less stringent time-limit on the other.

## Table I

Correlations Between Scores Obtained with Different Time-limits

| Experimenter | Test r | $r$ between | r |
| :---: | :---: | :---: | :---: |
| May (24) | Army Alpha st | standard and unlimited time | . 965 |
| Ruch and Koerth (33) | * | " | . 945 |
|  | st | standard and double time | . 966 |
| Freeman (11) | N. I. T. st | standard and unlimited time | . 83 |
|  | Otis Advanced | " | . 58 |
|  | Terman Group | " | . 93 |
| Davidson and | Arithmetical Reasoning | g | . 80 |
| Carroll (7) | Same-Opposite | " | . 62 |
|  | Number Series | " | . 77 |
|  | Verbal Analogies | " | . 39 |
|  | Directions | " | . 57 |
|  | Disarranged Morphemes | es | . 78 |
|  | Letter Grouping | 》 | . 73 |

The results given in Table I are only a small sample, but perhaps a representative sample of the results of studies concerning the correlation between scores in a time-limited and a free situation, when tests containing difficult items are used and when the time- mit is the standard time-limit for the test in question. These corre ations are quite high, and this can be thought to be dependent on the fact that a part-whole relationship is in question. The time-limit score for this reason measures almost the same as the score obtained in an unlimited situation.

Correlations are sometimes found using a score in an easy test as a measure of speed and a score in a difficult test as a measure of level. This was done for example by Lord (22). From his study we have selected the correlations between different versions of a vocabulary test. The correlations are sho $\therefore \mathrm{n}$ in Table 2. Lord also used spatial and arithmetical tests, obtaining results very similar to those presented in Table 2.

## Table 2

Correlations Between Different Versions of a Vocabulary Test, According to Lord

| 1 | Vocabulary, Level | 1 | 2 | 3 | 4 | 5 | 6 | 7 |
| :--- | :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 2 | $"$ | $"$ | .669 |  |  |  |  |  |
| 3 | $"$ | Moderately speeded | .706 | .690 |  |  |  |  |
| 4 | $"$ | Highly speeded | .620 | .648 | .660 |  |  |  |
| 5 | $"$ | $"$ | .693 | .697 | .745 | .775 |  |  |
| 6 | $"$ | $"$ | .641 | .650 | .700 | .757 | .855 |  |
| 7 | $"$ | Last item attempted | .324 | .343 | .393 | .531 | .671 | .609 |

It is seen, that though the correlations between the different speed tests are perhaps a little higher than those between the level and the speed tests, all these correlations are quite high. The correlations between the level and the last-item-attempted variables are lower. Thus it seems that if speed and level are measured in terms of the amount of items correctly solved, whether these are difficult or easy, these two variables correlate quite highly.

A third way to measure speed has been to take the time spent in solving a problem or doing a task. Some correlations between level and speed as obtained by this measure are shown in Table 3.

Table 3
Some Correlations Between Level Measures and Working Rate Measures of Speed
Experimenter
Davidson and Carroll (7)

Tate (41)

Nummenmaa (30)

| Test | r |
| :--- | :---: |
| Arithmetical Reasoning | .44 |
| Same-Opposite | .33 |
| Number Series | .48 |
| Verbal Analogies | .34 |
| Directions | .42 |
| Disarranged Morphemes | .14 |
| Letter Grouping | -.070 |
| Arithmetical Reasoning | .008 |
| Number Series Completion | -.071 |
| Spatial Relations | -.025 |
| Sentence Completion | -.02 |
| Cube Test |  |

Davidson and Carroll used the time spent to work through the test as a measure of speed, whereas Tate used a score obtained on the basis of the response times to the individual items. This score was independent of the accuracy of the answer. Nummenmaa used the mean time used in correct responses as a speed score. The correlations between the time used on the test form and level of performance as given by Davidson and Carroll are certainly lo ser than the correlations between scores in time-limited and free situations as given by Table 1. The results of experiments, in which the speed has been measured in terms of the time used for each item show about zero correlations to level measures.

In conclusion, some of the main results obtained in the above mentioned and in similar studies will be summarized; the discussion on pp. 65-68 is relevant to the interpretation of these results.

Measures of speed and level in a test correlate quite highly when a score obtained in a time-limited situation is used as measure of speed, at least if the time-limit in question is a »standard» time-limit of the test. Speed measures that have been determined by using the number of correct ansvers in an easy test correlate also quite highly with level scores. It should be noticed that it seems to be difficult to obtain very easy tests which would completely satisfy Gulliksen's criteria (17). Errors tend to appear through carelessness. The speed measures of this kind correlate somewhat higher with each other than with level measures. Working-rate measures of speed give much lower correlations with level scores than the measures mentioned above. When speed is measured as the time to work through the answer sheet or as the number of items attempted in an easy test (disregarding accuracy) the correlations will be small and positive; whereas when speed is measured taking the working time item by item the correlations will be about zero.

## Factor analytical studies on the problem of level and speed

As the number of factor analytical studies on the problem of level and speed is not very great, it is possible to describe briefly the studies that are most relevant to the present investigation.

Bernstein's (4) study was planned to investigate the problem of whether there exists a group factor of mental speed. He presented two groups of tests to his subjects, who were school children. The first group of tests was given in conditions, in which there was enough time to work; the second group was given in conditions in which the time was limited. These two groups of tests were referred to as leisure tests and haste tests. In addition to these test measures of intelligence, ratings of the slowness and intelligence of the subjects made by the teachers were obtained. No specific group factor was found. Both the leisure tests and the haste tests correlated in about the same way to the ratings of intelligence and slowness. A variable which was derived as a difference between the scores in leisure and haste conditions failed to show any correlations apart from zero to any other variables. This investigation was part of the evidence used by Spearman against the assumption that there is a mental speed factor independent of the $g$ factor.

McFarland's (26) study is also one of those which support the view that speed und level of performance are positively correlated. His investigation consisted of three parts. In the first experiment 11 differ-
ent kinds of test material were presented to 4 subjects. The test materials varied greatly with respect to the complexity of tasks; there were measures of reaction time as well as of intelligence. The tests were presented individually and timed item by item. The speed scores were obtained keeping accuracy constant, which was done by excluding all items which any of the subjects had done wrong. It was found that the rankings of the subjects with respect to speed were similar in the different tests, with only two exceptions. It was also found that the relative rankings with respect to the level tended to be about the same as the rankings with respect to the speed. The level scores were determined as the number of correct responses. The rankings were exactly the same when total measures of level and speed were considered. The second part of the investigation was for the most part similar to the first one, only a greater number of tests and items was used. 15 different tests were presented to 5 subjects in conditions similar to those used in the first part. Speed rankings in different tests correlated very highly with each other, with the exception of one test, a simple mathematical test. Also, the relative rankings with respect to speed were quite similar to those with respect to level. In the third part of the experiment a larger number of subjects, 34 in number, was used. These subjects were given 10 different tests. The correlations between the different speed measures were obtained and the order of tests was so arranged as to form a hierarchy. The hierarchy was explained with reference to the general factor of Spearman. Thus, in this investigation a general factor was found in speed measurements, and the correlations between the speed and level measurements were found to be positive.

Dahlgren (6) has made a factor analysis on the basis of the data presented by McCall (25). Seven tests of simple performances with stringent time-limits were included in Dahlgren's analysis, also four verbal tests, one arithmetical test, one general intelligence test, and one variable of school marks. Four factors were extracted. These were interpreted on the basis of the rotated matrix of loadings. There was a factor of intelligence, in which the variables of intelligence and the school marks and to some extent the verbal tests were loaded, and a factor of speed (in simple tasks), in which all of the seven time-limited tests of simple performances had loadings. In addition to these, there were two other, less clear factors, which were interpreted as factors of carefulness and of rated performances. This last factor had loadings in variables of intelligence rating and school marks.

Slater (35) presented to his subjects CAVD tests and some other
tests of intelligence. CAVD tests were presented in a group situation with no time-limit. The conditions were so arranged that each of the subjects, who were school children, could by means of a large clockapparatus time his own performance putting down himself the time at which he started each item. The subjects were instructed to use as much time as they wanted. In the speed measurements only the times for correct solutions were used. The level scores were obtained in the usual way from the number of right answers. In addition to the CAVD tests five non-verbal intelligence tests were used. These tests were presented in time-limited situations. Several groups of subjects were used. The directions test of the CAVD series was finally left out altogether and every group was scored on two speed and two level variables in CAV tests and on some non-verbal intelligence tests, a maximum total of not more than eight variables. The analysis of the results was carried out by means of Spearman's tetrad analysis. As a result of this analysis Slater concludes that there is a special factor which probably is verbal in nature in the C, A and V level measures, and that there is a factor of speed, which he calls the factor of speed preferences in the speed-rate measures. The intercorrelations between the level and the speed-rate measures, and the correlations of both of these with the non-verbal intelligence tests, and the intercorrelations of these nonverbal tests were due to one common factor $g$. The speed-rate measures had very low loadings in the $g$-factor.

Myers (29) prepared 100 items to a non-verbal reasoning test. These items were printed on ten pages, ten items on each page. As the test was presented, it was divided into five parts with a time-limit of 12 minutes in each part. The parts consisted of one to three pages of items. Thus, the parts were differently speeded. In order to control practice and fatigue effects, three forms of the test were prepared, the order of pages and the division of pages into the different parts of the test being different. The subjects were midshipsmen of a naval academy. Nineteen scores were obtained for each form of the test. These were: the number of correct answers on each of the ten pages, the number of correct answers in each two page part (there was one in each form) and the number of correct, incorrect, skipped and attempted items in each three page part (there were two in each form). In addition to these, scores were obtained on several criterion variables. The subjects were graded in the academy on seven courses and in addition to these, they got an average score of all the grades. A factor analysis of the intercorrelations of the variables was carried out by means of Thurstone's grouping meth-
od and it revealed two factors. In the first factor the first pages of al! parts of the test had loadings, and in the second factor the last pages. of the speeded parts had loadings. Thus, the first factor was identified as an ability factor and the second one as a rate-of-answering factor. The criterion scores had higher loadings in the ability factor than in the rate-of-answering factor, in which the loadings were very close to zero.

Porebski's (32) battery of tests was planned to include two speed measures and one power measure of verbal, spatial, and numerical abilities. The speed tests contained very easy items, the power tests consisted of 2-3 difficult problems. The subjects were allowed to take these problems home and use time freely in solving them. As a result of the factor analysis two factors emerged. These were interpreted as speed and level factors. On these grounds Porebski suggests what he calls a triad theory of intelligence. According to this theory, intelligence would consist of three factors, a general power factor, a general speed factor, and a specific speed factor.

Vincent (54) has criticized Porebski's work on several grounds. He claims that the speed tests are not necessarily tests of verbal, spatial, and numerical abilities. These tests, especially the verbal and spatial tests present the same tasks only with different symbols; there are for example a test of verbal analogies and a test of picture analogies. Thus, Vincent claims, it seems natural that these tests have loadings in one factor only. To reveal the nature of Porebski's power factor Vincent repeated the experiment with the same tests and conditions. Subsequent interviews with the subjects revealed that almost everybody, who had seriously tried to solve the power problems, had succeeded. The factor analysis resulted in the same factors as were found in $P_{0}$ rebski's study. Vincent gives a different interpretation, however. He interpretes Porebski's speed factor as a factor of general intelligence and the previous power factor as a factor of persistence.

Sutherland (39) has dealt with several problems concerning the speed factor. He tried to find out whether there is a speed factor independent of the general factor $» g »$, and made an attempt to clarify the nature of the speed of intellectual performances. In the first part of his experiment two measures of level and three measures of speed were obtained. Scores obtained in Kuhlmann-Anderson group tests of intelligence and Drever-Collins performance tests were used as level measures. The performance tests were given in standard conditions with certain time-limits in different tests. The total score in the test was taken
as being the level measure. To obtain the speed measures, the times were taken for each test solved in the given time-limit. Only such tests, that had been solved correctly by all or nearly all of the subjects were taken into consideration. The speed scores were determined for tests of block-design, cube-construction, and form board. The average of the total correlations between the speed measurements was .406, and when the level component was partialled out (using both Kuhl-mann-Anderson and Drever-Collins measures) it was .116. It is concluded that no speed factor independent of level exists. In the second part of the experiment the subjects were instructed to work on some problems of moderate difficulty with the greatest possible speed and accuracy. Five different tests were given with stringent time-limits to obtain speed scores. The Otis Advanced examination was used to find a level score. The average intercorrelation of four of the speed measurements (one was excluded because of negative correlations with the other four) was .38 , and when level was partialled out it was .30. Sutherland concludes that this might mean that there exists a speed factor in simple intellectual tasks. To investigate this further, a group of tests of a still easier level of difficulty was presented to a group of subjects. The average intercorrelation of these tests was .26 , and when level (Otis score) was partialled out it was .21. Thus, a factor of speed separate from level is taken to be involved in these performances. On the basis of an inspection of the intercorrelations of the variables Sutherland argues that both the factors of level and the factors of speed are determined by the same general factor "g». In the third part of the study Sutherland shows that the speed measurements and the number of moves in the performance tests correlate very highly with each other. The three measures of speed earlier mentioned correlate with the number of moves in the corresponding tests as highly as $.70-.90$. Kohs (21) has earlier reported a corresponding correlation of .7 in a block test. As a final conclusion in the speed versus level problem Sutherland suggests that the preferred rates of performance might be independent of the level of performance, but the ability for speed would not. Only in problems of a low level of difficulty would a speed factor come into operation.

Ruoppila (34) has studied the problem of the influence of different. time-limits on the factor structure of tests. The battery was planned to include tests of memory, verbal, numerical, visual, and reasoning abilities. The method of obtaining scores with different time-limits was to ask the subjects to work with pencils of different colours in the differ-
ent phases of testing. Factor analyses were carried out on the basis of two sets of scores: scores with »optimal» time-limits and scores with very stringent time-limit. In the former case, on the basis of graphical orthogonal rotation five factors were interpreted, those of memory, verbal, visual, reasoning and numerical abilities. With Ahmavaara's cosine method of rotation the reasoning factor dropped out. In the latter case the following factors were interpreted: speed, visual, verbal and memory. In the speed factor there are tests of numerical and verbal nature and it is characterized as the speed of reading easy materials and as the speed of answering easy items quickly. Ruoppila identifies this factor with the speed factor described by French (12, p. 241242). The change in the factor structure of tests as a function of timelimits was studied by special methods. There are changes in many individual tests. For example, a test of block counting at an easy level of difficulty and highly speeded seems to have equal loadings in numerical and visual factors, but at a more difficult level it has much more loading in the visual factor than in the numerical factor.

Davidson and Carroll (7) in a factor analytical investigation obtained three kinds of score from the same tests. Speed scores were obtained "as the number of seconds taken by the subject to work from the beginning to the end of the test, attempting every item once». The second type of scores were level scores, which were defined »as the number of items correctly answered when the subject is allowed to take all the time he desires to try every item and to check over his work». Thirdly, „time-limit scores were defined as the number of items correctly answered within a prescribed time limit» (7, p.415). The battery of tests was planned to include measures of verbal, numerical and reasoning factors, as well as a measure of perceptual speed and a measure of speed of reading. Some tests were discarded from the final analysis, among these the measure of perceptual speed which had low correlations with the other variables. Only the level and the speed scores were included in the factor analysis; the correlations of time-limit scores being analyzed separately and related to the main analysis by means of special methods. As a final result six factors emerged, which after an oblique rotation were interpreted as being a general speed factor, a level of reasoning and a specd of reasoning factor, two verbal factors which included level and time-limit variables, and a numerical factor which included speed, time-limit and some level variables.

Tate (41) has in an experimental investigation dealt with several problems concerning mental speed. His study was made to determine
whether there are individual differences in speeds of response to mental test materials of varying degrees of difficulty, and whether there is a factor of speed that is independent of altitude and of the task in which it is measured. He used four tests; these were tests of arithmetical reasoning, number series completion, sentence completion and spatial relations. The tests were presented in an individual situation and the subject's speed of response was measured separately for each item. The results were handled by means of an application of the analysis of variance. Tate found, first of all, that when the level of difficulty of the items and the accuracy of the response effects were controlled, there were extremely significant individual differences in the speed of response. This seems quite natural, though it has probably never been proved with equal rigour. In the second place, it was found when the accuracy of the response effect was controlled, that the subjects who were fast at one level of difficulty tended to be fast at other levels of difficulty, and that the subjects who were fast in one function tended to be fast in the other functions (tests). The intraclass coefficients of correlation, i.e. the fractions of the total variance of the true speed estimates ascribable to between-subject differences were substantially greater in the three levels of difficulty used in the study than they were in the four tests. This was interpreted as meaning that in addition to a general speed factor there exist special abilities of speed, which are linked to different kinds of tasks. As the intraclass coefficients of correlation between the speed scores at different levels of difficulty averaged to about .70 only, Tate concluded that there possibly might be co-variation of speed with altitude at the upper levels of difficulty. The correlations between measures of speed of response independent of accuracy and measures of altitude which were obtained by means of scaled items, were about zero in all of the four tests, as has already been shown in Table 3 on page 26.

Lord (22) in a factor analytical study of high precision gave his subjects, who were students in the first class of a naval academy, a battery of tests which was planned so as to include measures of verbal, spatial and arithmetic reasoning abilities. There were seven tests of each factor; i.e. two level, one moderately speeded and three highly speeded tests and one additional slightly speeded test. Six referencefactor tests of the factors of number speed, perceptual speed and word fluency were also included in the study, as well as several measures of the performance in courses in the naval academy. The correlations between the test variables were obtained as product moment coefficients
using normalized scores. The factor analysis was carried out by Lawley's maximum likelihood method. The interpretation of factors was done on the basis of a rotated factor matrix. The following factors were found: verbal factor, space factor, mathematical-reasoning factor, numberspeed factor, perceptual-specd factor, verbal-specd factor, spatial-speed factor, a factor of academic grades and a verbal-academic-grade factor. In the factors of verbal, spatial and mathematical reasoning abilities both tests of level and of speed had loadings, but in the speed factors only the speeded tests had loadings. The primary-vectors of the four speed factors were positively correlated, thus showing a second order factor of speed. Scores based on the last item attempted were more pure measures of speed than were the number-of-correct-answers variables in the speed tests. Correlations between course grades and speed factors were positive, though not as high as the correlations between course grades and the factors also involving level measures. Lord concludes that the speeds play some part in the success of the subject in a course, which can be measured in tests of a highly speeded nature only.

In conclusion some of the main results of the factor analytical studies reviewed above will be summarized. The interpretation of these results will be further discussed on pp. 65-68.

It is difficult to give a concise summary of the outcome of the factor analytical studies under discussion because of the multiplicity of results and views. Several factors account for this multiplicity. First of all, differences in plans and purposes of the investigations naturally cause the results to be different. Secondly, differences in measures, in particular in speed measures, lead to different results. It was shown earlier (on pages 24-27) that different speed measures behave differently. And thirdly, there are differences that are due to technical weaknesses.

We will first consider Spearman's two factor theory. This theory supposes that level and speed are the same thing. The investigations of McFarland (26) and Bernstein (4) seem to give support to this view. In these investigations only one general factor was found and it was identified with "g».
Secondly, there is the »triad theory» of Porebski (32). This theory assumes one general level and one general speed factor and, in addition, specific speed factors. This study was criticized by Vincent (54), who argues that the level factor of Porebski is a persistence factor and criticizes the assumption that level and speed are independent of each other. Distinct factors of level and speed have been found, however, in
other investigations as well, e.g. in the investigation by Dahlgren (6) on the basis of McCall's (25) data and also by Myers (29).

Then, there are studies which suggest that speed and level could be correlated highly on more difficult levels of difficulty and less on easy levels of difficulty, as for example the study of Sutherland (39). This hypothesis is also presented by Tate (41). In addition, there are a great number of factors interpreted as factors of speed in simple tasks of different kinds, as is seen in the survey of French (12), and in the results presented by Ruoppila (34), who found a speed factor in which easy tests had loadings.

In the study of Davidson and Carroll (7) one general speed factor and one specific speed factor plus several level factors were found. Tate (41) by means of an application of the analysis of variance was able to show that there possibly can be found several factors of speed.

And finally, we have results of yet a different type in the investigation of Lord (22). He found several speed factors independent of stim-ulus-content factors (but not of level factors).

On the basis of the above-mentioned results it seems likely that at least under some conditions several speed factors will be found. The nature of these speed factors will depend upon the kind of the measurement of the speed variables. It can be suggested, that when the speed measures are determined on the basis of working-rate, the speed factors will be distinct from level factors, and when the speed measures are determined by means of the number of correct answers in very easy speeded tests the speed factors will be distinct from stimulus-content: factors.

# CHAPTER IV 

## THE PRESENT INVESTIGATION

The purpose of the present investigation has been to carry out a factor analytical study on the basis of speed and level measurements made in nine different tests and in two different situations. The tests were so selected, that several factors, whether level or speed, would be likely to appear. The tests are, in fact, some of the factor tests most commonly used, and they were selected to represent three factors identified with great confidence. The main question in the investigation is: will the speed and level measurements have loadings in the same or in different factors and what will these factors be?

## The experimental situations

There were two experimental situations: an individual situation and a group situation.

In the individual situation the tests were presented in the same order to all the subjects. The order is indicated by the numbers of tests $1-9$ in Table 12. The tests were presented item by item. The answer of the subject was recorded as was the time used in every item. The time was taken with a stop watch to the accuracy of one second. In the verbal and numerical tests the answer was given orally, in the visual tests it was drawn on small papers containing one item each. The time was measured from the moment the subject got the item to the moment the answer was finished. In the general instruction the experimenter said that he was going to give the subject some tasks to do and that there would be a great number of these taking a long time (which was about 4-8 hours). This was necessary to keep the motivation as constant as possible during the test situation; in the psychology on work and effort it has been shown that the rate of work is dependent on the hours of work, and consequently the subject has to know something about how long the experiment will take to adapt a rate of work suited
for the situation. To keep the factor of persistence as constant as possible the subjects were told to perform every item only once. In the case of numerical items this would mean that the subject should perform the calculation through and give the result without checking. The meaning of performing every item only »once» is less clear in the verbal and visual items, but in these an answer given was taken as the final one. In the visual items in particular there is the possibility of checking whether the response meets the requirements given, but the subjects were encouraged not to analyze the solution after completing it, and when an item was finished the experimenter gave the next one. The subjects were told, that if some item turned out to be too difficult, they could leave it and take the next. This possibility was, however, used to a very small extent. The testing was interrupted in the ordinary lunch-hours and recesses of the school. It may be noted that no statement was made by the experimenter in favour of aiming at either speed or level of performance. It was, of course, plain to the subjects that both of these features are desirable and they could themselves decide, which way they wanted to weight each of these two. After these general instructions the experiment began. Specific instructions were given in connection with every test.

The group situation was arranged in order to measure the speed of performance in a different way, as an ability to perform quickly easy items. Two versions were prepared for each of the tests, one very easy and the other quite difficult. There were two parallel easy tests of each kind for the determination of reliability. In the general instructions the subjects were told that they would be given a group of tests. One part of the tests would be very easy and in these speed is required, while the other part of the tests would consist of difficult tests in which speed would play practically no part. The speed tests were given first, after which there was a pause, and then the level tests were given. The order of the presentation of the tests is indicated by the numbers of tests in Table 12.

## The subjects

The subjects were pupils of a trade school, the Central Trade School of Central Finland. Two groups of subjects were needed, one for the individual situation and another for the group situation. Pupils from nine different courses were selected to form the groups. There are great differences in the level of intellectual performance of the pupils of differ-
ent courses, according to the opinion of the rector of the school. The division of the courses into experimental groups was made in such a way that pupils studying courses of similar kinds were placed in the same group. Thus, the experimental group for the individual situation consisted of pupils studying courses in filer-mechanics, the mechanics of agricultural machines, and motor-car mechanics. The number of subjects in this group was 63. The experimental group for the group situation consisted of pupils following courses in cabinet making, carpentry, bricklaying, cement-working, iron-plate working, and plumbing. The number of subjects in this group was 70 . The testing was carried out in the working hours of the pupils, and the subjects were, as far as the experimenter is aware, ready to take part in the experiment.

## Description of the tests

The battery includes several tests, which have often been used in factorial experiments. In the following description a sample item is given from each test. These items also give some impression of the degree of difficulty of the tests; precise information concerning the difficulty of the tests is presented in Tables 4 and 5.

Punched holes. In this test successive drawings show one, two or three folds made in a square sheet of paper. In this paper one or two holes are punched. This is shown in a drawing. The subject has to show where the holes would be in the opened sheet. There were 40 items in the individual situation, $2 \times 10$ items in the speed test of the group situation, and 20 items in the power test of the group situation. An item of about $50 \%$ level of difficulty is shown in Figure 3. This test has earlier been used by Thurstone (45), Fruchter (12, pp. 98-99), Harrell (12, pp. 109-113), Michael, Zimmermann and Guilford (12, pp. 132-133), and Woodrow (12, pp. 196-199). This test has been identified as having loadings in factors of visualization or space.

Minnesota paper form board. In this test several small figures are shown to the subject and he has to illustrate by pencil lines how these figures could be made to fit together and form a square. There were 40 itcms in the individual situation, and $3 \times 8$ itcms in the speed test, and 24 items in the power test of the group situation. An item of about 50 \% level of difficulty is shown in Figure 4. This test has earlier been used by Chein (12, pp 72-73), Garrett (using Anastasi's data; 12, pp. 101102), and Harrell (12, pp. 109-113). It seems to be a measure of the space factor.


Figure 3


Figure 4


Figure 5

Piece test. In this test a drawing is shown to the subject and he has to indicate how this figure could with one straight line be divided into two or three parts, which could then be made to fit together to form a square. An item of about $50 \%$ level of difficulty is shown in Figure 5. There were 40 items in the individual situation, $3 \times 8$ items in the speed test and 24 items in the power test of the group situation. This test has been used by for example Takala, Siro and Toivainen (40), and it seems to be a measure of the space factor.

These three tests, Punched holes, Minnesota paper form board and Piece test were supposed to have loadings in the same factor.

Verbal analogies. In this test the subject has to discover a word missing in a verbal proportion so that in meaning the relation of the first word to the second will be the same as that of the third word to the fourth. There were 40 items in the individual situation, $2 \times 15$ in the speed test and 30 in the power test of the group situation. An example:
room: suite of rooms = cow: $\qquad$
This test has been used in a great many studies, e.g. in those of Thurstone (45), Chein (12, pp. 72-73), Carroll (12, pp. 79-81) and (12, pp. 74-78), Langsam (12, pp. 126-128), and Woodrow (12, pp. 196-199). This test obviously is a measure of the verbal factor.

Synonyms. In this test the subject had to choose a word out of four alternatives, the meaning of which was the same or almost the same as the meaning of a given stimulus word. There were 40 items in the individual situation, $2 \times 21$ in the speed test and 42 in the power test of the group situation. An example:
big, wide, tall, large, long
This test has been used by Thurstone and Thurstone (49) and Taylor (12, pp. 143-145). In the former study this test had a loading in the factor of word fluency, in the latter it had loadings in the factors of perceptual speed and fluency of expression. It may be supposed that if the test were made to consist of difficult items, it would be more highly loaded in the verbal factor.

Word grouping. In this test groups of five words are shown to the subject and he has to find out which of the words does not belong to the same category as the other four. There were 40 items in the individual situation, $2 \times 20$ in the speed test and 40 in the power test of the group situation. An example:
bark, gaiety, speech, cheep, growl

This test has been used by Thurstone (45, 46). In the former study, (45), this test had loadings in the factors of perceptual speed and in the verbal factor, in the latter study, (46), it had a loading in the verbal factor.

These three tests, Verbal analogies, Synonyms and Word grouping were supposed to have loadings in the same factor. The tests of synonyms and of word grouping were given as multiple-choice tests because it was thought to be more practical in the present study.

Addition. In this test the subject was asked to do additions of two figures in the individual situation, and additions of three figures in the group situation. The calculation was carried out without using paper and pencil. There were 40 items in the individual situation, and 75 in the speed test and 30 in the power test of the group situation. The following is an example of an item of about $50 \%$ level of difficulty; $12716+35681$. Tests of addition have been used in different forms in several studies, e.g. by Thurstone (45, 46), Coombs (12, pp. 82-85), and Woodrow (12, pp. 196-199). These tests have had loadings in the number factor.

Multiplication. In this test the subject had to perform the multiplication of two figures without using paper and pencil. There were 40 items in the invidual situation, 75 in the speed test and 30 in the power test of the group situation. An example of an item of about $50 \%$ level of difficulty; $19 \times 12$. This test has been used in different studies, e.g. Thurstone (45), Bechtold (12, pp. 62-67), Garrett (using Schneck's data; 12, p. 100), and Coombs (12, pp. 82-85). This test seems mainly to have loadings in the number factor.

Division. In this test the subject performed ordinary divisions without pencil and paper. There were 40 items in the individual situation, and 75 items in the speed test and 30 in the power test of the group situation. An example of $50 \%$ level of difficulty; 3816: 72. The test of division has been used in studies of Thurstone $(45,46)$. This test has had loadings in the number factor.

These three tests, Addition, Multiplication and Division were supposed to have loadings in the same factor.

## Hypothesis for the study

The hypothesis for the study follows from three assumptions, all of which have been substantiated in earlier studies. First of all, we assume that in factor analytical studies quantitative, verbal and visual domains (to use Ahmavaara's terminology, see page 15) are found independent of each other. Secondly, we assume that if speed measurements are
made in terms of working rate, speed factors will be independent of level factors. And thirdly, we assume that if speed measurements are made in terms of the number of correct answers in easy highly speeded tests, there can be found speed factors independent of factors which are describable in terms of stimulus content and in which both the level and speed measurements will have loadings.

Thus, it is suggested, that a factor analysis of the correlations of the variables of the individual situation will produce six factors, corresponding to the six possible arrangements of three values of stimulus content (quantitative, verbal, and visual) and two values of response quality (speed, level). A factor analysis of the correlations of the variables of the group situation should also produce six factors, but with the difference that instead of three level factors, three factors containing both level and speed variables and thus describable in terms of stimulus content only should be found.

## The scoring

The level scores of the tests of the individual situation were obtained simply by counting the number of correct responses. As all subjects performed all items in this situation, the level scores fulfill the requirements presented by Gulliksen (17, p. 231). The usual summation method was used, as no additional advantage can be obtained by using more complex methods as was indicated in Ch. II.

In the scoring of the power tests of the group situation the procedure of counting the number of right answers could not directly be applied, because the tests do not fulfill the requirement that all subjects have attempted every item. In the group situation it was necessary to use time-limits which, though liberal, caused some items to remain wholly unattempted. For this reason only items which all or almost all subjects had attempted were taken into consideration in the scoring (skipped items, i. e. items passed over, being regarded as attempted items). The level scores were then obtained in the usual way by counting the number of right answers. Some characteristics of these tests are shown in Table 4.

Coefficients of reliability for the level scores of the tests of the individual as well as of the group situation were obtained by using the split-half technique. These coefficients are shown in Tables 8 and 9.

The scoring of the speed of performance was more elaborate, especially in the individual situation. For scoring purposes the items
of each test of the individual situation were first divided into four levels of difficulty. At each level of difficulty two divisions, i. e. two splitted halves, were obtained for the determination of coefficients of reliability. As each test contained 40 items, in each such division there were five items. The means of percentages of correct answers of the items of different levels of difficulty in the divisions are shown in Table 5.

Several types of speed score were now tried, first of all in one test only, i. e. Punched holes. The first type of speed score was obtained in the following way. For each subject the mean response time of the correct responses was obtained at each level of difficulty, and the sum of these was the subject's score. As many subjects had no correct solutions at the most difficult level, only the mean response times of correct responses of the three first levels were added up to make the score. This method may have one drawback, however, and it is that in this way the more difficult levels of difficulty are weighted more, because the time spent in finding solutions is usually greater in more difficult items. Even though it could be assumed that the correlations of speeds at successive levels of difficulty are perfect, this is not the case in practice because of errors of measurement. And it may be that the errors of measurement are greater at higher levels of difficulty, because the number of correct responses there is smaller and thus the number of measurements smaller. For this reason a second type of speed score was obtained by normalizing the mean response times at each level of difficulty before adding them up.

In the preceding two types of score use is only made of the correct responses. As it was thought possible to increase the reliability by using also the speed of response of the incorrect solutions, two other speed scores were computed. The third type of score resembles closely that of the first type. In this case the mean response times of both the correct and incorrect solutions obtained at all levels of difficulty were added together to make the score. There was one question to be settled, however. In case the subject did not have responses of each kind at some level of difficulty, the response time of the existing kind of responses was taken twice. This way the responses at all four levels of difficulty could be used. In this procedure it is assumed that the response times of correct and incorrect answers would be the same, which always is not the case as will be shown later (on pages 7579). As the effect of the accuracy of response on the response time is relatively small as compared with the individual differences, this source of error does not presumably affect the scores much. The fourth
and last type of score was obtained by normalizing the mean response times of both the correct and incorrect answers before adding them up.

The coefficients of reliability of the different kinds of speed scores in Punched holes test are shown in Table 6, and the coefficients of correlation between these scores are shown in Table 7. It may be seen that the coefficients of reliability are about equal and that the coefficients of correlation are about equal to these reliabilities. Thus, for all practical purposes the scores seem to be similar. It was, however, decided to carry out the scoring in two ways, i. e. those numbered 1 and 4 in Table 6 . Of these the score of type 1 is thought to be, in principle, the best one, but it was thought that for variables with lower reliability the inclusion of the speed of responding in the incorrect solutions might have an effect of raising the reliability.

The scoring of the speed was more straightforward in the case of the speed tests of the group situation. The speed tests were scored by counting the number of correct answers in accordance with the usual practice. This was also the procedure adopted by Lord (22). Some characteristics of these speed tests are given in Table 4. It may be pointed out that the errors made in these speed tests are almost all careless errors. In designing the items of these tests the results of the tests of the individual situation were used as a basis. The percentages of correct solutions to items given in both situations were as a rule much higher in the individual situation, in which the instruction did not explicitly mention speed, than in the group situation, in which an instruction concerning speed was given.

The split-half reliability coefficients of the speed scores of the individual situation are shown in Table 8. The number of subjects is indicated in connection with the speed scores of type 1, because a few of the subjects did not have any correct solutions at some level of difficulty in both halves. It may be noted that the coefficients of reliability of the speed scores of type 4 are in general higher than those of type 1 . The coefficients of reliability of the speed tests of the group situation as calculated on the basis of parallel tests are given in Table 9.

## Factor analyses

As the speed scores had been obtained in two ways in the individual situation, two matrices of intercorrelations of the variables were obtained. Both of these contain the same level variables but different speed variables. There are 17 variables in both cases, as the level
variable of the Word grouping test was unreliable and had to be left out. The correlations vere calculated as product-moment coefficients from normalized scores. The matrices of intercorrelations are given in Tables 10 and 11. The intercorrelations between the different tests of the group situation were obtained in the same way as the productmoment coefficients from normalized scores. This matrix is given in Table 12.

Table 4
Some Characteristics of the Tests in the Group Situation

|  | Level tests |  | Speed tests |  |
| :---: | :---: | :---: | :---: | :---: |
|  | The proportion of unattempted items included in the test | The proportion of correct solutions of items included in the test | The proportion of all unattempted items | The proportion of correct solutions of attempted items |
| Minnesota | . 010 | . 39 | . 11 | . 82 |
| Synonyms | . 006 | . 65 | . 24 | . 91 |
| Addition | . 022 | . 68 | . 63 | . 88 |
| Punched holes | . 007 | . 40 | . 27 | . 83 |
| Word grouping | . 020 | . 55 | . 35 | . 84 |
| Multiplication | . 009 | . 73 | . 57 | . 90 |
| Piece test | . 001 | . 51 | . 14 | . 75 |
| Verbal analogies | . 007 | . 58 | . 31 | . 76 |
| Division | . 010 | . 71 | . 43 | . 95 |

## Table 5

The Levels of Difficulty of the Tests in the Individual Situation. Mean Pergentages of Correct Solutions in the Divisions.

| Level of difficulty | Punched holes | Piece test | Minnesota | Addition | Multiplic. | Division | Synonyms | Word group | Verba anal. |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | 2 | 12 | 12 | 12 | 12 | 12 | 12 | 12 | 12 |
| I | 9696 | 9796 | 9999 | 9594 | 9698 | 9696 | 9898 | 9595 | 8488 |
| II | 8385 | 8485 | 8887 | 7979 | 5958 | 8587 | 8888 | 7574 | 6464 |
| III | 6869 | 6162 | 6362 | 5759 | 3231 | 6970 | 7171 | 4748 | 4748 |
| IV | 4242 | 2829 | 2222 | 2019 | 1010 | 3941 | 5048 | 2423 | 2931 |

Table 6
Coefficients of Reliability of the Different Speed Score Variables in the Punched Holes Test

1. Sum of the mean times used in correct responses at difficulty levels 1-3.

Rel. S—B corrected
2. Sum of the normalized mean-time-scores of correct responses at difficulty levels $1-3$.
3. Combined score of the mean times used in correct and incorrect responses at difficulty levels $1-4$.
4. Combined score of the normalized mean-time-scores of correct and incorrect responses at difficulty levels $1-4$.

Table 7
Coefficients of Correlation Between Different Speed Score Variables in the Punched Holes Test. The Variables Are Numbered as in Table 6.

|  | 1 | 2 | 3 | 4 |
| :---: | :---: | :---: | :---: | :---: |
| 1 |  |  |  |  |
| 2 | .88 |  |  |  |
| 3 | .88 | .85 |  |  |
| 4 | .86 | .87 | .92 |  |

## Table 8

The Coefficients of Reliability of the Tests in the Individual Situation

|  | Level scores |  |  | Speed scores, type 1 Speed scores, type 4 |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | N | Rel. | Rel. | N | Rel. | Rel. | N | Rel. | Rel. |
|  |  |  | S-B |  |  | S-B |  |  | S-B |
| Punched holes | 63 | . 57 | . 73 | 60 | . 83 | . 91 | 63 | . 83 | . 91 |
| Piece test | 63 | . 66 | . 80 | 60 | . 61 | . 76 | 63 | . 75 | . 86 |
| Minnesota | 63 | . 66 | . 80 | 59 | . 68 | . 81 | 63 | . 78 | . 88 |
| Addition | 63 | . 66 | . 80 | 56 | . 72 | . 84 | 63 | . 8.3 | . 91 |
| Multiplication | 63 | . 76 | . 86 | 61 | . 39 | . 56 | 63 | . 76 | . 86 |
| Division | 63 | . 67 | . 80 | 60 | . 67 | . 80 | 63 | . 68 | . 81 |
| Synonyms | 63 | . 33 | . 50 | 58 | . 85 | . 92 | 63 | . 85 | . 92 |
| Word grouping | 63 | . 05 | . 10 | 57 | . 68 | . 81 | 63 | . 81 | . 90 |
| Verbal analogies | 63 | . 37 | . 54 | 56 | . 50 | . 67 | 63 | . 62 | . 77 |

Table 9
The Coefficients of Reliability of the Tests in the Group Situation. $N=70$, Except in the Synonyms Level, in Which it is 69.

|  | Level | tests | Speed | tests |
| :--- | :--- | :--- | :--- | :--- |
| Rel. | Rel. S—B | Rel. | Rel. S—B |  |
| Minnesota | .39 | .56 | $.43^{1}$ | .69 |
| Synonyms | .45 | .62 | .65 | .79 |
| Addition | .64 | .78 | .72 | .83 |
| Punched holes | .50 | .66 | .54 | .70 |
| Word grouping | .53 | .69 | .70 | .82 |
| Multiplication | .43 | .60 | .79 | .88 |
| Piece test | .61 | .76 | $.43^{1}$ | .69 |
| Verbal analogies | .49 | .66 | .57 | .73 |
| Division | .74 | .85 | .67 | .80 |

${ }^{1}$ ) The mean of the intercorrelations of three parallel tests

The extraction of factors was carried out by Thurstone's centroid method. The communalities were estimated by the centroid formula given by Thurstone (48, p. 300). In deciding when the factorization should be stopped Ahmavaara's (52) suggestion was used, i.e. that one should extract as many factors as are certainly enough, and then take those factors as real, which can be given an interpretation. The point when the factors that can be interpreted have been extracted may be determined by some rough rule, according to Ahmavaara, who says that factorization should in general cease, when the loadings in the last factor are smaller than 20 . A second basis for ceasing the factorization was in this particular case the hypothetized number of factors, which was six. On these grounds, six factors were extracted from the matrices of the individual situation. In the analysis of the matrix of the group situation it was obvious after the fifth factor that no more interpretable factors could emerge, and consequently only five factors were extracted. The orthogonal factor matrices are given in Tables 13, 14, and 15.

The rotations were at first performed graphically with orthogonal axes in all three analyses. A fairly simple structure was obtained. The rotated factor matrices of the three analyses are given in Tables 16 , 17 , and 18.

Table $10^{\circ}$
Matrix of Intercorrelations of the Variables of the Individual Situation. Speed Scores Are of Type I. The Table Also Gives Sixth Factor Residuals.

|  |  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 Punched holes | level |  | . 00 | -. 01 | -. 02 | -. 01 | . 03 | -. 03 | . 00 | -. 01 | . 04 | -. 03 | . 00 | . 03 | . 00 | . 00 |  | -. 02 |
| 2 Piece test | » | 3 |  |  | -. 09 | -. 04 | . 05 | -. 06 | . 08 | . 03 | -. 10 | . 03 | . 02 | .C1 | . 03 | . 02 |  | -. 06 |
| Minnesota | " | . 41 | 0 |  | . 02 | . 02 | -. 04 | . 05 | -. 02 | . 00 | . 00 | -. 01 | . 04 | -. 01 | . 00 | -. 02 | . 00 | . 0 |
| Addition | " | . 20 | . 13 | . 26 |  | . 05 | -. 02 | . 00 | -. 04 | . 03 | . 04 | -. 04 | -. 02 | -. 64 | . 03 | . 01 | -. 03 | . 0 |
| 5 Multiplication | " | . 40 | . 32 | . 38 | 45 |  | . 00 | . 06 | . 02 | -. 06 | . 09 | -. 01 | . 01 | .C.5 | . 00 | -. 04 | . 00 | . 0 |
| 6 Division | " | . 40 | . 25 | . 33 | . 33 | . 48 |  | . 03 | -. 01 | . 01 | -. 04 | . 04 | -. 01 | .C2 | . 02 | . 01 |  | -. 03 |
| 7 Synonyms | " | . 17 | . 17 | . 35 | -. 02 | . 20 | . 07 |  | . 04 | . 04 | . 08 | . 04 | -. 04 | . 44 | -. 01 | . 0 | -. | -. 0 |
| 8 Verbal analogies | " | . 31 | . 44 | . 28 | . 12 | . 47 | . 18 | . 40 |  | -. 03 | -. 04 | . 01 | . 04 | -. C 5 | . 03 | -. 01 | . 01 | . 0 |
| 9 Punched holes | spee | -. 11 | . 09 | -. 1 | -. 27 | -. 08 | -. 30 | . 14 | . 08 |  | -. 04 | . 06 | . 00 | . 13 | -. 02 |  | -. 01 |  |
| 0 Piece test | " | -. 11 | -. 18 | -. 2 | -. 28 |  | -. 30 | . 13 | -. 02 | 49 |  | -. 03 | . 02 | -. 11 | -. 03 | -. 05 | . 07 |  |
| 1 Minnesota | " | -. 16 | -. 07 | -. 29 | $-.30$ |  | -. 12 | -. 13 | . 04 | . 53 | . 44 |  | . 00 | . 11 | . 07 | . 05 |  | -. 0 |
| 12 Addition | " | -. 09 | -. 11 | -. 08 | -. 12 | . 27 | . 00 | . 05 | . 13 | . 33 | . 38 | . 40 |  | . 14 |  | -. 02 | . 00 |  |
| 13 Multiplication | " | -. 14 | -. 20 | -. 19 | $-.12$ |  | -. 02 |  | -. 14 | . 15 | 19 | . 21 | . 52 |  | -. 03 | -. 03 |  | -. 03 |
| 4 Division | " | . 18 | . 07 | . 15 | . 07 | .49 | . 31 | . 18 | . 33 | . 06 | . 15 | . 31 | . 53 | . 28 |  |  | -. 03 | . 03 |
| 5 Synonyms | * | -. 21 | . 02 | -. 15 | -. 15 | -. 08 | $-.28$ | . 23 | . 06 | . 40 | . 28 | . 21 | . 39 | . 33 | . 15 |  | . 04 | . 04 |
| 6 Word grouping | " | -. 13 |  | -. 16 | -. 04 |  | -. 17 | . 09 | . 08 | . 46 | . 39 | . 23 | . 41 | . $\leq 1$ | . 06 | . 74 |  | -. |
| 7 Verbal analogies | " | -. 05 | -. 02 |  | -. 08 |  | . 1 |  |  | . 32 | . 35 | 1 | 3 | 2 | . 1 | 45 |  |  |

Table 11

Matrix of Intercorrelations of the Variables of the Individual Situation. Speed Scores Are of Type 4. The Table Also Gives Sixth Factor Residuals.

|  |  |  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | Punched holes | level |  | . 01 | . 01 | -. 05 | -. 03 | . 02 | -. 01 | . 00 | . 00 | -. 02 | -. 02 | . 01 | . 04 | -. 04 | -. 03 | . 02 | . 03 |
| 2 | Piece test | " | . 33 |  | . 08 | -. 03 | -. 02 | . 02 | -. 05 | . 03 | . 00 | -. 08 | -. 02 | . 00 | . 05 | -. 01 | . 07 |  | -. 02 |
| 3 | Minnesota | " | . 41 | . 40 |  | . 04 | -. 01 | -. 03 | -. 01 | -. 04 | . 05 | . 00 | -. 05 | . 11 | . 01 | -. 04 | . 03 | -. 04 | . 01 |
| 4 | Addition | " | . 20 | . 13 | . 26 |  | . 08 | -. 01 | . 01 | -. 04 | . 00 |  | -. 01 | -. 04 | -. 02 | -. 04 | -. 02 | . 06 | -. 01 |
| 5 | Multiplication | " | . 40 | . 32 | . 38 | . 45 |  | -. 01 | . 03 |  | -. 05 | . 11 | . 03 | . 01 | -. 02 | . 00 | -. 03 | . 03 | . 02 |
| 6 | Division | " | . 40 | . 25 | . 33 | . 33 | . 48 |  | . 07 | -. 01 | . 01 | -. 08 | . 00 | . 04 | -. 03 | . 05 | . 01 | -. 01 | . 00 |
| 7 | Synonyms | " | . 17 | . 17 | . 35 | -. 02 | . 20 | . 07 |  | . 07 | . 05 | . 04 | . 02 | -. 06 | . 00 | -. 01 | -. 01 |  | -. 03 |
| 8 | Verbal analogies | " | . 31 | . 44 | . 28 | . 12 | . 47 | . 18 | . 40 |  | -. 03 | -. 01 | . 00 | . 00 | -. 06 | . 08 | . 03 | -. 02 | . 02 |
| 9 | Punched holes | speed | -. 07 | . 09 | -. 10 | -. 27 | -. 04 | -. 17 | . 16 | . 00 |  | -. 01 | . 03 | . 02 | . 08 | -. 03 | -. 03 |  | -. 05 |
|  | Piece test | * | -. 11 | -. 14 | -. 06 | -. 23 | . 09 | -. 26 | . 21 | -. 05 | . 55 |  | . 02 | -. 05 | -. 02 | . 00 | -. 08 | . 07 | . 06 |
|  | Minnesota | " | -. 07 | . 07 | -. 27 | -. 21 | . 09 | -. 10 | -. 02 | . 00 | . 66 | . 45 |  | -. 01 | -. 05 | . 05 | . 05 | . 01 | . 01 |
| 12 | Addition | " | -. 09 | -. 11 | -. 12 | -. 16 | . 21 | -. 08 | -. 02 | . 06 | . 49 | . 43 | . 41 |  | . 04 | -. 03 | -. 01 | . 00 | . 06 |
|  | Multiplication | * | -. 11 | -. 22 | -. 10 | -. 21 |  | -. 17 | . 12 | -. 16 | . 33 | . 47 | . 11 | . 64 |  | . 00 | . 02 | . 03 | -. 03 |
|  | Division | * |  | -. 03 |  | -. 11 | . 32 | . 12 | . 15 | . 16 | . 32 | . 46 | . 34 | . 55 | . 54 |  | . 08 | -. 03 | . 01 |
|  | Synonyms | " | -. 13 | . 03 | -. 06 | -. 12 | -. 04 | -. 23 | . 16 | . 00 | . 46 | . 36 | . 36 | . 40 | . 31 | . 36 |  | -. 05 | . 02 |
|  | Word grouping | * | -. 12 | -. 02 | -. 27 | -. 04 |  | -. 26 | . 09 | . 00 | . 57 | . 47 | . 46 | . 53 | . 34 | . 28 | . 61 |  | -. 03 |
|  | Verbal analogies | " | . 02 | -. 08 |  | -. 02 |  | -. 08 |  | -. 05 | . 36 | . 51 | . 26 | . 45 | . 35 | . 39 | . 66 | . 54 |  |

Table 12
Matrix of Intercorrelations of the Variables of the Group Situation. The Table Also Gives Fifth Factor Residuals.

|  |  |  | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | Minnesota le | level |  | . 06 | . 01 | -. 05 | . 02 | . 00 | . 01 | . 11 | . 02 | . 04 | -. 05 | . 01 | -. 06 | -. 09 | . 00 | . 05 | . 07 | -. 02 |
| 2 | Synonyms | " | . 03 |  | . 00 | -. 11 | . 00 | . 08 | $-.01$ | -. 14 | $-.05$ | . 01 | . 14 | -. 12 | . 01 | -. 07 | . 05 | . 02 | . 03 | . 00 |
| 3 | 3 Addition | " |  | -. 06 |  | -. 01 | . 09 | -. 04 | . 04 | . 10 | . 04 | -. 01 | . 01 | $-.03$ | $-.05$ | . 00 | . 06 | -. 03 | -. 03 | -. 04 |
| 4 | Punched holes | " | . 34 | . 02 | . 08 |  | . 07 | . 04 | . 00 | -. C 2 | -. 01 | -. 04 | -. 02 | . 06 | . 04 | . 05 | . 01 | $-.02$ | . 03 | -. 06 |
| 5 | Word grouping | » | . 13 | . 25 | . 08 | . 24 |  | . 01 | $-.02$ | -. 12 | . 02 | -. 12 | $-.03$ | . 02 | $-.09$ | . 05 | . 00 | . 04 | -. 04 | -. 08 |
| 6 | Multiplication | " | . 11 | . 24 | . 16 | . 16 | . 09 |  | . 09 | -. 13 | $-.02$ | -. 05 | . 02 | . 03 | . 02 | . 01 | . 09 | . 02 | -. 11 | . 00 |
| 7 | Piece test | " |  | -. 03 | . 15 | . 56 | -. 05 | . 18 |  | -. 013 | . 010 | . 07 | . 09 | . 00 | -. 04 | $-.01$ | -. 04 | . 02 | $-.04$ | . 01 |
|  | Verbal analogies | 》 | . 10 | . 35 | . 02 | . 27 | . 36 | . 20 | . 05 |  | . 01 | . 05 | -. 03 | . 04 | . 00 | $-.03$ | -. 04 | . 00 | . 09 | . 03 |
| 9 | Division |  | -. 02 | . 03 | . 23 | . 05 | . 11 | . 26 | . 00 | . 18 |  | -. 02 | . 01 | . 02 | . 02 | . 05 | $-.02$ | . 00 | . 03 | -. 01 |
| 10 | Minnesota sp | speed | . 50 | -. 09 | . 05 | . 37 | . 00 | $-.17$ | . 45 | . 09 | $-.24$ |  | $-.01$ | . 05 | . 05 | . 02 | . 00 | -. 02 | . 07 | . 06 |
| 11 | Synonyms | " | . 05 | . 3 | -. 07 | . 34 | . 26 | $-.05$ | . 31 | . 29 | . 05 | . 23 |  | $-.05$ | . 01 | . 05 | . 09 | -. 03 | -. 14 | -. 05 |
| 12 | Addition | " | . 29 | -. 08 | . 20 | . 25 | . 30 | . 20 | . 05 | . 29 | . 31 | . 26 | . 15 |  | . 02 | . 03 | -. 03 | -. 06 | . 08 | . 02 |
| 13 | 3 Punched holes | " | . 45 | -. 03 | . 12 | . 48 | -. 03 | . 13 | . 42 | . 09 | -. 05 | . 54 | . 09 | . 21 |  | . 08 | -. 01 | . 03 | -. 07 | . 02 |
| 14 | 4 Word grouping | " | . 10 | . 16 | . 07 | . 35 | . 49 | . 09 | . 03 | . 42 | . 28 | . 24 | . 51 | . 55 | . 19 |  | -. 03 | -. 03 | -. 04 | -. 02 |
|  | 5 Multiplication | 》 | . 10 | . 02 | . 31 | . 16 | . 13 | . 32 | . 05 | . 13 | . 50 | -. 03 | . 26 | . 48 | . 00 | . 40 |  | -. 02 | . 00 | $-.01$ |
| 16 | 6 Piece test | " |  | -. 08 | . 05 | . 39 | . 05 | $-.02$ | . 45 | . 00 | $-.09$ | . 43 | . 17 | . 08 | . 45 | . 10 | . 05 |  | . 01 | . 04 |
| 17 | 7 Verbal analogies | ) » | . 21 | . 40 | -. 02 | . 49 | . 34 | . 08 | . 22 | . 63 | . 25 | . 21 | . 35 | . 36 | . 07 | . 49 | . 28 | . 15 |  | . 04 |
| 18 | Division | ) | . 15 | . 00 | . 18 | . 18 | . 13 | . 18 | . 14 | . 23 | . 41 | . 15 | . 19 | . 53 | . 13 | . 44 | . 63 | . 19 | . 36 |  |

## Table 13

Orthogonal Factor Matrix: Analysis of the Variables of the Individual Situation. Speed Scores Are of Type I.

|  | I | II | III | IV | V | VI | h $^{2}$ |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 1 | .27 | .52 | .08 | .03 | .02 | .20 | .39 |
| 2 | .33 | .39 | .33 | -.09 | -.21 | .07 | .43 |
| 3 | .27 | .58 | .22 | -.08 | .23 | .17 | .55 |
| 4 | .09 | .46 | -.15 | -.35 | -.18 | -.08 | .40 |
| 5 | .65 | .47 | -.27 | -.12 | -.22 | .05 | .78 |
| 6 | .22 | .57 | -.31 | -.12 | .05 | .16 | .51 |
| 7 | .37 | .17 | .36 | .11 | .27 | -.16 | .41 |
| 8 | .49 | .41 | .19 | .29 | -.16 | -.32 | .66 |
| 9 | .40 | -.52 | .26 | .26 | -.22 | .15 | .64 |
| 10 | .33 | -.51 | .06 | .31 | -.05 | .11 | .48 |
| 11 | .30 | -.45 | -.19 | .39 | -.24 | .16 | .56 |
| 12 | .59 | -.38 | -.36 | .06 | .13 | -.08 | .65 |
| 13 | .31 | -.39 | -.33 | -.15 | .23 | -.11 | .44 |
| 14 | .61 | .10 | -.43 | .22 | .21 | -.04 | .66 |
| 15 | .46 | -.53 | .26 | -.24 | .12 | -.31 | .73 |
| 16 | .57 | -.51 | .20 | -.43 | -.21 | -.16 | .88 |
| 17 | .41 | -.35 | .12 | -.20 | .15 | .21 | .41 |

Table 14
Orthogonal Factor Matrix: Analysis of the Variables of the Individual. Situation. Speed Scores Are of Type 4.

|  | I | II | III | IV | V I | VI | h $^{2}$ |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 1 | .27 | .52 | .01 | -.05 | .05 | .14 | .37 |
| 2 | .29 | .47 | .28 | -.07 | -.22 | .09 | .44 |
| 3 | .27 | .60 | .11 | .17 | .30 | .10 | .57 |
| 4 | .07 | .44 | -.21 | -.31 | .08 | -.13 | .36 |
| 5 | .56 | .52 | -.31 | -.12 | -.10 | .05 | .71 |
| 6 | .17 | .54 | -.29 | -.20 | .07 | .25 | .51 |
| 7 | .37 | .22 | .27 | .39 | .14 | -.11 | .44 |
| 8 | .38 | .48 | .13 | .16 | -.29 | -.17 | .53 |
| 9 | .56 | -.47 | .31 | -.07 | -.19 | .22 | .72 |
| 10 | .52 | -.48 | .09 | .21 | .08 | .19 | .60 |
| 11 | .43 | -.41 | .17 | -.19 | -.37 | .34 | .67 |
| 12 | .60 | -.46 | -.33 | .08 | -.22 | -.10 | .75 |
| 13 | .40 | -.47 | -.38 | .36 | .15 | -.05 | .68 |
| 14 | .61 | -.22 | -.33 | .19 | .04 | .12 | .58 |
| 15 | .53 | -.43 | .25 | -.17 | .21 | -.26 | .67 |
| 16 | .54 | -.50 | .14 | -.27 | -.08 | -.29 | .72 |
| 17 | .59 | -.35 | .06 | -.20 | .43 | -.14 | .72 |

Table 15
Orthogonal Factor Matrix: Analysis of the Variables of the Group Situation

|  | I | II | III | IV | V | h $^{2}$ |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
|  | .48 | -.44 | .10 | .23 | .04 | .49 |
| 1 | .22 | .31 | -.32 | .17 | -.28 | .35 |
| 2 | .23 | -.06 | .36 | .05 | -.06 | .19 |
| 3 | .65 | -.31 | -.21 | -.14 | -.15 | .60 |
| 4 | .39 | .27 | -.22 | .26 | .09 | .35 |
| 5 | .29 | .15 | .27 | .13 | -.42 | .37 |
| 6 | .47 | -.51 | -.08 | -.27 | -.21 | .60 |
| 7 | .51 | .33 | -.28 | .19 | -.21 | .53 |
| 8 | .32 | .39 | .38 | -.20 | -.15 | .46 |
| 9 | .42 | -.54 | -.17 | .14 | .31 | .61 |
| 10 | .48 | .12 | -.44 | -.20 | .20 | .52 |
| 11 | .59 | .19 | .30 | .17 | .30 | .59 |
| 12 | .46 | -.55 | .05 | .19 | -.06 | .56 |
| 13 | .67 | .35 | -.14 | .10 | .31 | .70 |
| 14 | .53 | .34 | .48 | -.28 | .15 | .73 |
| 15 | .39 | -.49 | -.05 | -.12 | .13 | .43 |
| 16 | .39 | .26 | -.35 | -.08 | -.15 | .67 |
| 17 | .67 | .23 | .34 | -.20 | .20 | .58 |
| 18 | .58 |  |  |  |  |  |

Table 16
Graphically Rotated Orthogonal Factor Matrix: Analysis of the Variables of the Individual Situation. Speed Scores Are of Type $I$.

|  | I | II | III | IV | V | VI | $\mathrm{h}^{2}$ |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 1 | -.16 | .21 | .35 | -.19 | .04 | .40 | .39 |
| 2 | .01 | .16 | .50 | .03 | -.19 | .33 | .42 |
| 3 | -.10 | .04 | .46 | -.38 | .02 | .42 | .54 |
| 4 | .11 | .47 | .22 | -.29 | -.18 | .02 | .40 |
| 5 | .12 | .65 | .44 | -.03 | .25 | .27 | .77 |
| 6 | -.08 | .48 | .17 | -.38 | .15 | .26 | .50 |
| 7 | -.03 | -.25 | .55 | -.05 | .18 | .09 | .41 |
| 8 | -.24 | .20 | .72 | .17 | .12 | .00 | .66 |
| 9 | .17 | -.23 | .06 | .69 | .18 | .20 | .63 |
| 10 | .11 | -.21 | -.06 | .55 | .34 | .09 | .49 |
| 11 | .00 | .06 | -.20 | .62 | .37 | .08 | .57 |
| 12 | .34 | .10 | .04 | .25 | .68 | -.07 | .66 |
| 13 | .42 | .00 | -.10 | .03 | .48 | -.18 | .45 |
| 14 | -.01 | .29 | .22 | .00 | .73 | .07 | .67 |
| 15 | .61 | -.33 | .30 | .28 | .22 | -.20 | .74 |
| 16 | .79 | -.05 | .28 | .41 | .06 | -.04 | .88 |
| 17 | .47 | -.20 | .05 | .15 | .25 | .26 | .42 |

In the following interpretation loadings that are above an arbitrary limit .30 are taken into account. The results given in Table 16 will first be considered.

The following tests have loadings in the first factor.
Word grouping speed . 79
Synonyms speed . 61
Verbal analogies speed . 47
Multiplication speed . 42
Addition speed . 34
This factor seems to be a speed factor. The highest loadings are certainly the loadings of the speed variables of the Word grouping test and of the Synonyms test. Therefore, the factor can perhaps be named Verbal speed. Both of these tests were multiple-choice tests, but one can notice, that the Verbal analogies speed variable has also a loading in this factor.

The following tests have loadings in the second factor.
Multiplication level . 65
Division level . 48
Addition level . 47
Word grouping speed - 33
This factor clearly is a factor of Number level. This as a pure number factor, because the tests presuppose only manipulation of numbers.

In the third factor the following tests have loadings.
Verbal analogies level . 72
Synonyms level . 55
Piece test level . 50
Minnesota level . 46
Multiplication level . 44
Punched holes level . 35
The third factor is a level factor, in which both the verbal tests and the visual tests plus one number test have loadings. This factor could be called Verbal-and-visual level.

In the fourth factor the following tests have loadings.
Punched holes speed . 69
Minnesota speed . 62
Piece test speed . 55
Word grouping speed . 41
Minnesota level -. 38
Division level -. 38

All the visual tests' speed variables have high and positive loadings in this factor, as well as one additional speed variable. The factor may be called Visual speed.

In the fifth factor the following tests have loadings.
Division speed . 73
Addition speed . 68
Multiplication speed . 48
Minnesota speed . 37
Piece test speed . 34
All the number speed variables have loadings in this factor, and these loadings are much higher than the loadings of the two visual speed variables. Accordingly, the factor is called Number speed.

In the sixth factor the following tests have loadings.
Minnesota level
.42
Punched holes level . 40
Piece test level . 33
All the variables are visual level variables. These variables also had loadings in the third factor. Therefore, a clear and independent visual level factor is not found.

Table 17
Graphically Rotated Orthogonal Factor Matrix: Analysis of the Variables of the Individual Situation. Speed Scores Are of Type 4.

|  | I | II | III | IV | V | VI | $\mathrm{h}^{2}$ |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 1 | -.04 | .43 | .38 | -.01 | -.13 | .14 | .37 |
| 2 | .12 | .29 | .51 | -.20 | -.20 | -.08 | .45 |
| 3 | -.28 | .32 | .54 | .05 | -.09 | .27 | .56 |
| 4 | -.22 | .56 | .00 | -.07 | -.01 | -.07 | .37 |
| 5 | .15 | .71 | .35 | .24 | .01 | -.03 | .71 |
| 6 | -.01 | .63 | .11 | .08 | -.19 | .23 | .50 |
| 7 | -.15 | -.09 | .61 | .15 | .10 | .01 | .44 |
| 8 | .01 | .27 | .57 | .08 | -.15 | -.31 | .52 |
| 9 | .67 | -.23 | .28 | -.08 | .36 | .03 | .72 |
| 10 | .43 | -.30 | .23 | .24 | .41 | .19 | .59 |
| 11 | .78 | -.10 | .09 | -.10 | .16 | .00 | .66 |
| 12 | .49 | -.01 | .03 | .48 | .46 | -.23 | .74 |
| 13 | .18 | -.19 | -.02 | .65 | .44 | .06 | .69 |
| 14 | .36 | .09 | .18 | .53 | .35 | .11 | .59 |
| 15 | .19 | -.14 | .19 | -.11 | .74 | -.06 | .66 |
| 16 | .38 | -.07 | .07 | -.10 | .68 | -.27 | .71 |
| 17 | .13 | .03 | .15 | .01 | .80 | .18 | .71 |

We will now consider the results presented in Table 17.
In the first factor of this analysis there are the following loadings.

$$
\begin{array}{ll}
\text { Minnesota speed } & .78 \\
\text { Punched holes speed } & .67 \\
\text { Addition speed } & .49 \\
\text { Piece test speed } & .43 \\
\text { Word grouping speed } & .38 \\
\text { Division speed } & .36
\end{array}
$$

This factor is clearly a speed factor, but there are loadings of speed variables of many kinds. Two of the visual variables have by far the highest loadings and even the third has a moderate loading. Thus, this factor can perhaps be called Visual speed, though it is not a pure visual factor.

In the second factor the following loadings are found.
Multiplication level 71
Division level . 63
Addition level . 56
Punched holes level . 43
Minnesota level . 32
Piece test speed -. 30
The highest loadings are those of the level variables of the number tests, and consequently this factor may be called Number level.

In the third factor there are the following loadings.

$$
\text { Synonyms level . } 61
$$

Verbal analogies level . 57
Minnesota level . 54
Piece test level . 51
Punched holes level . 38
Multiplication level . 35
This factor is a Verbal-and-visual level factor.
In the fourth factor there are the following loadings.
Multiplication speed . 65
Division speed . 53
Addition speed . 48
This is a Number speed factor.

In the fifth factor the following variables have loadings.

| Verbal analogies speed | .80 |
| :--- | :--- |
| Synonyms speed | .74 |
| Word grouping speed | .68 |
| Addition speed | .46 |
| Multiplication speed | .44 |
| Punched holes speed | .36 |
| Division speed | .35 |

Almost all the speed variables have loadings in this factor, but there is a considerable difference between the loadings of the verbal speed variables and the others, and this factor may perhaps be called Verbal speed.

The sixth factor remains uninterpreted in this analysis.

Table 18
Graphically Rotated Orthogonal Factor Matrix: Analysis of the Variables of the Group Siluation

|  | I | II | III | IV | V | $\mathrm{h}^{2}$ |
| ---: | ---: | ---: | ---: | ---: | ---: | ---: |
| 1 | .64 | -.20 | .03 | .10 | .15 | .48 |
| 2 | -.11 | .08 | .29 | .50 | -.05 | .36 |
| 3 | .19 | -.13 | .20 | -.14 | .28 | .19 |
| 4 | .64 | .30 | .19 | .25 | .04 | .60 |
| 5 | .03 | .00 | .01 | .53 | .24 | .34 |
| 6 | .06 | -.11 | .56 | .06 | .20 | .37 |
| 7 | .69 | .28 | .19 | -.05 | -.10 | .60 |
| 8 | .05 | .12 | .31 | .62 | .16 | .52 |
| 9 | -.11 | .18 | .40 | -.08 | .50 | .46 |
| 10 | .69 | -.06 | -.32 | .18 | .03 | .62 |
| 11 | .20 | .48 | -.15 | .45 | .15 | .52 |
| 12 | .21 | -.08 | .00 | .20 | .70 | .58 |
| 13 | .72 | -.18 | .08 | .07 | .02 | .56 |
| 14 | .14 | .19 | -.07 | .55 | .57 | .69 |
| 15 | .05 | .26 | .20 | -.11 | .78 | .73 |
| 16 | .63 | .12 | -.13 | -.03 | .03 | .43 |
| 17 | .21 | .41 | .27 | .58 | .22 | .67 |
| 18 | .17 | .23 | .11 | .01 | .70 | .58 |

The results of the analysis of the variables of the group situation will now be considered. These results are shown in Table 18. In the first factor the following variables have loadings.

Punched holes speed . 72
Minnesota speed . 69
Piece test level . 69
Minnesota level . 64
Punched holes level . 64
Piece test speed . 63
This factor is a Visual factor.
In the second factor the following loadings are found.

$$
\text { Synonyms speed . } 48
$$

Verbal analogies speed . 41
Punched holes level . 30
It is difficult to characterize this factor, as so few variables have loadings and as the loadings are so low. But perhaps this factor has something to do with Verbal speed. It may, however, be a residual factor.

In the third factor the following loadings can be found.

| Multiplication level | .56 |
| :--- | ---: |
| Division level | .40 |
| Minnesota speed | -.32 |
| Verbal analogies level | .31 |

This factor is also difficult to interpret. There are two number level tests in this factor so that, perhaps, this factor has something to do with Number level.

In the fourth factor there are the following loadings.
Verbal analogies level . 62
Verbal analogies speed . 58
Word grouping speed . 55
Word grouping level . 53
Synonyms level . 50
Synonyms speed . 45
This factor is obviously a Verbal factor.
In the fifth factor the following loadings are found.
Multiplication speed .78
Addition speed . 70
Division speed $\quad 70$
Word grouping speed . 57
Division level . 50

This factor seems to be a Number speed factor. It is remarkable that apart from the Division test the number level tests do not have loadings in this factor.

In the hope that a rotation with oblique axes would throw some additional light on the interpretation of the factors, and, in particular, in order to obtain the correlations between the primary factors, the rotation was also performed by the method of extended vectors ( $48, \mathrm{Ch}$. XI). Then, however, rotations were only made of one of the matrices of the individual situation, and of the matrix of the group situation. Of the factor matrices of the individual situation the matrix given in Table 13 was chosen to be rotated. There vere two reasons for this choice. First of all, this matrix was obtained by using the speed scores of type 1, which were thought to be in principle the best ones. And secondly, there seemed to be a possibility of obtaining six interpretable factors from this matrix (instead of five from the matrix given in Table 14). The reference factor matrices are shown in Tables 19 and 21, and the factor plots in Figures 6 and 7. From these figures it can be seen that simple structure is not completely obtained, especially not in the case of the individual situation.

Table 19
Factor Matrix Rotated by the Method of Extended Vectors: Analysis of the Individual Situation. Speed Scores Are of Type I.

|  | $\mathrm{V}_{5}=\mathrm{F}_{0} \cdot \wedge{ }_{05}$ |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  | I | II | III | IV | V | VI |
| 1 | -. 09 | . 38 | . 48 | . 16 | . 00 | -. 02 |
| 2 | . 08 | . 40 | . 50 | . 15 | -. 26 | . 20 |
| 3 | -. 01 | . 26 | . 54 | . 01 | . 08 | . 07 |
| 4 | . 20 | . 41 | . 00 | -. 19 | . 00 | -. 03 |
| 5 | . 19 | . 83 | . 37 | . 29 | . 19 | . 02 |
| 6 | . 00 | . 48 | . 20 | . 00 | . 24 | -. 22 |
| 7 | . 00 | . 01 | . 43 | . 20 | . 13 | . 45 |
| 8 | -. 19 | . 43 | . 46 | . 53 | -. 03 | . 55 |
| 9 | . 08 | . 08 | . 31 | . 48 | -. 20 | . 16 |
| 10 | . 01 | . 00 | . 17 | . 44 | . 02 | . 08 |
| 11 | -. 10 | . 18 | . 07 | . 54 | . 00 | -. 09 |
| 12 | . 28 | . 24 | -. 01 | . 31 | . 52 | . 08 |
| 13 | . 36 | . 00 | -. 23 | -. 03 | . 50 | . 00 |
| 14 | . 01 | . 42 | . 20 | . 43 | . 59 | . 05 |
| 15 | . 56 | -. 11 | . 01 | . 00 | . 19 | . 48 |
| 16 | . 76 | . 20 | . 05 | . 00 | . 00 | . 33 |
| 17 | . 43 | . 04 | . 21 | . 00 | . 16 | . 01 |


|  | $\mathbf{M}^{\prime}$ |  |  |  |  |  |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: |
| I | .37 | -.31 | .02 | -.88 |  | -.03 |
| II | .63 | .42 | -.31 | -.08 | -.54 | .17 |
| III | .56 | .36 | .54 | .28 | .03 | .43 |
| IV | .51 |  |  | .82 | -.28 |  |
| V | .33 | -.04 | -.57 | -.05 | .72 | -.20 |
| V I | .32 | .02 | .50 | .14 | .03 | -.80 |

In the analysis of the individual situation (Table 19), the variables that have the highest and most pure loadings in the first factor are the variables number $15-17$, which are the verbal speed variables. This factor consequently is that of Verbal speed.

In the second factor the variables 4-6, which are the number level variables, have loadings. In this factor a small degree of loading is also shown for variables $1-3$, which are the visual level variables, and variable 8 , one of the two verbal level variables, and variable 14, one of the number speed variables. The number level variables have the relatively most pure loadings, and thus this factor may, perhaps, be tentatively identified as that of Number level.

In the third factor variables $1-3$, the visual level variables, and variables $7-8$, the verbal level variables, have loadings. This factor seems to be that of Verbal-and-visual level.

In the fourth factor there are loadings of variables 9-11, the visual speed variables, variable 8 , one of the verbal level variables, and to some extent variables 12 and 14 , two of the number speed variables. The variables 9-11 are, however, the most pure representatives of this factor which can, perhaps, be identified as Visual speed.

In the fifth factor only variables $12-14$, the number speed variables, have appreciable loadings, and consequently this factor may be identified as that of Number speed.

In the sixth factor there are loadings of variables $7-8$, the verbal level variables, and 15-16, two of the verbal speed variables. As all of these variables have loadings in other factors too, it is difficult to interpret this factor. The situation is similar to that obtained in the orthogonal rotation, where one verbal-and-visual factor was found and in addition to this some traces of a visual factor. There is a difference, however, in that in the present case one verbal-and-visual factor is found but in addition some traces of a verbal factor.

The intercorrelations between the primary factors are shown in Table 20.

60
















Factor Plots of the Rotation Carried Out by the Method of Extended Vectors; Analysis of the Individual Situation. Speed Scores of Are Type 1.

Figure 6.

Table 20
Intercorrelations of the Factors Obtained by the Method of Extended Vectors: Analysis of the Individual Situation. Speed Scores Are of Type I.

|  | I | II | III | IV | V | VI |
| :--- | ---: | ---: | ---: | ---: | ---: | ---: |
| I | 1.00 |  |  |  |  |  |
| II | -.54 | 1.00 |  |  |  |  |
| II I | -.12 | -.12 | 1.00 |  |  |  |
| IV | .71 | -.57 | -.36 | 1.00 |  |  |
| V | -.23 | .10 | .24 | -.20 | 1.00 |  |
| V I | -.37 | .39 | -.02 | -.45 | .07 | 1.00 |

As was seen in Figure 6, a simple structure was not completely obtained in this rotation, and the interpretation of the factors showed that in particular the level factors were not clear-cut. The level variables did not have loadings in the factors in the way anticipated. The rotation does not succeed in giving three clear-cut level factors. This fact has to be borne in mind when the intercorrelations of the factors are considered. Because the level factors are not clear-cut, the main interest lies in the intercorrelations of the speed factors. An inspection of Table 20 shows that two of the intercorrelations of the level factors are about zero, the third is somewhat positive. The factors of Verbal speed and Visual speed correlate highly, approximately .70, whereas the third speed factor, Number speed, has a slight negative correlation, about -. 20 , with both the other speed factors. Thus, a second order speed factor is not supported by the data. The correlations between the speed and level factors are, on the whole, negative rather than positive.

The interpretation of the factors obtained in the oblique rotation of the data from the group situation (see Table 21) offers no difficulties with regard to the first three factors, which are obviously Verbal, Visual and Number factors. The last two are difficult to interpret. In the fourth factor six speed and three level tests have loadings. In the fifth factor most of the higher positive loadings are loadings of level tests. However, any specific interpretation does not seem possible. Intercorrelations of the factors are given in Table 22. The uninterpreted factors are, of course, not of interest here. The correlation between the Verbal factor and the Visual factor as well as between the Number factor and the Visual factor is about zero; the correlation between the Verbal factor and the Number factor is positive and low.

Table 21
Factor Matrix Rotated by the Method of Extended Vectors: Analysis of the Group Situation

|  | $\mathrm{V}_{3}=\mathrm{F}_{0} \cdot \wedge_{03}$ |  |  |  |  |
| ---: | ---: | ---: | ---: | ---: | ---: |
|  | I | II | III | IV | V |
| 1 | .02 | .62 | .19 | -.06 | -.10 |
| 2 | .50 | -.11 | .00 | -.01 | .46 |
| 3 | -.19 | .13 | .38 | .01 | .09 |
| 4 | .32 | .47 | .08 | .38 | .12 |
| 5 | .50 | .01 | .16 | -.04 | .19 |
| 6 | .00 | .00 | .42 | -.01 | .52 |
| 7 | .03 | .52 | .00 | .40 | .01 |
| 8 | .61 | -.01 | .20 | .08 | .49 |
| 9 | -.05 | -.32 | .59 | .31 | .39 |
| 10 | .15 | .68 | -.09 | .01 | -.40 |
| 11 | .57 | .03 | -.03 | .42 | -.01 |
| 12 | .13 | .09 | .62 | .07 | .03 |
| 13 | .00 | .70 | .10 | -.04 | -.08 |
| 14 | .56 | -.02 | .39 | .22 | .11 |
| 15 | -.07 | -.23 | .76 | .45 | .17 |
| 16 | .01 | .52 | .00 | .23 | -.26 |
| 17 | .66 | .01 | .20 | .39 | .42 |
| 18 | .04 | -.08 | .65 | .40 | .09 |


|  | $\mathrm{M}_{\mathbf{0 3}}^{\prime}$ |  |  |  |  |
| :--- | ---: | ---: | ---: | ---: | ---: |
| I | .45 | .37 | -.79 | .20 |  |
| II | .36 | -.86 | -.07 | .35 |  |
| III | .55 | .36 | .75 | .04 |  |
| IV | .40 | .08 | -.09 | -.91 |  |
| V | .27 | .50 | .02 | .11 | -.82 |

Table 22
Intercorrelations of of the Factors Obtained by the Method of Extended Vectors: Analysis of the Group Situation

|  | I | II | III | IV | V |
| :--- | ---: | ---: | ---: | ---: | ---: |
| I | 1.00 |  |  |  |  |
| II | -.09 | 1.00 |  |  |  |
| II I | .36 | .00 | 1.00 |  |  |
| IV | -.16 | .25 | -.16 | 1.00 |  |
| V | -.41 | .28 | -.41 | .12 | 1.00 |












Factor Plots of the Rotation Carried Out by the Method of Extended Vectors: Analysis of the Group Situation

Figure 7

The results obtained in the individual situation were submitted to two factor analyses, both of which were performed on the basis of eight level and nine speed variables. Both analyses were first interpreted on the basis of an orthogonal rotation and one of them was, in addition, interpreted on the basis of a rotation carried out by the method of extended vectors. The picture obtained has the same features in each of these three cases. 'Ihree speed factors are oblained in each case. These factors are Verbal, Visual and Number speed, and they thus are the expected factors (cf. p. 41). The correlations between the Number speed factor on the one hand, and the other two speed factors on the other were negative but low, while the two latter factors had a fairly high positive intercorrelation. The level factors were not as clear-cut as the speed factors. In no rotation could three clear-cut level factors be found. Verbal and visual tests tended to have loadings in the same factor, whereas the number tests formed a factor of their own. It has to be noted, however, that there were only two verbal level variables, and both of these were relatively unreliable, so that this technical detail may cause that only two level factors are found.

From the results of the group situation one factor analysis was calculated. This analysis was interpreted on the basis of an orthogonal and an oblique rotation. In the former, factors of Verbal and Visual abilities (in which both speed and level tests had loadings) as well as Number speed and Number level factors were found. In the latter, only three factors could be interpreted; these were a Verbal, a Visual, and a Number factor.

## CHAPTER V

## DISCUSSION

The results of the present study give rise to a number of questions which seem to merit discussion. This chapter is primarily concerned with a discussion of the correlation between the level and the speed of performance in different tests and of the results obtained in the individual and in the group situation, the latter of which failed to show the expected effects. Particular attention will be paid to the difference between the results obtained in the two situations. Finally the question of the definition and concept of intelligence is considered from the point of view of the speed and level controversy.

The correlation between speed and level components in different tests

The correlations between speed and level components in different tests and in different situations have been selected out from Tables 10-12 and are given again in Table 23. It may be seen that no considerable correlations are found in the individual situation, in which speed is measured as working speed, i.e. as the time spent in working out individual items. The correlations cluster around zero; there are both negative and positive correlations. This result is in agreement with some earlier results, e.g. those of Tate (41) and Nummenmaa (30). In the group situation the correlations are moderately positive, though not high. This is in agreement with earlier results reported by Lord (22), though the correlations reported by him are higher than those of the present study. The difference between the correlations obtained in the different situations will be discussed more closely in connection with the factor analysis of the results of the material of the group situation. Some comments should be made concerning the nature of the scores that are obtained when speed is measured in terms of working rate, as was done in the individual situation.

[^0]Table 23

## Correlations Between the Speed and Level Components in Different Tests and in Different Situations.

| Test | Individual | Situation | Group Situation |
| :--- | :---: | :---: | :---: |
| Punched holes | -.11 | -.07 | .48 |
| Piece test | -.18 | -.14 | .45 |
| Minnesota | -.29 | -.27 | .50 |
| Addition | -.12 | -.16 | .20 |
| Multiplication | .12 | .02 | .32 |
| Division | .31 | .12 | .41 |
| Synonyms | .23 | .16 | .34 |
| Word grouping |  |  | .49 |
| Verbal analogies | -.07 | -.05 | .63 |

The problem here is whether these scores give some information about the subject's ability for speed, or whether they merely reflect preferred working speed rates connected with personal tempo. This question cannot be answered on the basis of the present data. In the present case, no instruction in favour of either speed or level was given by the experimenter. It is obvious that all subjects in a situation, where speed is measured by a stop watch, aim at speed at least to some extent, but this extent may vary. It should be remarked, however, that the solution to the problem is not as simple as to ask the subjects to work with „maximum speed». This maximum speed, to be a measure of the same thing in case of all subjects, should be mmaximum speed without any loss in level of performance» and this is not reached by an instruction of the kind indicated. This question will be discussed later in connection of the factor analytical results of the material of the group situation.

The factor analysis of the variables of the individual situation
In the analysis of the variables of the individual situation the speed and level factors were found to be independent. This is a consequence of the about zero correlations between the level and speed variables. Independent factors of level and speed have earlier been found by e.g. Slater (35), Davidson and Carroll (7), Dahlgren (6), and Myers (29). In the present study several speed factors were found. This is what also Tate could expect on the basis of his experiments. The speed factors obtained are the customarily found Verbal, Visual and Number factors; the
present factors are, however, pure speed factors. In each domain (see page 15) the subject thus has a speed rate characteristic to the domain in question. The level factors were less clear, and did not completely give the structure that was expected on the basis of earlier investigations (45, 2, 12), but this may be due to the lower reliability coefficients of the level variables in general and of the verbal level variables in particular.

## The factor analysis of the variables of the group situation

The results obtained in the individual situation could be interpreted as supporting the hypothesis, although they are not entirely clear-cut. The results of the group situation, on the other hand, did not show the expected speed factors. Instead, the factors were formed only on the basis of the nature of the stimuli given. Only in the number tests was there some differentiation between level and speed factors. This outcome of the analysis gives rise to two questions: Why did the expected speed factors not appear? And what is the nature of the empirically found factors from the point of view of the problem of speed and level? In connection with this second question the difference between the results of the individual and the group situation will also be discussed. The second question will be considered first.

In the individual situation the speed was measured at three levels of item-difficulty; in group situation it was measured using very easy items only. There is, however, no reason why speed in easy items should correlate higher with level than speed in difficult items; an explanation cannot be built on this ground. The explanation may be that in the group situation speed and level were not measured independently of each other. The speed of performance, one could imagine, can be increased if the subject wants to do so. For instance, the speed may be dependent on motivation and other factors. It is suggested that the increase in speed may occur in two ways. One can imagine that when the subject performs some intellectual task, it is done in several „parts». The performance includes several phases. It has been shown by Sutherland (39) that in performance tests the time of response is very much dependent on the number of the moves the subject makes. These moves» cannot easily be shown in the case of intellectual performance in a paper and pencil test, but it could be thought that they exist in the same way, e.g. in the Minnesota paper form board one »move» would be an attempt to place (in mind) a piece somewhere in the square. Now
if one wants to make the performance in less time this can be achieved in two ways. First, one may perform each phase (move) faster, or secondly one may leave out some phases, or one may do both. It can be supposed that the former way is used up to a certain point, and when it fails to work, then the second way is used. It is very possible that the probability of obtaining a correct solution is dependent of the number of phases left out. It may be that in a group situation, where the subjects know that they have a limited time at their disposal, the slow subjects try to perform the task with the same speed as the fast ones. In other words, there arises a norm with regard to the speed of working. When subjects try to attain this speed, the slow ones are frequently forced to use the method of leaving out mmoves» and thus the level of their performance goes down; in this way level scores may be dependent on speed even when they fulfill the formal requirements for level scores presented by Gulliksen (17) as they did in the present study. In the group situation of the present study the experimenter himself further produced such norms of working in the visual tests, by giving some directions with regard to the speed of working, in order to prevent the subjects from using an excessive amount of time in any one item. On account of this, the level scores of the group situation are not pure level scores but contain also some elements of speed. On the other hand, the speed tests were scored counting the number of correct responses, and consequently it is clear that they contain an element of level to some extent. Therefore, neither of these types of test measured exactly what they were intended to measure. This analysis suggests the following answer to the question of the nature of the factors found in the analysis of the group situation: As the tests do not exclusively measure either speed or level, but to some extent both, the resulting factors must correspondingly be described in terms of both speed and level, in addition to the description in terms of stimulus content.

To the first question presented on p. 67, i.e. why could clearcut speed factors not be found in the analysis of the group situation, no exact answer can be given here. One possible reason is, that the errors of measurement and sampling were too great to allow these factors to appear. Thus, though these results do not confirm those of Lord (22), this cannot be taken as any proof of the opposite.

## On the question of validity

With respect to the validity the two following things can be noted. First of all, as the speed of working as measured in the individual situation and the level of performance are independent of each other, they act as predictor-variables of different kinds. However, in this present experiment no attempt was made to study the relative efficiency of these two aspects of performance for purposes of prediction. Secondly, it is obvious that some features characteristic of the group situation and of the scoring of tests can cause the level and speed tests to give very similar results, and when they do so, the differences between speed and level variables as predictors are consequently smaller than in the former case. If speed factors can be found independently of stimulus content factors, the former may have some predictive value of their own. This is what Lord (22) found.

## On the definition and concept of intelligence

There have been many attempts to define what intelligence »really" is. Some of these definitions were presented in Chapter I. The only point concerning the definitions of intelligence on which most of the writers agree is that none of the definitions suggested seems to be really satisfactory.

The earlier attempts to define intelligence were speculative in nature. Speculative definitions have been criticized on two grounds. First the meaning of concepts tends to be different for different writers, and secondly, on the basis of speculations an empirical science cannot be built so as to be able to give predictions of any kind.

In practice, it is possible to see what an investigator means by »intelligence» by looking at the method - the test - with which he attempts to measure intelligence (operational definition). As a consequence of this the popular definition of intelligence has evolved: "Intelligence is what is measured by intelligence tests.». However, this definition obviously has not much to do with scientific definitions, as is pointed out by Feigl (10). It has in fact no meaning as such, because of the tautology involved. If the definition is to be satisfactory, there should be a selection of tests officially or by general consensus named nintelligence tests». As it stands, the definition seems to involve a misuse of operationism. An operational definition generally presupposes a corresponding or supplementary definition at a mere general level,
but no such definition is connected with the above mentioned definition of intelligence.

It is the very multiplicity of tests commonly used as measures of intelligence which makes it so difficult to find what could be common to them all. Spearman's studies were the first attempt to show empirically that a concept of general intelligence can really be defined. Later, however, it was shown that a general factor could not be found in the form suggested by Spearman. The multiple factor theory will, however, give a system for the description of intelligence. An old observation, namely that all tests of intelligence correlate positively with each other, made it possible for Thurstone to suppose that the general factor would be existing »in the form of some central parameter which has positive influence on all the special mental abilities . . .» (50, p. 8). Some writers, e.g. Guilford (16), are of the opinion that to no second order factor can be given the name of mintelligence». Consequently, it seems that the concept of intelligence will remain unclear and shifty. Guilford as well as Ahmavaara (3) suggest, that it could possibly be used in a popular or semipopular sense.

In the factor theory the situation at the moment seems to be the the following. Several independent features of intelligent behaviour or "parts of intelligence», several factors, have been isolated. Descriptions have been given to these factors; for instance a definition or description of the space factor is as follows: -»The space factor is found in tests which require that the subject manipulate an object imaginally in two or three dimensions» (49, p. 21). Obviously, as the parts to be described are small the descriptions are easier to do than if a very large area or class of mental operations is in question.

In this way, one uniform definition of intelligence is not found, but instead a great many definitions of the parts of intelligence, i.e. of the factors. It should be noted, that the factor tests are not necessarily operational definitions of some concepts earlier presented at a general level; it may be that these factors are found first of all and a definition or a description is given afterwards. In this way, the process of dividing intelligence into smaller and smaller parts continues; it is not necessarily assumed that the primary factors will be finite in number. The factors seem to be divisible into factors of a quite small area. This is empirically shown, for example by Guilford's (16) investigations.

With regard to the speed and level controversy in the definition of intelligence, it may be stated first, that if speed is measured as rate-of-working, speed and level are independent of each other and con-
sequently they both have their own factors; in some other circumstances speed factors independent of stimulus content factors can be found. Obviously, the speed and the level cannot be held as the same thing, but a theory of intelligence must take both of them into account. The nature of level factors can perhaps be more easily described, as it may be done in terms of the tasks involved. Number ability for instance is an ability to perform correctly calculations of several kinds. The speed factors are found in tasks which require that the subject performs tasks of certain types quickly. Thus, the factor of number speed is found in tests that require that the subject performs (correct) calculations quickly. The importance of the abilities for speed is, of course, to make the use of level abilities more or less efficient.

## SUMMARY

The problem of the relationship between the speed and the level components of intelligence is introduced in Chapter I. There are two opposite hypotheses concerning this relationship, both of which have been vigorously defended. One is that speed and level are independent of each other, the other that speed is an essential characteristic of intelligent behaviour and that consequently the two components are closely interrelated. These two hypotheses are also reviewed in connection with the definitions and factor theories of intelligence.

Problems connected with the measurement of speed and level are discussed in Chapter II.

Chapter III gives a review of earlier investigations of the relationship between speed and level. First of all, studies concerning the correlation of these two aspects of performance in one and the same test are considered. On the basis of these earlier studies it is suggested that the amount of correlation essentially depends on the way in which these variables, particularly that of speed, are measured. It is suggested that when speed is measured in terms of rate of working, the correlation between speed and level will be about zero or just possibly positive but low, and when it is measured by means of easy highly speeded tests (which are scored by counting the number of correct answers) the correlation will be positive and even quite high. A more detailed discussion of this problem is given later in Chapter V.

A review of the factor analytical studies shows a great multiplicity of results. This multiplicity is obviously partly due to the differences in the plans of the studies and perhaps to technical weaknesses in some studies, but it is also, and this is important, due to the fact that the ways in which the speed and the level have been measured have been different in different studies. On the basis of the earlier investigations the hypothesis is presented, that when speed variables are measured in terms of working rate, speed factors independent of level factors are found, if the level variables are measured independently of speed. Also, the hypothesis is presented that when speed variables are measured by means of easy tests (which are scored by counting the number of right answers) and level variables are measured under the usual conditions of group testing, speed factors will be found which are independent of factors describable in terms of stimulus content (in which both the speed and level variables have loadings). The investigation was planned to test these hypotheses.

The present investigation is described in Chapter IV. Measurements of the subjects' speed and level of performance were obtained in situations of two different kinds. In the individual situation nine tests were presented item by item and the response times as well as the quality (accuracy) of the responses were recorded. In the group situation speed and level versions of nine similar tests were presented.

Speed was measured by means of easy highly speeded tests and level was measured by difficult tests with liberal time-limits. In the former situation a factor analysis interpreted on the basis of graphic rotation with orthogonal axes revealed three speed factors (Visual, Verbal and Number speed) and two level factors (Number level and Verbal + visual level). Thus, the expected result was obtained, with the exception that the level side was not differentiated into three factors, probably because of the low reliability of the verbal tests. A rotation carried out by the method of extended vectors gave an essentially similar picture. In the group situation a factor analysis interpreted on the basis of a graphic rotation with orthogonal axes revealed four interpretable factors. In two of these, Verbal and Visual factors, both the level and the speed tests had loadings. In addition there were two number factors, Number speed and Number level. However, oblique rotation carried out on the basis of the results of this situation gave only three interpretable factors, a Verbal, a Visual, and a Number factor. Thus, the expected speed factors were not found, possibly because the errors of measurement and the sampling errors were too great to allow these factors to emerge.

Chapter V gives a discussion, which is concerned mainly with the nature of speed and level measurements in different situations and with the definition and concept of intelligence.

Following a brief discussion of the correlation between speed and level of performance in the different tests, the results obtained in the factor analyses of the individual and the group situation are considered. The hypotheses concerning the results to be expected in the two situations, presented in Chapter IV, were simply based on the outcome of earlier investigations and did not contain any interpretation or »explanation» of the expected differences. In Chapter V such an interpretation is attempted, particular attention being paid to an analysis of the stimulus content factors of the group situation, and to an analysis of the conditions under which the tests were performed. With respect to the level tests it is pointed out that in the group situation time-limits were introduced, though these were liberal. The group situation, it is suggested, may give rise to a norm concerning the working speed. Such a norm may cause some subjects to work at a higher speed than they would do in the absence of it, this being reflected in a decrease in the level of performance. To the extent that this occurs, the effect would be greatest in the case of the slowest subjects and thus tend to give rise to a positive correlation between the speed and the level scores. It should be pointed out that this may be the case even when the level scores fulfill all the formal criteria presented by Gulliksen (17). Correspondingly, when speed tests are scored in terms of the number of correct answers, the speed scores will not be independent of level. It thus appears that in conditions like those of the present group situation, level is not measured independently of speed, nor speed independently of level. This causes the correlation between the speed and the level scores to be positive, and, consequently, the tests of speed and those of level tend to have loadings in the same factors. It is still possible that the speed variables in addition have loadings in some factors of their own, but this was not the case in the present investigation.

In the final part of the discussion various types of definition of intelligence, speculative definitions, operational definitions, and different conceptions of general intelligence by factor analysts, are briefly considered. Each definition has had its share of criticism from the research workers in the field. At the moment the posi-
tion in the factor theory seems to be the following. The term of general intelligence is used mainly in a popular or semipopular sense. Instead, smaller specific areas or features of intelligence, i.e. factors, are isolated and described. It is suggested that a theory of intelligence should take into consideration both the speed and the level aspects of intelligence. In the factor theory of intelligence both types of factor can be described primarily in terms of stimulus content, but in addition to this a level factor can be described as an ability to do tasks of a certain type correctly, and a speed factor as an ability to do tasks (correctly) quickly.

APPENDIX

## Analysis of some factors affecting the response times

In the individual situation the response times were taken item by item to the accuracy of one second. As has been explained previously (p. 43) the mean response times were obtained at four levels of difficulty. In this Appendix an analysis of some factors affecting these response times is reported. The main purpose in this respect was to study whether there is any difference between the response times of correct and incorrect solutions. To obtain some additional information it was decided to study also the effect of the level of difficulty on these response times. This was done by means of analyses of variance. The independent variables were as follows. 1) Accuracy of response. This variable has two values according to whether a response is correct or incorrect. 2) Level of difficulty. In each case those two successive levels of difficulty at which the number of subjects having both correct and incorrect solutions was greatest were selected for inclusion in the analysis. In most cases the third and fourth levels of difficulty fulfilled this criterion, the only exception being the Addition test in which the relevant levels were the second and the third. 3) Subjects (individual differences). As subjects not having both correct and incorrect responses at the levels of difficulty in question were omitted from the analysis, the total number of subjects varied from 45 to 63 in the different tests. The analysis was carried out separately for each test.

First of all, the equality of variances in the different combinations of the variables accuracy and level of difficulty was studied. This was done by means of Bartlett's test (18). The results of these tests are shown in Table 24. In most cases the inequality of the variances is obvious. Only in the case of tests of Word grouping and Verbal analogies is this less clear. It is known, that the distributions of the speed scores tend to be skew. A transformation, which very often has been

Table 24
Bartlett's Test of the Equality of Variances: Raw Scores (in Seconds)

|  | N | $s^{2}$ |  |  |  | Chi ${ }^{2}$ | $\mathrm{P}<$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Correct |  | Incorrect |  |  |  |
|  |  | 3 | 4 | 3 | 4 |  |  |
| Punched holes | 58 | 2096 | 3458 | 3704 | 8697 | 31,32 |  |
| Piece test | 56 | 4836 | 12927 | 30200 | 14906 | 167,94 |  |
| Minnesnta | 4.5 | 5877 | 19283 | 69443 | 22050 | 62,21 |  |
| Addition | 51 | 102,2 | 566,6 | 151,5 | 1539,0 | 111,32 |  |
| Multiplication | 43 | 147,6 | 1011,3 | 959,7 | 686,2 | 37,43 |  |
| Division | 52 | 614,7 | 2077,0 | 1228,0 | 2395,0 | 25,50 |  |
| Synonyms | 62 | 101,0 | 144,8 | 427,2 | 262,8 | 36,38 |  |
| Word grouping | 59 | 271,5 | 229,4 | 383,4 | 219,2 | 5,86 | 90,0 |
| Verbal analogies | 63 | 578,8 | 955,6 | 1214,1 | 739,7 | 9,39 | 99,0 |

found to produce a normal distribution in such cases, is the logarithmic transformation. At the same time the variances may become more equal. This transformation was applied in cases other than Word grouping and Verbal analogies. The Rartleft's tests of the equality of variances are shown in Table 25 . It can be seen that the transformation markedly reduced the heterogeneity of the variances; although some heterogeneity still remains, it is clearly significant only in the case of the Piece test. The analyses of variance were performed using the scores in seconds in the case of the Word grouping test and the Verbal analogies test, and using logarithmic scores in the other cases with the exception of the Piece test, which was omitted from the analysis. The results of the analyses of variance are shown in Table 26, and the means of the response times at the different levels of difficulty are given in Table 27.

The tests of the significance of the main effects and interactions are presented separately in Table 28 which thus contains a summary of the outcome of analyses. It is seen that the effect of individual differences (subject) is significant in each test, as might be expected. The interactions $\mathrm{A} \times \mathrm{S}$ and $\mathrm{L} \times \mathrm{S}$ are not significant in any test. This shows that the relative standings of the individuals with respect to speed are not significantly altered by either accuracy or level of difficulty. Instead, there are two significant interactions between accuracy and level of difficulty, namely in tests of Addition and Multiplication, and one possibly significant interaction, in the test of Synonyms. An inspection of the means given in Table 27 reveals, that all these interactions are of different kinds. In the Addition test the time of the cor-
rect responses changes less from level 2 to level 3 than does the time of the incorrect responses, whereas in the Multiplication test the time of the correct responses changes more from level 3 to level 4 than the time of the incorrect responses.

Table 25
Barlett's Test of Equality of Variances: Logarithmic Scores

|  | N |  |  | $\mathrm{s}^{2}$ |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Correct |  | Incorrect |  |  |  |
|  |  | 3 | 4 | 3 | 4 | Chi ${ }^{2}$ | $\mathrm{P}<$ |
| Punched holes | 58 | . 0233 | . 0302 | . 0309 | . 0377 | 3,06 | 70,0 |
| Piece test | 56 | . 064 | . 161 | . 144 | . 072 | 17,20 |  |
| Minnesota | 45 | . 0439 | . 0682 | . 0996 | . 0436 | 10,63 | 99,0 |
| Addition | 51 | . 0282 | . 0410 | . 0394 | . 0538 | 5,12 | 90,0 |
| Multiplication | 43 | . 0310 | . 0300 | . 0614 | . 0293 | 8,87 | 97,5 |
| Division | 52 | . 0312 | . 0284 | . 0620 | . 0500 | 10,60 | 99,0 |
| Synonyms | 62 | . 0507 | . 0567 | . 0675 | . 0626 | 1,42 | 30,0 |

Table 27

Means of the Response Times of Correct and Incorrect Solutions at Difficulty Levels 3 and 4. In the Addition Test the

Difficulty Levels Are 2 and 3.

|  |  | Correct <br> level 3 | solutions <br> level 4 | Incorrect <br> level 3 | solutions <br> level 4 |
| :--- | :---: | :---: | :---: | :---: | :---: |
| Test, type of score | Log X | 2,07 | 2,09 | 2,06 | 2,07 |
| Punched holes | l | 1,97 | 1,89 | 2,18 | 2,12 |
| Piece test | $"$ | 2,17 | 2,25 | 2,33 | 2,38 |
| Minnesota | $"$ | 1,37 | 1,64 | 1,38 | 1,79 |
| Addition | $"$ | 1,47 | 1,83 | 1,62 | 1,79 |
| Multiplication | $"$ | 1,69 | 1,97 | 1,72 | 2,00 |
| Division | $"$ | 1,23 | 1,27 | 1,38 | 1,34 |
| Synonyms | $"$ | 30,83 | 27,07 | 34,51 | 31,49 |
| Word grouping | X | 39,06 | 47,67 | 46,11 | 58,52 |

Table 26
The Results of the Analyses of Variance

|  |  | Accuracy | Level of difficulty | Subject | A < L | $\mathrm{A} \times \mathrm{S}$ | $\mathrm{L} \times \mathrm{S}$ | $\mathrm{A} \times \mathrm{L} \times \mathrm{S}$ | Total |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Punched holes | SS | 0,0111 | 0,0122 | 4,9344 | 0,0001 | 0,8514 | 0,5253 | 0,6474 | 6,9819 |
|  | df | 1 | 1 | 57 | 1 | 57 | 57 | 57 | 231 |
|  | var. est. | 0,0111 | 0,0122 | 0,0866 | 0,0001 | 0,0149 | 0,0092 | 0,0114 |  |
| Minnesota | SS | 0,9046 | 0,2027 | 6,7382 | 0,0180 | 1,6630 | 1,4348 | 1,3845 | 12,3457 |
|  | df | 1 | 1 | 44 | 1 | 44 | 44 | 44 | 179 |
|  | var. est. | 0,9046 | 0,2027 | 0,1531 | 0,0180 | 0,0378 | 0,0326 | 0,0315 |  |
| Addition | SS | 0,3377 | 6,0255 | 5,5461 | 0,2402 | 0,7171 | 1,1408 | 0,7194 | 14,7268 |
|  | df | 1 | 1 | 50 | 1 | 50 | 50 | 50 | 203 |
|  | var.est. | 0,3377 | 6,0255 | 0,1109 | 0,2402 | 0,0143 | 0,0228 | 0,0144 |  |
| Multiplication | SS | 0,1215 | 3,0512 | 3,4467 | 0,3694 | 1,2086 | 0,5334 | 1,1806 | 9,9117 |
|  | df | 1 | 1 | 42 | 1 | 42 | 42 | 42 | 171 |
|  | var.est. | 0,1215 | 3,0512 | 0,0821 | 0,3694 | 0,0288 | 0,0127 | 0,0281 |  |
| Division | SS | 0,0402 | 4,0016 | 4,6031 | 0,0019 | 0,8039 | 1,7048 | 1,6348 | 12,7903 |
|  | df | 1 | 1 | 51 | 1 | 51 | 51 | 51 | 207 |
|  | var.est. | 0,0402 | 4,0016 | 0,0903 | 0,0019 | 0,0158 | 0,0334 | 0,0321 |  |
| Synonyms | SS | 0,7613 | 0,0003 | 11,7573 | 0,0875 | 0,9988 | 0,9497 | 0,7825 | 15,3373 |
|  | df | 1 | 1 | 61 | 1 | 61 | 61 | 61 | 247 |
|  | var.est. | 0,7613 | 0,0003 | 0,1927 | 0,0875 | 0,0164 | 0,0156 | 0,0128 |  |
| Word grouping | SS | 968,15 | 677,97 | 47354,80 | 8,20 | 4649,89 | 6969,07 | 5025,76 | 65653,84 |
|  | df | 1 | 1 | 58 | 1 | 58 | 58 | 58 | 235 |
|  | var.est. | 968,15 | 677,97 | 816,46 | 8,20 | 80,17 | 120,16 | 86,65 |  |
| Verbal analogies | SS | 5049,14 | 6956,25 | 138256,65 | 228,57 | 31007,86 | 18609,75 | 28395,43 | 228503,65 |
|  | df | 1 | 1 | 62 | 1 | 62 | 62 | 62 | 251 |
|  | var.est. | 5049,14 | 6956,25 | 2229,95 | 228,57 | 500,13 | 300,16 | 457,99 |  |

Table 28
Results of the Tests of Significance of the Analyses of Variance ${ }^{1}$

|  | Accuracy | Level of <br> difficulty | Subject | $\mathrm{A} \times \mathrm{L}$ |
| :--- | :---: | :---: | :---: | :---: |
| Punched holes | .001 | .05 | .001 |  |
| Minnesota |  |  | .001 |  |
| Addition |  |  | .001 | .001 |
| Multiplication |  | .001 | .01 | .001 |
| Division | .01 | .01 | .001 | .05 |
| Synonyms | .01 | .001 | .01 |  |
| Word grouping | .001 |  |  |  |
| Verbal analogies |  |  |  |  |

It is difficult to account for these interactions. In the case of the Multiplication test the following observation may be relevant: in this test the subjects tended to give rough, approximate figures as responses at the most difficult items more frequently than they did it in the Addition test. Such approximate responses evidently took comparatively little time and they were scored as errors; the interaction in question may have been a consequence of the occurrence of this type of responses. However, in the case of the Addition test an explanation is difficult to find. The interaction is even more unexpected in the case of the Synonyms test and may not in fact be genuine. (There is the possibility that the data did not fully meet the assumptions of the analysis of variance and that this influenced the results.) In the three above mentioned tests neither accuracy nor level of difficulty effects are significant when tested against the $\mathrm{A} \times \mathrm{L}$ interaction variance. In four tests, Minnesota paper form board, Division, Word grouping and Verbal analogies the effect of the level of difficulty was significant. The accuracy effect was significant in three tests, Minnesota paper form board, Word grouping and Verbal analogies, the incorrect responses tending to take more time than correct ones.

[^1]
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[^1]:    ${ }^{1}$ In the tests of significance the first order interaction terms were tested against the corresponding second order terms. The main effects were similarly tested against the second order interaction terms except when the relevant first order interactions were significant. In the latter case the significant interactions were used as error terms.

