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Heavy flavour production in the SACOT-m_T scheme

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The hadroproduction of heavy-flavoured mesons has recently attracted a growing interest e.g. within the people involved in global analysis of proton and nuclear parton distribution functions, saturation physics, and physics of cosmic rays. In particular, the D- and B-meson measurements of LHCb at forward direction are sensitive to gluon dynamics at small x and are one of the few perturbative small-x probes before the next generation deep-inelastic-scattering experiments. In this talk, we will concentrate on the collinear-factorization approach to inclusive D-meson production and describe a novel implementation — SACOT-m_T — of the general-mass variable flavour number scheme (GM-VFNS). In the GM-VFNS framework the cross sections retain the full heavy-quark mass dependence at $p_T = 0$, but gradually reduce to the ordinary zero-mass results towards asymptotically high p_T. However, the region of small (but non-zero) p_T has been somewhat problematic in the previous implementations of GM-VFNS, leading to divergent cross sections towards $p_T \to 0$, unless the QCD scales are set in a particular way. Here, we provide a solution to this problem. In essence, the idea is to consistently account for the underlying energy-momentum conservation in the presence of a final-state heavy quark-antiquark pair. This automatically leads to a well-behaved GM-VFNS description of the cross sections across all p_T without a need to fine tune the QCD scales. The results are compared with the LHCb data and a very good agreement is found. We also compare to fixed-order based calculations and explain why they lead to approximately a factor of two lower D-meson production cross sections than the GM-VFNS approach.

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1. Motivation

The potential of D- and B-meson production as a constraint for parton distributions (PDFs) has been recently under active investigation [1, 2, 3]. The heavy-quark mass provides a hard scale offering a possibility to use perturbative QCD for production of heavy-flavoured mesons even down to zero transverse momentum, $P_{\mathrm{T}} = 0$. While the general-purpose PDFs commonly used for LHC phenomenology are defined in general-mass variable flavour number schemes (GM-VFNS) [4], there are no public GM-VFNS tools for heavy-flavoured meson hadroproduction available. This was the motivation for our study [5] which we summarize here.

2. Heavy-flavour production in fixed flavour-number schemes

In fixed flavour-number schemes (FFNS), the heavy quarks Q are produced in three partonic processes $g+g \to Q+X, q+\overline{q} \to Q+X, q+g \to Q+X$. The rapidity- ($y$) and transverse-momentum (p_{T}) differentiated cross section for producing heavy quarks can be written as a convolution of PDFs $f_i^h(x_1,\mu^2_{\mathrm{fact}})$ and partonic cross sections $d\hat{\sigma}$ as

$$\frac{d\sigma(h_1+h_2\to \mathrm{Q}+X)}{dp_{\mathrm{T}}dy} = \sum_{ij}\int dx_1 dx_2 f_i^{h_1}(x_1,\mu^2_{\mathrm{fact}}) \frac{d\hat{\sigma}^{ij\to \mathrm{Q}+X}(\tau_1,\tau_2,m^2,\mu^2_{\mathrm{ren}},\mu^2_{\mathrm{fact}})}{dp_{\mathrm{T}}dy} f_j^{h_2}(x_2,\mu^2_{\mathrm{fact}}),$$

where $\tau_1 \equiv p_1\cdot p_3/p_1\cdot p_2 = m_{\mathrm{T}}e^{-y}/(\sqrt{s}x_2)$, $\tau_2 \equiv p_2\cdot p_3/p_1\cdot p_2 = m_{\mathrm{T}}e^{y}/(\sqrt{s}x_1)$, and m_{T} represents the transverse mass $m^2_{\mathrm{T}} = p^2_{\mathrm{T}} + m^2$. Here $p_{1,2}$ refer to the momenta of the incoming partons, p_3 is the momentum of the outgoing heavy quark Q, and m denotes the heavy-quark mass. The renormalization and factorization scales are denoted by μ^2_{ren} and μ^2_{fact}. At high p_{T} the FFNS cross section diverges logarithmically $d\sigma \sim \log(p^2_{\mathrm{T}}/m^2)$, so the framework is reliable only at low p_{T}.

To convert the parton-level cross sections to hadronic ones, the partonic spectrum is typically folded with a $Q\to h_3$ fragmentation functions (FFs) $D_{Q\to h_3}(z)$, fitted to e^+e^- data. For this we must define a fragmentation variable z which is, however, ambiguous in the presence of massive partons and hadrons. As a working assumption, we shall define z as the fraction of fragmenting heavy-quark's energy carried by the outgoing hadron h_3 in the hadronic center-of-mass frame, $z \equiv E_{\mathrm{hadron}}/E_{\mathrm{parton}}$. Together with the assumption of collinear fragmentation, this leads to

$$\frac{d\sigma(h_1+h_2\to h_3+X)}{dP_{\mathrm{T}}dY} = \sum_{ij}\int \frac{dz}{z} dx_1 dx_2 f_i^{h_1}(x_1,\mu^2_{\mathrm{fact}}) \frac{d\hat{\sigma}^{ij\to \mathrm{Q}+X}}{dp_{\mathrm{T}}dy} f_j^{h_2}(x_2,\mu^2_{\mathrm{fact}}) D_{Q\to h_3}(z)$$

where the partonic (lower case) and hadronic variables (upper case) are related as

$$p^2_{\mathrm{T}} = \frac{M^2_{\mathrm{T}}\cosh^2 Y - z^2 m^2}{z^2}\left(1+\frac{M^2_{\mathrm{T}}\sinh^2 Y}{P^2_{\mathrm{T}}}\right)^{-1} \xrightarrow{P_{\mathrm{T}}\to\infty} \left(\frac{P_{\mathrm{T}}}{z}\right)^2$$

$$y = \sinh^{-1}\left(\frac{M_{\mathrm{T}}\sinh Y}{P_{\mathrm{T}}}\frac{p_{\mathrm{T}}}{m_{\mathrm{T}}}\right) \xrightarrow{P_{\mathrm{T}}\to\infty} Y$$

where $M_{\mathrm{T}} = \sqrt{P^2_{\mathrm{T}} + M^2_{h_3}}$ is the hadronic transverse mass. A framework very similar to this has been compared with the LHCb data e.g. in Ref. [6], and the typical situation is that the calculations undershoot the data by a factor of two or so, though within the large scale uncertainties there is still a fair agreement.

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3. From FFNS to GM-VFNS heuristically

The GM-VFNS framework can be derived from FFNS by resumming the $\log(p_{\rm T}^2/m^2)$ terms present in the FFNS partonic cross sections. The diagram (a) in Figure 1 shows an NLO diagram in which an incoming gluon splits into a $Q\overline{Q}$ pair giving rise to a collinear logarithm $\sim \log(p_{\rm T}^2/m^2)$. This is just the first term of the whole tower of terms that are in variable flavour number scheme resummed into the heavy-quark PDF $f_Q^{h_1}$. This resummation can be effectively done by including

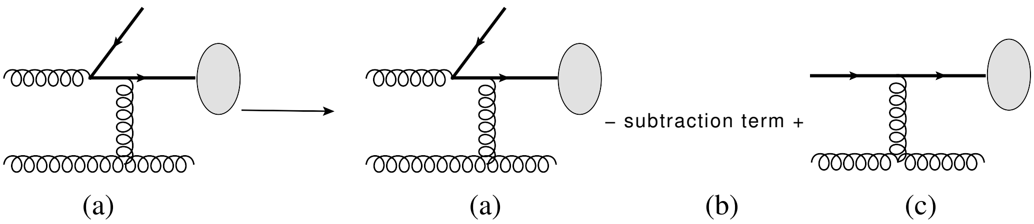

Figure 1: A schematic representation of how to deal with the initial-state logarithms.

the heavy-quark initiated contribution (c) and a term (b) that subtracts the overlap between diagrams (a) and (c). We may write the contribution from the $Qg \to Q+X$ channel as

$$\int \frac{dz}{z} dx_1 dx_2 \, f_Q^{h_1}(x_1, \mu^2_{\rm fact}) \frac{d\hat{\sigma}^{Qg\to {\rm Q}+X}(\tau_1,\tau_2)}{dp_{\rm T}dy} f_g^{h_2}(x_2, \mu^2_{\rm fact}) D_{Q\to h_3}(z) \, .$$

The compensating subtraction term is obtained from the above expression by swapping the heavy-quark PDF with its perturbative expression to first order in α_s,

$$f_Q(x, \mu^2_{\rm fact}) = \left(\frac{\alpha_s}{2\pi}\right) \log\left(\frac{\mu^2_{\rm fact}}{m^2}\right) \int_x^1 \frac{d\ell}{\ell} P_{qg}\left(\frac{x}{\ell}\right) f_g(\ell, \mu^2_{\rm fact}) \, ,$$

where P_{qg} is the standard gluon-to-quark splitting function. As is well known [4], the GM-VFNS framework contains an inherent scheme dependence which leaves us with some freedom to choose the exact form of $d\hat{\sigma}^{Qg\to {\rm Q}+X}(\tau_1,\tau_2)$ in the above expressions. In practice, the only requirement is that we must recover the zero-mass expressions at high $p_{\rm T}$,

$$\frac{d\hat{\sigma}^{Qg\to {\rm Q}+X}(\tau_1,\tau_2)}{dp_{\rm T}dy} \xrightarrow{p_{\rm T}\to\infty} \frac{d\hat{\sigma}^{qg\to {\rm q}+X}(\tau_1,\tau_2)}{dp_{\rm T}dy} \quad ({\rm q} = \text{light quark}) \, .$$

The simplest option is clearly to use the zero-mass expressions from the outset, $d\hat{\sigma}^{Qg\to {\rm Q}+X}(\tau_1,\tau_2) \equiv d\hat{\sigma}^{qg\to {\rm q}+X}(\tau_1,\tau_2)$ and also to forget completely about the heavy-quark mass in the kinematics, $\tau_{1,2} \to p_{\rm T}e^{\mp y}/(\sqrt{s}x_{2,1})$. This defines the so-called SACOT scheme [7]. The problem of this scheme is that since the partonic cross sections behave as $d\hat{\sigma}^{qg\to {\rm q}+X}/d^3p \xrightarrow{p_{\rm T}\to 0} (\tau_{1,2})^{-n}$, it leads to infinite (positive or negative) production cross sections towards $P_{\rm T} \to 0$. This unphysical behaviour can be neatly avoided in what we call here the SACOT-$m_{\rm T}$ scheme [5]: The idea is to retain the $Q\overline{Q}$-pair kinematics also for the $Qg \to Q+X$ channel, implicitly understanding that the final state must still contain the $\overline{Q}$. With this physical motivation, we define $d\hat{\sigma}^{Qg\to {\rm Q}+X}(\tau_1,\tau_2) \equiv d\hat{\sigma}^{qg\to {\rm q}+X}(\tau_1,\tau_2)$ taking $\tau_{1,2} = m_{\rm T}e^{\mp y}/(\sqrt{s}x_{2,1})$ as in the massive FFNS case. This automatically leads to finite cross sections in the $P_{\rm T} \to 0$ limit.

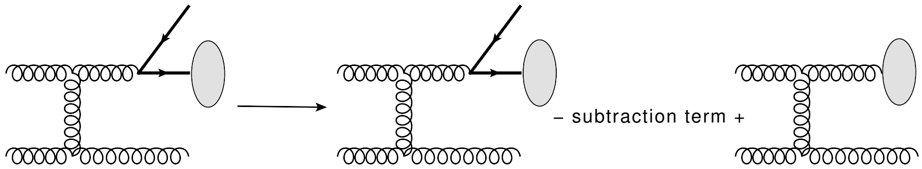

Figure 2: A schematic representation of how to deal with the final-state logarithms.

There are also collinear logarithms coming from the final-state e.g. when — as in Figure 2 above — an outgoing gluon splits into a $Q\overline{Q}$ pair. In this case the $\log(p_T^2/m^2)$ terms are resummed into the scale-dependent gluon FFs, $D_{g\rightarrow h_3}(z,\mu^2_{\rm frag})$. Thus, in GM-VFNS one has also the contribution of the $gg \rightarrow g+X$ channel,

$$\int \frac{dz}{z} dx_1 dx_2\, f_g^{h_1}(x_1,\mu^2_{\rm fact}) \frac{d\hat{\sigma}^{gg\rightarrow g+X}(\tau_1,\tau_2)}{dp_T dy} f_g^{h_2}(x_2,\mu^2_{\rm fact}) D_{g\rightarrow h_3}(z,\mu^2_{\rm frag})\,.$$

The compensating subtraction term is the same expression, but now with the gluon FF replaced by its perturbative form to first order in α_s,

$$D_{g\rightarrow h_3}(x,\mu^2_{\rm frag}) = \left(\frac{\alpha_s}{2\pi}\right) \log\left(\frac{\mu^2_{\rm frag}}{m^2}\right) \int_x^1 \frac{{\rm d}\ell}{\ell} P_{qg}\left(\frac{x}{\ell}\right) D_{Q\rightarrow h_3}(\ell)\,.$$

Consistently with our choice of scheme, also here we use the well-known zero-mass matrix elements for $d\hat{\sigma}^{gg\rightarrow g+X}(\tau_1,\tau_2)$ with the massive expressions for $\tau_{1,2}$. The latter accounts for the fact that even if the heavy quarks do not explicitly appear in the $gg \rightarrow g+X$ process, the origins of these contributions are in diagrams where the $Q\overline{Q}$ pair is created. Without going into more details, our final expression in the GM-VFNS is eventually

$$\frac{d\sigma}{dP_T dY} = \sum_{ijk} \int \frac{dz}{z} dx_1 dx_2 f_i^{h_1}(x_1,\mu^2_{\rm fact}) \frac{d\hat{\sigma}^{ij\rightarrow k}(\tau_1,\tau_2,m,\mu^2_{\rm ren},\mu^2_{\rm fact},\mu^2_{\rm frag})}{dp_T dy} f_j^{h_2}(x_2,\mu^2_{\rm fact}) D_{k\rightarrow h_3}(z,\mu^2_{\rm frag})\,,$$

where the sum runs over all parton flavours and the fragmentation function is also scale dependent. Towards $p_T \rightarrow 0$ the partonic cross sections tend to FFNS ones, but in the $p_T \rightarrow \infty$ limit to the zero-mass $\overline{\rm MS}$ expressions. In our numerical implementation, we have taken the light-parton$\rightarrow Q$ expressions up to $\mathcal{O}(\alpha_s^3)$ from the MNR code [8], and all the remaining processes from the INCNLO code [9], up to $\mathcal{O}(\alpha_s^3)$ as well.

4. Results and discussion

Figure 3 presents a comparison between the LHCb 13 TeV proton-proton data on D^0 mesons and our GM-VFNS theory calculation. The PDF uncertainty from NNPDF3.1 (pch) [11] is shown in darker colour and the combined scale+PDF uncertainties in light blue. The FFs used are those of Ref. [12]. The agreement is quite excellent though the scale uncertainties are large at small P_T. We also compare to an approach in which the partonic $c\overline{c}$ events from POWHEG event generator [13] are showered and hadronized with PYTHIA 8 [14]. Similarly to the FFNS calculations discussed earlier, the POWHEG+PYTHIA setup tends to underpredict the experimental results by a factor of two. We believe the most significant reason for this is that by starting with $c\overline{c}$ pairs generated

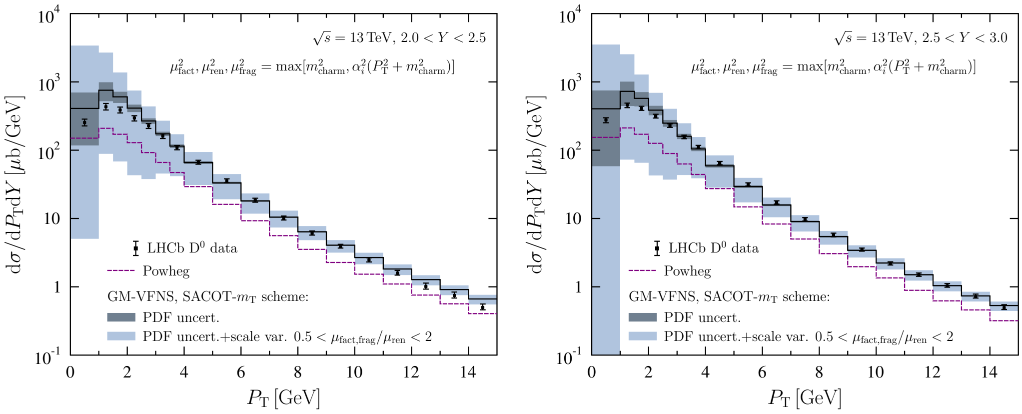

Figure 3: LHCb D^0 data [10] in proton-proton collisions compared with our GM-VFNS calculation and POWHEG+PYTHIA framework.

by POWHEG one misses the contributions in which the $c\bar{c}$ pair is created only later in the parton shower. Contributions like these are resummed in GM-VFNS to the scale-dependent FFs and, at high P_T, e.g. the gluon-to-D contribution is around 50% of the total cross section. In comparison to FFNS, we have also found that these contributions significantly alter the regions where the PDFs are sampled. Therefore, the use of FFNS-based calculations when fitting D-meson data with PDFs poses a potential bias.

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