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Hill's Equation in Arm Push of Shot Put and in Braking of Arm Rotation

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ABSTRACT

This chapter consists of the earlier study of shot put where A.V. Hill's force-velocity relationship was transformed into a constant maximum power model consisting of three different components of power. In addition, the braking phase of the arm rotation movement was examined where Hill's equation was applied for accelerated motions. Hill's force-velocity relationship was tested by fitting it into two arm push measurements of shot put experiments and one braking phase of whole arm rotation. Theoretically derived equation for accelerated motions was in agreement with the measured data of shot put experiments and the braking phase of the whole arm rotation experiment. Maximum power in these experiments was also tested by three different equations and two of them seemed to function well. The progress of movement in the studied experiments was concluded to be as follows: 1) the state of low speed and maximal acceleration which applies to the hypothesis of constant force, 2) the state of high speed and maximal power which applies to the hypothesis of constant power.

Keywords: Muscle mechanics; muscle power; force-velocity relationship; Hill's equation; arm movement; arm push in shot put.

1. INTRODUCTION

British Nobel laureate A.V. Hill invented the famous model of muscle mechanics which describes the force-velocity relationship of skeletal muscle contraction (Fig. 1). The equation of this model is $(F + a)(v + b) = b(F_0 + a)$, where F is maximum force in muscle contraction, a is constant force and b is constant velocity, F_0 is isometric force of muscle or the constant maximum force generated by muscle with zero velocity and v is velocity [1,2]. This equation was based on the laboratory measurements in which the force (F) of activated muscle was measured as the muscle was contracting at a constant speed in an isolated condition. In the equation the vectors of forces and velocities have the same direction and therefore Hill's equation can be presented in a scalar form. Other early experiments of force-velocity relationship of skeletal muscle were done by e.g. Fenn and Marsh [3] and good reviews are also available [4,5].

The arm rotation experiments of Rahikainen et al. [6] followed the theory, where movement was described to have four (4) different phases: 1) start of motion 2) movement proceeds at constant maximum rotational moment during the first part of the movement 3) movement proceeds at constant maximum muscular power during the second part of the movement 4) stopping of motion. For validation of these assumptions the equation was solved for angular velocity-time function:

$$I\,\frac{d\,\dot{\varphi}}{d\,T} = \frac{P}{\dot{\varphi}} - C\,\dot{\varphi}$$

This theoretically derived equation with constant maximum power (phase 3 above) was in good agreement with the experimentally measured results.

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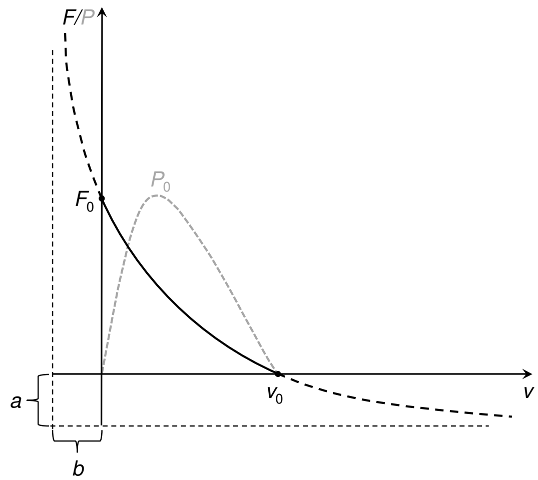


Fig. 1. Hill's force-velocity curve with the corresponding power P

The study of Rahikainen and Virmavirta [6] continued the experiments of the previous study [7] and further developed its theory of mechanics resulting in further solution of Hill's equation. The results were based on the assumption that in muscle mechanics there is a constant maximum power which the muscle is able to generate within a certain range of velocity. The principle of constant maximum power is also in Hill's equation and in this respect the two models can be considered the same. In the left side of Hill's equation the term $(F + a)(v + b)$ is muscles' total power including Fv, which is the power of moving the external load. The right side of the equation, $b\,(F_0 + a)$, includes only constants and thus the equation can be considered as a constant power model. However, the constant maximum power in the study of Rahikainen and Virmavirta [6] is a characteristic of whole muscle group instead of separate muscle fibers as in the Hill's equation. The model was based on the muscular system's ability to transfer chemical energy and, therefore, it is not necessary to know the contribution of the individual muscles involved.

The constant power of Hill's equation presented by Rahikainen and Virmavirta [6] is not the power of Hill's original curve as it is usually considered in biomechanics, but it is the sum of three different power components. It was inferred that the constant power model of the study of this paper acts during high speed movements with no external load, where Hill's equation does not seem to fit the experimental points [2, p. 32, Fig. 3.2] very well. As an explanation for this mismatch Hill mentioned that "sharp rise at the end of the curve in the region of very low tension was due to the presence of a limited number of fibers of high intrinsic speed and no such equation could fit the observed points below P/P_0 = 0.05".. Because Hill's equation is also a constant power model, it is acting only in a certain state of motion, which is constant power movement at low speed of motion decelerated by counter force. Therefore it is not a model of motion which is suitable for every state of muscle motion.

Although Hill's force-velocity relationship has been an important part of muscular mechanics models, it has deficiencies which to a great extent restrict its application for the real muscular mechanics of human motion. Hill's equation is a constant power model and it has no term for the power of acceleration and therefore it cannot be applied within accelerated motions. Also at the point of maximum speed, where force is zero, Hill's equation does not seem to fit the experimental points well. The third reason for the deficiencies of the models based on Hill's equation is that they take no account of the effect of the elastic properties of muscle-tendon unit.

The present chapter consists of the earlier study of the arm push of the Olympic shot put winner where the further develop of Hill's equation was applied. As an example of the accelerated motion, the braking phase of the whole arm rotation was also examined. Maximum power in these experiments was also tested by three different equations.

2. SHOT PUT

2.1 Hill's Equation in Accelerated Motion

In order to find out the function of Hill's force-velocity relationship in accelerated motion, equation of motion was derived from Hill's equation. In the present study Hill's equation is presented in a form:

$$(F+a)(v_{\mathrm{H}}+b)=(F_0+a)b \tag{1}$$

where

F is the muscular force in Hill's equation. The force F must be constant during the whole muscle contraction. If it is not, the equation of motion of muscle contraction will be much more complex.

v_H is constant velocity in Hill's equation, and during the muscle contraction the muscular force F corresponds to the velocity v_H in Hill's equation.

The results of Hill's experiments could be transformed into hyperbola equation describing force-velocity dependence of the movement. The left side of Hill's equation represents the maximum total power consumed into muscle contraction, and the right side of Hill's equation indicates that this maximum total power is constant. In the following this maximum total power is divided into three power components. Hill's equation, the motion with maximum constant power is the second state of motion. The first state of motion is with the maximum constant force, and in that state of motion acceleration is constant. Fig. 2 represents a further development of Hill's force-velocity relationship. Hill's equation, (F + a) (v_H + b) = constant, implies that the area of the rectangle (F + a) (v_H + b) is constant. The total power of the muscle is comprised of three different components represented by rectangles A, B and C. The area of rectangle A = Fv_H represents the power needed from muscle against an external load (see the power curve in Fig. 1). If there is no external load, this power is consumed by acceleration. The area of rectangle B = (F + a) b represents the power of muscle's internal loss of energy. This power creates a counter force against an external load. As the velocity is zero, this power B is highest and, therefore, it is not related to external movement. When velocity increases, this power decreases rapidly initially, then slowly at higher velocities. The area of rectangle C = v_H a represents the power of friction due to the motion of the muscle – load system. Because power is force multiplied by velocity, the force of friction is a. This is not force directly proportional to velocity, generally known as liquid friction (which is the friction used in the present study in paragraph 2.3), but constant force of friction which is known as glide friction. Now we can see that there are three different states of motion: 1) at the beginning of motion characterized by a state of low speed constant maximal acceleration, then 2) as the motion continues a state of high speed, constant maximal power, which applies to motion of Eq. (21) and to Hill's equation. The maximum power is due to the fact that the transfer of energy within the muscle system must have a maximum rate and, therefore, muscle's power generation must also have a certain maximum rate.

Application of Hill's equation into human movement is problematic. Hill's force-velocity relationship has no power term for acceleration, and therefore it is not valid in accelerated motions. Hill's force -velocity relationship was measured with a measuring device in which the muscle force is measured at constant velocity. Its applications must also be constant velocity movements.

Herein theoretical experiment is performed: A mass m is accelerated by a muscle contraction. The force generated by the muscle is F and its counter force is – F. In the beginning velocity is zero, and the movement is at a state of high acceleration. As the movement continues, velocity v increases and acceleration decreases, and if the movement continues sufficient long distance at some point the movement can be regarded as constant. Then there is force F corresponding to velocity v_H as it is in Hill's force – velocity relationship. The total power of Hill's equation can be divided into three separate power components (Fig. 2): the power of the work done against counter force Fv_H, the power of friction av_H and the power consumed within generation of muscle force $(F + a)\,b$. Because the muscle force F is constant and a and b are also constants, the power $(F + a)\,b$ is also constant. The total power at the phase of constant velocity is the sum of the three rectangles A, B and C (Fig. 2) which is:

$$F\,v_H + av_H + (F + a)\,b = (F + a)(v_H + b) = (F_0 + a)\,b \tag{2}$$

At the phase of acceleration the power consumption into acceleration is:

$$P_{acc} = m\frac{dv}{dt}v \tag{3}$$

where v is general velocity in movement containing also accelerated phase of motion.

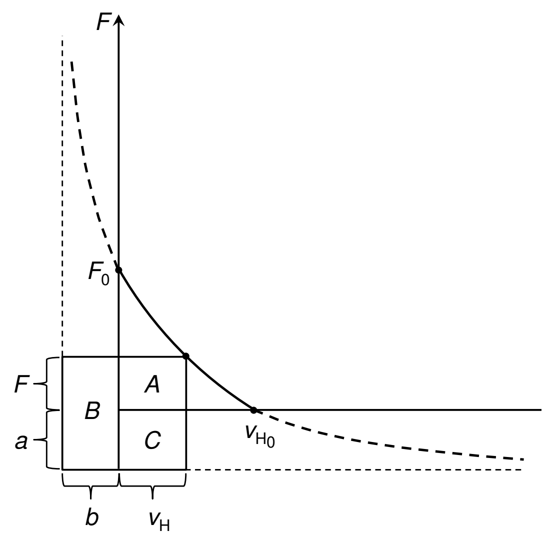


Fig. 2. Hill's force-velocity relationship presented with asymptotes (broken lines) and three rectangles of power. In traditional presentation of hyperbola *a* and *b* are negative, but here they refer to the positive constant terms of Hill' equation

Because at the phase of acceleration the velocity v is less than the constant velocity v_H, the power of the work done against counter force Fv and the power of friction av are less than that at the velocity v_H. The difference of these powers is equal to the power into acceleration. We obtain the equation of motion.

$$\begin{aligned} m\frac{dv}{dt}v &= Fv_H - Fv + av_H - av + \\ (F+a)b-(F+a)b &= F\left(v_H - v\right) + a\left(v_H - v\right) \end{aligned} \tag{4}$$

Solution

$$\frac{dv}{dt}v = \frac{F+a}{m}\left(v_H - v\right) \tag{5}$$

$$\frac{dv}{dt} = \frac{F+a}{m}\left(\frac{1-v/v_H}{v/v_H}\right) \tag{6}$$

$$\left(\frac{v/v_{\mathrm{H}} - 1 + 1}{1 - v/v_{\mathrm{H}}}\right) dv = \frac{F + a}{m}\, dt \tag{7}$$

$$\left(-1 + \frac{1}{1 - v/v_{\mathrm{H}}}\right) dv = \frac{F + a}{m}\, dt \tag{8}$$

$$-\int_0^v dv - v_{\mathrm{H}} \int_0^v -\frac{1}{v_{\mathrm{H}}} \ln\left(\frac{1}{1 - v/v_{\mathrm{H}}}\right) dv = \frac{F + a}{m} \int_0^t dt \tag{9}$$

$$-v - v_{\mathrm{H}}\left[\ln\left(1 - v/v_{\mathrm{H}}\right) - \ln\left(1\right)\right] = \frac{F + a}{m} t + C \tag{10}$$

constant C = 0

ln (1) = 0

$$t = -\frac{m}{F + a}\left[v + v_{\mathrm{H}} \ln\left(1 - v/v_{\mathrm{H}}\right)\right] \tag{11}$$

This is the equation of motion as mass m is accelerated by muscle contraction. Hill's velocity v_{H} corresponds to muscle force F in Hill's equation. Hill's velocity is the velocity after the phase of acceleration as the motion can be regarded as constant velocity movement. Calculation of the values of Hill's velocity v_{H} and muscular force F (Eq. 1) are:

$$\begin{aligned} &(F + a)(v_{\mathrm{H}} + b) = (F_0 + a) b \\ &F(v_{\mathrm{H}}) = \frac{F_0 b - a v_{\mathrm{H}}}{v_{\mathrm{H}} + b} \end{aligned} \tag{12}$$

Substituting $F = 0$ into Hill's equation

$$a v_{\mathrm{H0}} = F_0 b \tag{13}$$

$$b = \frac{a v_{\mathrm{H0}}}{F_0} \tag{14}$$

2.2 Numerical Calculations of Hill's Equation in Accelerated Motion

Theoretical velocity functions of the mass lifted against gravity force by muscle contraction are determined by selecting constant values F_0 = 1 N and v_{H0} = 1 m/s (for convenience), and a / F_0 = 0.27 (MacIntosh and Holash [8], p. 194). Moving mass m is equal to force divided by gravitational coefficient $m = F / g$.

First Hill's velocities are chosen for the curves of contraction equations which will be calculated (v_{H} = 0.2 m/s, 0.4 m/s, 0,6 m/s, 0.8 m/s, 0.9 m/s, Fig. 3). Then the values of constant force $a = 0.27\ F_0$ and constant velocity b using Eq. (14) are chosen (a = 0.27, b = 0.27). After that the corresponding values of force F using Eq. (12) are calculated (F = 0.460 N, 0.242 N, 0.124 N, 0.051 N, 0.023 N, Fig. 3).

Finally corresponding values of force F and Hill's velocity v_H are substituted into equation of muscle contraction (Eq.11), and the velocity curves of mass accelerated in muscle contraction resisted by constant counter force F are obtained, (Fig. 3).

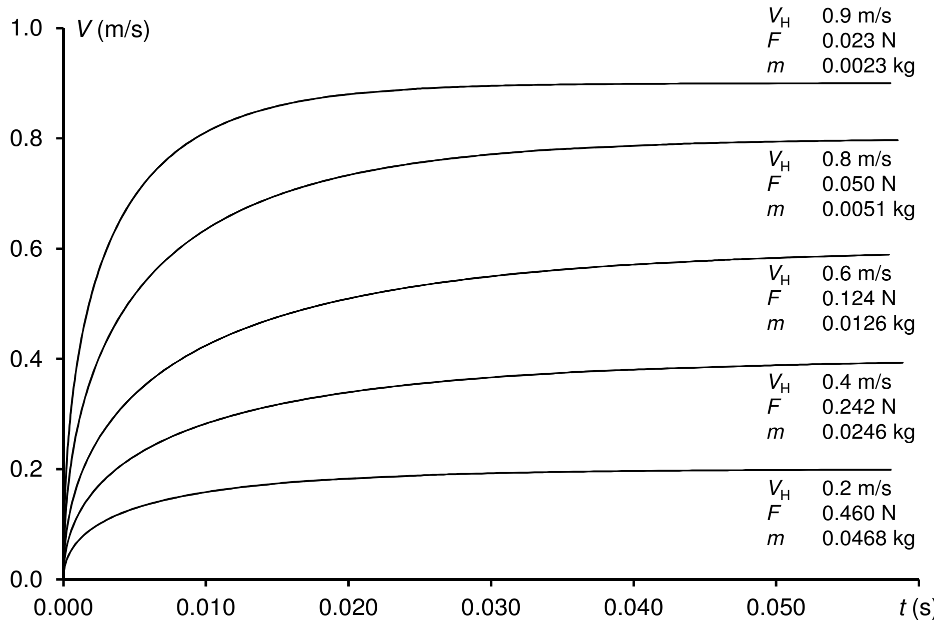


Fig. 3. Velocity curves of moving mass accelerated in muscle contraction resisted by counter force which is equal to the force of gravitation. Hill's velocity v_H is the limit velocity that the velocity of the moving mass approaches. Hill's velocity v_H corresponds to the velocity in the Hill's force velocity relationship and the force *F* corresponds to the force of Hill's force velocity relationship

2.3 Constant Power - Liquid Friction Model of Muscle Contraction

The model used in the present study is constructed according to Newton's II law, which was first used in linear motion of arm push in shot put (Rahikainen and Luhtanen [9]) and then applied to rotational motion by Rahikainen et al. [7] and Rahikainen and Virmavirta [6]. The theory of arm movement is as follows: At the beginning of the movement, velocity is naturally zero and it takes some time to generate force. At that phase of motion, passive elements of muscle-tendon unit have influence on the motion, but after reaching the full state of tension, they have no further dynamic effect. After that it can be assumed that a maximum muscle force takes action and at that phase of motion constant value glide friction acts. Because the muscle system is able to transfer only a certain quantity of chemical energy during the time of contraction, there must be a constant maximum power, which the muscle is able to generate within a certain range of velocity. As the velocity increases the motion reaches the point where the maximum power takes action and acting force is less than the maximum force. This way power remains constant as the velocity increases and the force decreases. At high velocity phase of motion, liquid friction, directly proportional to velocity, acts. The constant value glide friction decreases as forces at the joint decrease and it becomes indifferent. The model of arm movement during constant power phase in shot put study was constructed as follows: accelerating force is mass multiplied by acceleration which equals muscle force minus the force generated by inner friction of muscle. The effect of gravitational force is added afterwards.

$$m\frac{dV}{dT} = \frac{P}{V} - CV \tag{15}$$

Where

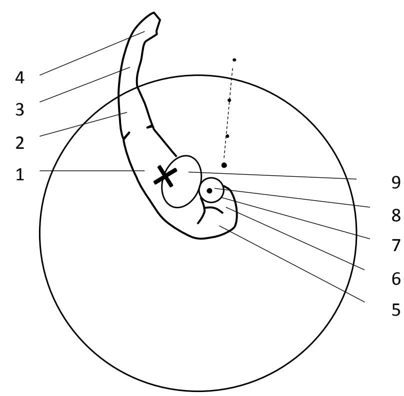
The weight of the shot 7.27 kg and the weight of the arm approximately 3.5 kg, or total weight 10.8 kg (Table 1).

Mass of shot and arm	m
Velocity of shot	V
Power generated by arm	P
Time of arm push	T
Pushing force	P / V
Internal friction in arm	$C\,V$

Internal friction of muscle is liquid friction inside muscle, which is directly proportional to velocity. The same liquid friction was also used in the study of Rahikainen et al. [7] which was initially adopted from Alonso and Finn [10].

Table 1. Body segment masses [11] and estimated moving mass of the shot putter (140 kg) in the present study

	% of total mass	mass (kg)	moving mass (kg)
1 trunk	34.70	48.6	
2 upper arm L	2.65	3.7	
3 forearm L	1.82	2.5	
4 hand L	0.50	0.7	
5 upper arm R	2.65	3.7	¼ = 0.925
6 forearm R	1.82	2.5	¾ = 1.875
7 hand R	0.50	0.7	0.7
8 shot		7.3	7.3
9 head	6.72	9.4	



Solution of velocity in Eq. (15)

$$m \frac{V}{P - CV^2}\, dV = d\,T \tag{16}$$

$$-\frac{m}{2C} \int_0^V -2CV \frac{1}{P - CV^2}\, dV = \int_0^T d\,T \tag{17}$$

$$\ln\left(P - CV^2\right) - \ln\left(P\right) = -\frac{2C}{m} T \tag{18}$$

$$\ln\left(\frac{P - CV^2}{P}\right) = -\frac{2C}{m} T \tag{19}$$

$$1 - \frac{C}{P} V^2 = e^{-\frac{2C}{m} T} \tag{20}$$

$$V = \sqrt{\frac{P}{C}\left(1 - e^{-\frac{2C}{m} T}\right)} \tag{21}$$

2.4 Effect of Gravitational Force on the Movement in Shot Put

The force that is induced by gravity was omitted from the motion model. The power generated by this gravity force is $P_{gr} = mg \cdot \sin(41^{\circ}) \cdot V = 69.5\ \mathrm{N} \cdot V$, where mg is gravitational force of moving mass (Fig. 4), V is velocity of arm movement. In Fig. 6, velocity (V in Eq. 21) coincides the measured velocity curve between 4 and 6 m/s and the best fit for power (4750 W, Fig. 6) is in the middle of these velocities, at 5 m/s. As the force of gravity is relatively small, the power induced by gravity was calculated in this study as a constant factor. It is included in the power P as follows; 1)

$$P_0 = P_{acc} + P_{fr} + P_{gr} \tag{22}$$

$$P = P_0 - P_{gr} = P_{acc} + P_{fr}$$

P is power in Eq. (21), P_0 is muscle power, acc is acceleration, fr is friction, gr is gravity, force of gravity is F. At the point B in Fig. 5 velocity is 4 m/s and the real power can be calculated as follows; 2)

$$P_{real} = P + (5 - 4)\ \mathrm{m/s} \cdot F = 4750\ \mathrm{W} + 69.6\ \mathrm{W} = 4819.6\ \mathrm{W}$$

The real velocity can be solved from the power ratio of Eq. (21); 3)

$$V_{real}/V = \sqrt{4819.6/4750} = 1.0073 \tag{23}$$

$$V_{real} = 4 \cdot 1.0073 = 4.029\ \mathrm{m/s}$$

which is within the accuracy of this study 4 m/s

In constant acceleration phase of movement

$$F_0 = F_{acc} + F_{fr} + F$$

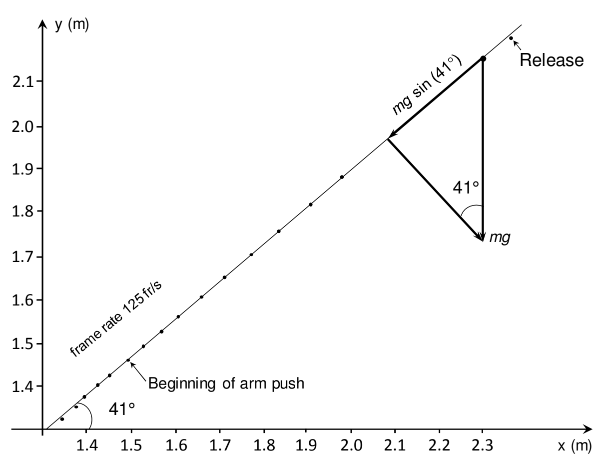


Fig. 4. Sideview of shot's path during the arm push (distance of the put 19.47 m)

2.5 Analysis of 19.47 m Put Using Eq. (21)

If the time of arm push is known, it is possible to determine the speed of shot during the arm push using speed curves from Rahikainen and Luhtanen [9], (Figs. 5 and 6). Thereby, the part corresponding to the time of arm push is separated from the end of the speed curve (e.g. Fig. 5). In the path of the shot (Fig. 6) it can be seen that in section A - B the arm push continues to generate speed with the maximal pushing force and the inclination of the speed curve is almost constant. This is because the maximal generation of speed is limited by the shot putter's maximal arm-pushing force. As the arm push continues, in section B - C - D, the pushing force accelerating the shot decreases and the inclination of the speed curve decreases as well. There are three different factors that cause the decrease in acceleration. First: as the speed of the shot increases, the rate of increase is not limited by a maximal pushing force, but by a maximal propulsive power, in which case force is power divided by velocity. Second: The internal friction of the pushing arm, which can be considered to be directly proportional to the velocity, decreases the velocity of the shot. Third: as the shot putter in the rotational motion turns sideways in respect to the direction of the arm push, the pushing force of the arm decreases and disappears and the arm just follows the shot without accelerating it. In Fig. 6 the broken line describes the effect of the first and second factor mentioned above. In section B – C, the measured speed curve and the broken line coincides. In this phase of the arm push, the two above-mentioned factors are the principal factors influencing the speed of the shot. In section C – D, the measured speed curve travels under the broken line. In this phase of the arm push, the shot putter turns so much sideways in respect to the direction of the arm push that the acceleration of the shot decreases further. If the shot putter would not turn (or rotate) during the arm push, the measured speed curve would combine with the broken line in section C - E. By fitting Eq. (21) into the measured speed curve in Fig. 6 values of internal friction and power are obtained C = 64.8 kg/s and P = 4750 W.

2.6 Analysis of Arm Push in Shot Put using Hill's Equation

In Hill's equation the velocity of muscle contraction v_H is measured, as the force $-F$ is resisting the motion. The muscle force is then F. In the beginning of movement the velocity of muscle contraction is zero, then the muscle force accelerates the motion, and the velocity increases. At some point it reaches maximum value, and at this constant speed phase of movement, velocity v_H in Hill's equation corresponds to the muscle force F. Theoretically the time of motion for the constant maximum velocity v_H is indefinite, but if the counter force $-F$ is strong enough, the movement decelerates and the constant maximum phase really exists in muscle mechanics. All the velocities v from zero to maximum velocity correspond to muscle forces greater than F (muscle force velocity = constant power) and therefore in this study the maximum velocity corresponding F is marked as Hill's velocity vH.

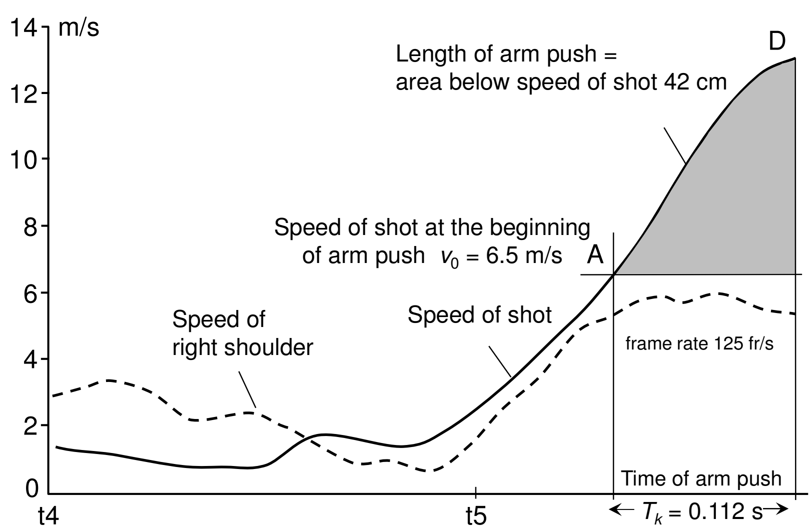
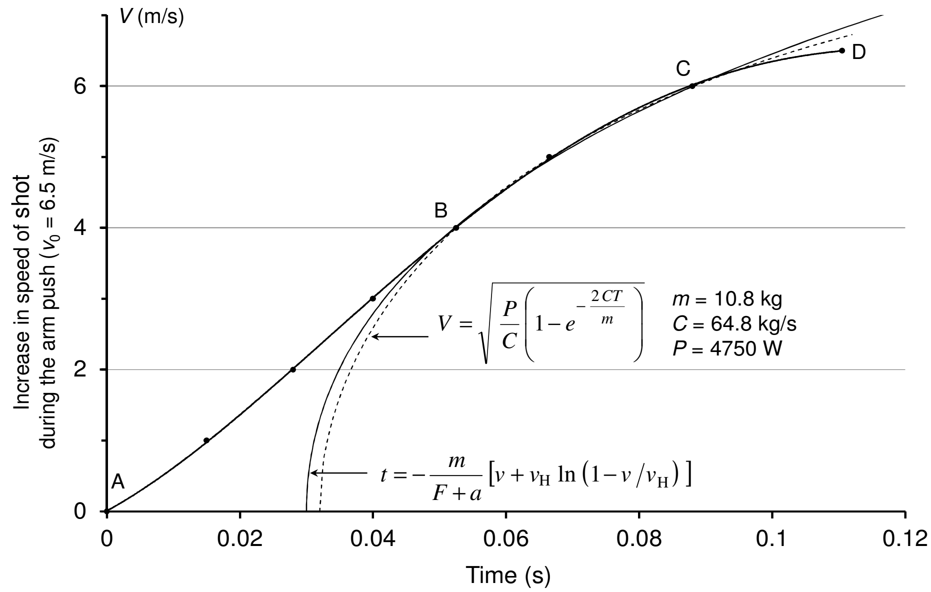


Fig. 5. The measured speed and the length of arm push (shaded area A–D) in 19.47 m shot put



$$F = \frac{F_0 b - a v_H}{v_H + b}, \quad m = 10.8\text{ kg},\ F_0 = 1260\text{ N},\ F = 69.6\text{ N},\ v_{H0} = 17.2\text{ m/s},\ v_H = 13.2\text{ m/s},$$

$$a = 0.24 \cdot F_0 = 302\text{ N},\ b = 0.24 \cdot v_{H0} = 4.12\text{ m/s}$$

Fig. 6. The measured speed of the 19.47 m shot put (curve A – B – C – D) and the theoretical speed curves, dashed line from Eq. (21) and solid line from Eq. (11). The zero time (*T* = 0) for theoretical curves is at 0.032 and 0.030 s, respectively

In arm push of shot put the corresponding progress of velocity does not reach the velocity value v_H because the range of movement is too short for that. In that movement all the velocity values are calculated from Eq. (11).

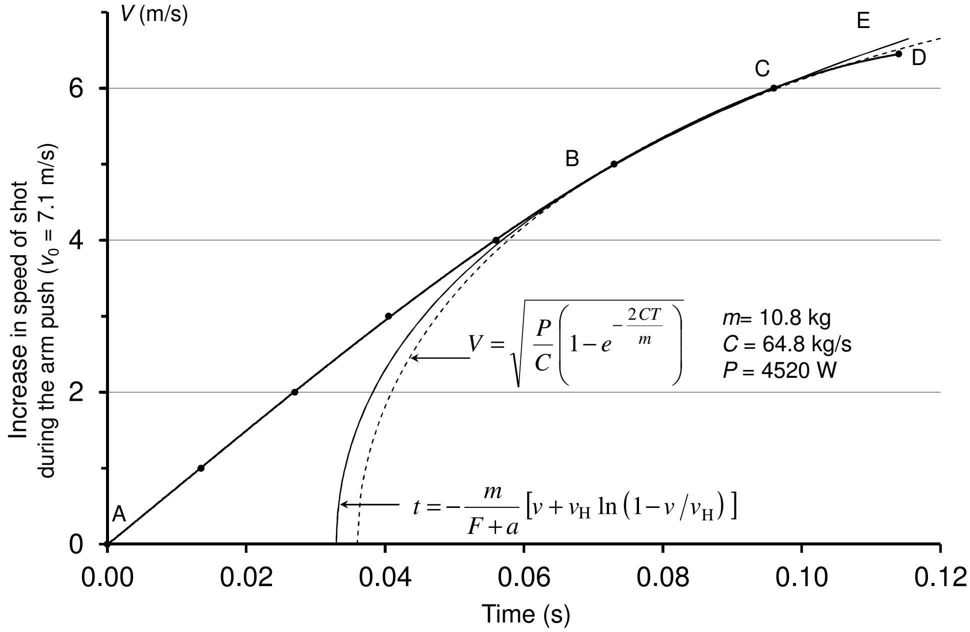
$$t = -\frac{m}{F+a}\left[v + v_H \ln\left(1 - v/v_H\right)\right]$$

2.7 Determining the Constants in Eq. (11)

The moving mass *m* is presented in Table 1. Gravitational force of the moving mass is *mg* = 10.8 · 9.82 = 106 N and the force resisting the acceleration of the shot is – *F* = – mg · sin(41º) = – 69.6 N, and the muscle force of Hill's equation is *F* = 69.6 N.

The maximum muscle force F_0 can be determined by two different ways. Shot putter's maximum bench press with two hands has been measured 260 kg and thus the estimated result for one hand is 130 kg. The maximum muscle force is then F_{01} = 130 kg · 9.82 m/s2 = 1280 N. The other way to determine the maximum muscle force F_{02} is to use Eq. (21) which yields the value of power 4750 W (Fig. 6). At the velocity of 4 m/s this power gives the force 4750 W / 4 m/s, and by adding the force of gravity of shot and hand the maximum muscle force is obtained F_{02} = 4750 W / 4 + 69.6 N = 1260 N.

Constant *a* in Eq. (11) is the constant value glide friction It can be determined in the phase of constant acceleration between A and B (V between ~1.5 - 3.5 m/s, Fig. 6) as follows: Force of friction *a* is equal to the maximum muscle force F_0 subtracted the gravitational force resisting acceleration *F* and the force of acceleration F_{acc}. The force of acceleration F_{acc} is equal to moving mass *m* multiplied by acceleration. The acceleration can be determined from the phase of constant acceleration by measuring the corresponding changes of velocity and time, dV/dT = 82 m/s2, and the force of acceleration is F_{acc} = 10.8 kg 82 m/s2 = 888 N. The force of friction (using F02) is a = 1260 N – 888 N – 69.6 = 302 N and the coefficient of friction is 302 N / 1260 N = 0.24.



$$F = \frac{F_0 b - a v_H}{v_H + b}$$, m = 10.8 kg, F_0 = 1200 N, F = 69.6 N, v_{H0} = 16.6 m/s, v_H = 12.7 m/s,

$a = 0.24 \cdot F_0$ = 288 N, $b = 0.24 \cdot v_{H0}$ = 3.99 m/s

Fig. 7. The measured speed of the 20.90 m shot put (curve A – B – C – D) and the theoretical speed curves, dashed line from Eq. (21) and solid line from Eq. (11). The zero time (T= 0) for theoretical curves is at 0.032 and 0.030 s, respectively

Hill's velocity v_H can be determined by iteration. Iteration can be done (for instance) so that Eq. (11) is calculated with proper value of v_H through the intersection point of the measured velocity curve and the line of the velocity 4 m/s (point B). The value of v_H is then found by iteration so that Eq. (11) matches the measured velocity curve at the velocity of 6 m/s (point C). Increase of the velocity value v_H results in increase of inclination of the iterated velocity curve. Constant b is calculated from Hill's equation.

2.8 Analysis of 20.90 m Put

Another shot put performance (20.90 m, Rahikainen and Luhtanen [9]) was analyzed in order to be able to compare the two puts and to learn more about the characteristics of the arm push. Another analysis was also needed to confirm the validity of the equation of motion. Measured speed curve of this put (dots) is presented in Fig. 7. The speed which is calculated with Eq. (21) coincides with the speed of further development of Hill's equation Eq. (11) in section B – C (Fig. 7). The propulsive power of the arm push is approximately P = 4520 W. The arm push in Fig. 6 has a greater pushing force 1260 N than in Fig. 7 1200 N, but at the end of the push the decrease in acceleration in Fig. 6 is so great that the total velocities generated during both arm pushes are almost equally high. The reason for the large decrease in acceleration is probably due to muscle's mechanics, maybe a pressure decrease in muscles as the turning of the body is getting larger. In the optimal arm push, this large acceleration decrease must be eliminated.

3. ARM ROTATION MOVEMENT

3.1 Model of Muscle Contraction in the Braking Phase of Arm Rotation

Two models of muscle contraction have been used for the braking phase of arm rotation. The first model is constructed according to Newton's II law, which was first used in linear motion by Rahikainen and Luhtanen [9] and then applied to rotational motion by Rahikainen et al. [7] and Rahikainen and Virmavirta [6]. The second model is Hill's equation, which has been solved for accelerated movement by Rahikainen et al. [12].

The theory of arm rotation in the braking phase is as follows: In the beginning of the braking phase, angular velocity has reached the maximum value. At this phase elasticity of the muscle-tendon unit has influence on the motion, which can be seen as a wave motion between points B – C in Fig. 8, but thereafter the muscle-tendon unit have full state of tension, and it has no further dynamic effect in the **braking phase of movement between points C – E**. The phase of braking movement between points C – D is the phase of maximum constant power.

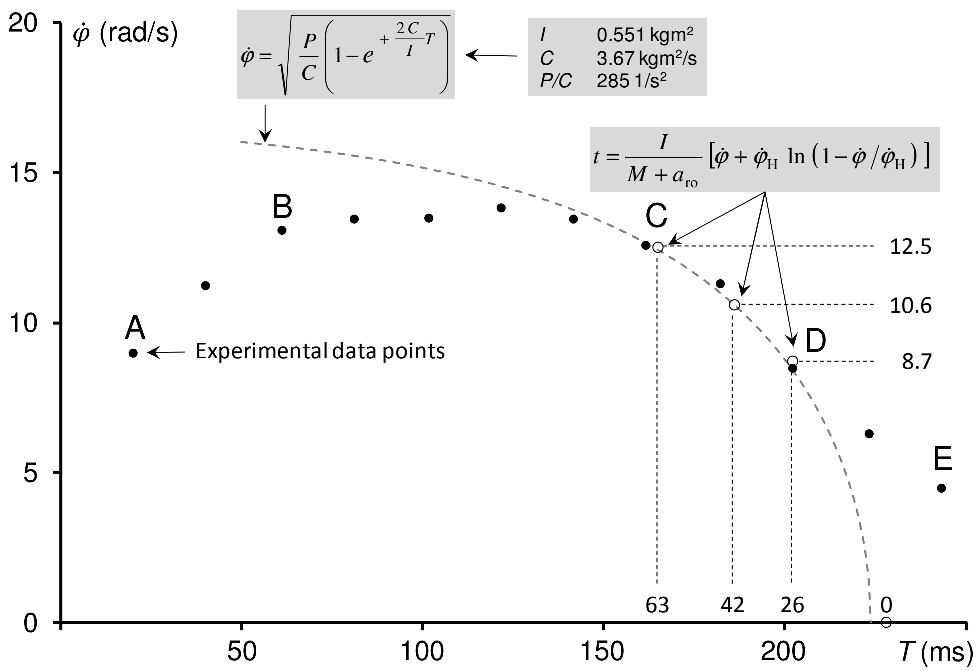


Fig. 8. The measured angular velocity values of whole arm rotation upwards are marked as filled circles (A – E), and the theoretical angular velocity values calculated from the equation Eq. (34) are shown as broken-line curve. Open circles are the points calculated from Hill's equation (Eq. 50). For convenience, the time for Eq. (50) is presented from right to left

Because the muscle system is able to transfer only a certain quantity of chemical energy during the time of contraction, it is obvious that arm rotation must have maximum power that cannot be exceeded. It can also be assumed that the maximum power acts within a certain range of velocity and it is a constant maximum power. As the braking of the movement starts, the maximum power takes action and the rotational moment is less than the maximum moment. This way power remains constant as the angular velocity decreases and rotational moment increases, until rotational moment is the maximum rotational moment at point D. In the braking phase of motion, moment of the frictional force of fluid friction $C\dot{\varphi}$, which is directly proportional to velocity, was used in the model of arm rotation. Within arm rotation experiments constant force of sliding friction turn out to be better friction approximation than fluid friction model. However, the fluid friction model was good enough within the range of Maximum constant power. Hill's equation is another model of Maximum constant power. The theoretical phase of movement is Maximum constant velocity which is the phase of the velocity of Hill's equation $\dot{\varphi}_{\mathrm{H}}$, which velocity in this study is called Hill's velocity. Usually arm rotation movements do not reach Hill's velocity within the range of movement. There must be strong enough counter moment *M*, Eq. (36), to resist the acceleration so that the movement reach Maximum constant velocity within the range of movement.

The model of deceleration is constructed according to Newton's II law as follows:

Phase of maximum constant power *P* decelerating movement

$$I\,\frac{d\,\dot{\varphi}}{d\,T} = -\frac{P}{\dot{\varphi}} + C\,\dot{\varphi} \tag{24}$$

Moment of inertia in arm rotation	I
Angular velocity	$\dot{\varphi}$
Maximum constant power generated by arm muscles	P
Time	T
Maximum constant moment generated by muscle force	$P/\dot{\varphi}$
Moment of fluid friction (inner friction of muscle)	$C\dot{\varphi}$
Coefficient of friction	C

Inner friction of muscle is fluid friction which is directly proportional to velocity. It can be seen in Fig. 8 that initially movement accelerates at Maximum constant moment M_0 between points A – B, which implies that the moment generated by muscle force is constant. After that the movement has reached maximum velocity between points B – C. As the velocity decreases, the movement decelerates at maximum constant power within the range of movement between points C – D in Fig. 8 and the power *P* is constant.

Solution

$$I\;\frac{\dot{\varphi}}{-P + C\dot{\varphi}^2}\,d\dot{\varphi} = d\,T \tag{25}$$

Substitution

$$U = -P + C\dot{\varphi}^2 \quad \Rightarrow \quad \frac{dU}{d\dot{\varphi}} = 2\,C\dot{\varphi}$$
$$d\dot{\varphi} = \frac{dU}{2\,C\dot{\varphi}} \tag{26}$$

$$I\,\frac{\dot{\varphi}}{U}\left(\frac{dU}{2C\dot{\varphi}}\right) = dT \tag{27}$$

$$\frac{I}{2C}\,\frac{1}{U}\,dU = dT \tag{28}$$

$$\frac{I}{2C}\int_0^U \frac{1}{U}\,dU = \int_0^T dT \tag{29}$$

$$\ln\left(U\right) - \ln\left(-P\right) = \frac{2C}{I}\,T \tag{30}$$

$$\ln\left(-P+C\dot{\varphi}^2\right)-\ln\left(-P\right)=\frac{2C}{I}T \tag{31}$$

$$\ln\left(\frac{-P+C\dot{\varphi}^2}{-P}\right)=\frac{2C}{I}T \tag{32}$$

$$1-\frac{C}{P}\dot{\varphi}^2=e^{\frac{2C}{I}T} \tag{33}$$

$$\dot{\varphi}=\sqrt{\frac{P}{C}\left(1-e^{\frac{2C}{I}T}\right)} \tag{34}$$

The theoretical phase of maximum constant velocity in Eq. (34) is

$$T=\infty \qquad \Rightarrow \qquad \dot{\varphi}=\sqrt{\frac{P}{C}} \tag{35}$$

The corresponding velocity in Hill's equation is $\dot{\varphi}_{\mathrm{H}}$, Eq. (36). The moment M in Hill's equation is the moment of gravitational force which resists the movement. Towards the end of the braking phase in Fig. 8 movement turns to horizontal direction, and therefore the moment of the gravitational force can be omitted (M = 0). In whole arm rotation the moment of gravitational force has notable effect on motion between points B – C in Fig. 8.

3.2 Constant Velocity Movement

Experiments of Rahikainen et al. [7] and Rahikainen and Virmavirta [6] verified that the conclusions of the theoretically derived equation with constant maximum power Eq. (21) were in agreement with experimentally measured results. As it is compared to Eq. (34), it can be seen that it is the same save the negative power. As Hill's equation is a constant power model, it is same as the model of Eq. (34) and Eq. (21) in that respect. Hill's force - velocity relationship was derived from the experiments in which the velocity of muscle contraction was measured against a certain constant force. The experiments of Hill's equation naturally started at zero velocity and continued in the same manner as the experiment of the present study through all the phases. Hill's equation was derived from muscle contraction experiments, in which moving masses were very small. The scale difference has a great effect on muscle forces and inertial forces of moving masses. For instance 10 times larger muscle has 100 times stronger muscle forces (related to cross-section area of the muscle) and 1000 times stronger inertial forces of moving masses (related to the volume of moving mass). Therefore Hill's equation is valid within the muscular mechanics of mice and mosquito, but within muscular mechanics of human being it must be solved into accelerated motion [13,14] and [15]. The measurement of the this study was made without external load and it did not reach the maximum theoretical velocity of Eq. (21).

$$\dot{\varphi}=\sqrt{\frac{P}{C}\left(1-e^{-\frac{2C}{I}T}\right)}$$

Maximum theoretical velocity of Eq. (21) and Eq. (34)

$$T=\infty \qquad \Rightarrow \qquad \dot{\varphi}=\sqrt{\frac{P}{C}}$$

Maximum theoretical velocity in Hill's equation is written here in the form $\dot{\varphi}_{\mathrm{H}}$

$$\left(M+a_{\mathrm{ro}}\right)\left(\dot{\varphi}_{\mathrm{H}}+b_{\mathrm{ro}}\right)=\left(M_0+a_{\mathrm{ro}}\right)b_{\mathrm{ro}} \tag{36}$$

where

M is the rotational moment resisting movement in Hill's equation and in real muscle movement. The moment *M* must be constant during the whole muscle contraction. If it is not, the equation of motion of muscle contraction will be much more complex.

$\dot{\varphi}_{\mathrm{H}}$ is the maximum constant angular velocity in Hill's equation, and during the muscle contraction the moment *M* corresponds to the angular velocity $\dot{\varphi}_{\mathrm{H}}$ in Hill's equation.

$\dot{\varphi}$ is general angular velocity in real muscle movement containing also accelerated phase of motion.

Three power components of Hill's equation are:

$$\begin{aligned}&\left(M+a_{\mathrm{ro}}\right)\left(\dot{\varphi}_{\mathrm{H}}+b_{\mathrm{ro}}\right)=\\&M\dot{\varphi}_{\mathrm{H}}+a_{\mathrm{ro}}\dot{\varphi}_{\mathrm{H}}+\left(M+a_{\mathrm{ro}}\right)b_{\mathrm{ro}}\end{aligned} \tag{37}$$

i) Power done against an external load $M\dot{\varphi}_{\mathrm{H}}$

ii) Power of friction due to the motion of muscle's internal structures $a_{\mathrm{ro}}\dot{\varphi}_{\mathrm{H}}$

iii) Power of muscle's internal loss of energy $\left(M+a_{\mathrm{ro}}\right)b_{\mathrm{ro}}$

These powers correspond to powers of translational movement related to areas A, B, C in Fig. 2. Hill's equation can be transformed into a form in which it is described by three power components Eq.(37). As the factors of muscles' maximum moment M_0, force of friction a_{ro} and constant factor b_{ro} are constants, it implies that the power $\left(M_0+a_{\mathrm{ro}}\right)b_{\mathrm{ro}}$ is also constant. The total power of the muscle is comprised of three different power components. The power $M\dot{\varphi}_{\mathrm{H}}$ represents the power, which the muscle does against external load (corresponding the area A in Fig. 2). If there is no external load, this power is consumed by acceleration. The power $\left(M+a_{\mathrm{ro}}\right)b_{\mathrm{ro}}$ represents the power of muscle's internal loss of energy (corresponding the area B in Fig. 2). This power creates a counter force against external load. As the velocity is zero, the force is highest. Thereafter, as velocity increases, initially this power decreases rapidly, then slowly at higher velocities. This power has no velocity variable, and therefore it is not related to movement. The power $a_{\mathrm{ro}}\dot{\varphi}_{\mathrm{H}}$ represents the power of friction due to the motion of muscle's internal structures (corresponding the area C in Fig. 2). Because power is force multiplied by velocity, the force of friction is a_{ro}. This is not the force directly proportional to velocity, generally known as fluid friction (which is the friction used in the upper model of muscle contraction), but constant force of friction which is known as sliding friction.

3.3 Model of Hill's Equation in the Braking Phase of Arm Rotation

The Hill's equation Eq. (36) for decelerating movement in Fig. 8 was solved between points C – D. The solution has been calculated from the theoretical maximum velocity phase of movement, in which Hill's equation Eq. (36) is valid.

Decelerated movement (braking phase)

At the phase of deceleration the power consumption into deceleration is

$$P_{\mathrm{dec}} = I\,\frac{d\dot{\varphi}}{dt}\,\dot{\varphi}\,. \tag{38}$$

Because at the phase of deceleration the velocity $\dot{\varphi}$ is less than the maximum constant velocity $\dot{\varphi}_{\mathrm{H}}$, the power of the work done against counter moment $M\dot{\varphi}_{\mathrm{H}}$ and the power of friction $a_{\mathrm{ro}}\dot{\varphi}_{\mathrm{H}}$ are less than that at the velocity $\dot{\varphi}_{\mathrm{H}}$. The difference of these powers is equal to the power into deceleration. As we know that equation Eq.(50) in curve fitting in Fig. 8 functions right with plus sign, we obtain the equation of motion as follows:

$$\begin{aligned}
&I\,\frac{d\dot{\varphi}}{dt}\,\dot{\varphi} = -M\,\dot{\varphi}_{\mathrm{H}} + M\,\dot{\varphi} - a_{\mathrm{ro}}\,\dot{\varphi}_{\mathrm{H}} + a_{\mathrm{ro}}\,\dot{\varphi} \\
&-\left(M + a_{\mathrm{ro}}\right)b_{\mathrm{ro}} + \left(M + a_{\mathrm{ro}}\right)b_{\mathrm{ro}} \\
&I\,\frac{d\dot{\varphi}}{dt}\,\dot{\varphi} = -M\left(\dot{\varphi}_{\mathrm{H}} - \dot{\varphi}\right) - a_{\mathrm{ro}}\left(\dot{\varphi}_{\mathrm{H}} - \dot{\varphi}\right)
\end{aligned} \tag{39}$$

and solution of rotational motion corresponding to Eq.(11) is

$$\frac{d\dot{\varphi}}{dt}\,\dot{\varphi} = \frac{-M - a_{\mathrm{ro}}}{I}\left(\dot{\varphi}_{\mathrm{H}} - \dot{\varphi}\right) \tag{40}$$

$$\frac{d\dot{\varphi}}{dt}\,\dot{\varphi}/\dot{\varphi}_{\mathrm{H}} = -\frac{M + a_{\mathrm{ro}}}{I}\left(1 - \dot{\varphi}/\dot{\varphi}_{\mathrm{H}}\right) \tag{41}$$

$$\frac{d\dot{\varphi}}{dt} = -\frac{M + a_{\mathrm{ro}}}{I}\left(\frac{1 - \dot{\varphi}/\dot{\varphi}_{\mathrm{H}}}{\dot{\varphi}/\dot{\varphi}_{\mathrm{H}}}\right) \tag{42}$$

$$\left(\frac{\dot{\varphi}/\dot{\varphi}_{\mathrm{H}} - 1 + 1}{1 - \dot{\varphi}/\dot{\varphi}_{\mathrm{H}}}\right)d\dot{\varphi} = -\frac{M + a_{\mathrm{ro}}}{I}\,dt \tag{43}$$

$$\left(-1 + \frac{1}{1 - \dot{\varphi}/\dot{\varphi}_{\mathrm{H}}}\right)d\dot{\varphi} = -\frac{M + a_{\mathrm{ro}}}{m}\,dt \tag{44}$$

Substitution

$$\begin{aligned}
&U = 1 - \dot{\varphi}/\dot{\varphi}_{\mathrm{H}} \quad \Rightarrow \\
&\frac{dU}{d\,\dot{\varphi}} = -1/\dot{\varphi}_{\mathrm{H}} \quad \Rightarrow \quad d\,\dot{\varphi} = -\dot{\varphi}_{\mathrm{H}}\,dU
\end{aligned} \tag{45}$$

$$-d\dot{\varphi} - \dot{\varphi}_{\mathrm{H}}\,\frac{1}{U}\,dU = -\frac{M + a_{\mathrm{ro}}}{I}\,dt \tag{46}$$

$$\int_0^{\dot{\varphi}} d\dot{\varphi} + \dot{\varphi}_{\mathrm{H}}\int_1^{U}\frac{1}{U}\,dU = \frac{M + a_{\mathrm{ro}}}{I}\int_0^{t} dt \tag{47}$$

$$\dot{\varphi} + \dot{\varphi}_{\mathrm{H}} \left[\ln \left(U \right) - \ln \left(1 \right) \right] = \frac{M + a_{\mathrm{ro}}}{I} t + C \tag{48}$$

$$\dot{\varphi} + \dot{\varphi}_{\mathrm{H}} \left[\ln \left(1 - \dot{\varphi} / \dot{\varphi}_{\mathrm{H}} \right) - \ln \left(1 \right) \right] = \frac{M + a_{\mathrm{ro}}}{m} t + C \tag{49}$$

Constant $C = 0$ and ln (1) = 0

$$t = \frac{I}{M + a_{\mathrm{ro}}} \left[\dot{\varphi} + \dot{\varphi}_{\mathrm{H}} \ln \left(1 - \dot{\varphi} / \dot{\varphi}_{\mathrm{H}} \right) \right] \tag{50}$$

3.4 Analysis of the Braking Phase of Whole Arm Rotation Movement

At the section of braking movement C – D deceleration decreases, and the movement decelerates at constant power movement. This is followed by stopping of the movement.

The theory of muscle contraction in arm rotation movement in Fig. 8 is as follows:

The phase of Acceleration

Maximum rotational moment between points A – B.

Maximum power acceleration the motion after point B (very short).

The phase of Maximum constant velocity

Maximum velocity between points B – C is less than the theoretical Maximum constant velocity in Hill's equation $\dot{\varphi}_{\mathrm{H}}$ and the Maximum velocity of Eq. (35).

The Phase of Deceleration

Maximum power decelerates the motion between points C – D. Fitting of Eq. (34) and Eq. (50) into the measured velocity curve yields two Maximum constant power values, *P* from Eq. (34) and $a_{\mathrm{ro}}\dot{\varphi}_{\mathrm{H}}$ from Eq. (50). At the end the movement turns to horizontal, and therefore the effect of gravity is about zero $M = 0$.

Eq. (34)

$$\dot{\varphi} = \sqrt{\frac{P}{C} \left(1 - e^{\frac{2C}{I} T} \right)}$$

Eq. (50)

$$t = \frac{I}{M + a_{\mathrm{ro}}} \left[\dot{\varphi} + \dot{\varphi}_{\mathrm{H}} \ln \left(1 - \dot{\varphi} / \dot{\varphi}_{\mathrm{H}} \right) \right]$$

The end of the deceleration with maximum constant power is the point D in Fig. 8. At this point the rotational moment generated by muscle is maximum constant moment, and maximum constant power is maximum constant moment multiplied by velocity.

The effect of the force of gravity, the moment M was omitted from the equation of arm rotation Eq.(24)

$$I\,\frac{d\,\dot{\varphi}}{d\,T}=-\frac{P}{\dot{\varphi}}+C\,\dot{\varphi}$$

Adding the effect of the force of gravity yields the equation

$$m\frac{d\dot{\varphi}}{dT}=\frac{-P-M\,\dot{\varphi}_C}{\dot{\varphi}}+M+C\dot{\varphi} \tag{51}$$

The real constant power decelerating the movement is approximately

$$P+M\,\dot{\varphi}_C \tag{52}$$

in which the velocity $\dot{\varphi}_C$ varies, and in Fig. 8 the constant power phase of decelerated motion is from 12.5 rad/s to 8.7 rad/s, and the average constant value for velocity $\dot{\varphi}_C$ is 10.6 rad/s.

Power consumption in decelerated motion between C – D in Fig. 8

Power consumption into deceleration

$$(M_0 - a_{ro} - M)\ \dot{\varphi} \tag{53}$$

Power consumption of friction $\quad a_{\mathrm{ro}}\dot{\varphi}$ (54)

Power consumption of force of gravity $\quad M\,\dot{\varphi}$ (55)

Total power consumption $\quad M_0\dot{\varphi}$ (56)

Power consumption in maximum constant velocity, phase of Hill's Equation (theoretical)

$$(M+a_{\mathrm{ro}})(\dot{\varphi}_{\mathrm{H}}+b_{\mathrm{ro}})=(M_0+a_{\mathrm{ro}})b_{\mathrm{ro}}$$

Left side of Hill's equation

$$(M+a_{\mathrm{ro}})(\dot{\varphi}_{\mathrm{H}}+b_{\mathrm{ro}})=M\,\dot{\varphi}_{\mathrm{H}}+a_{\mathrm{ro}}\,\dot{\varphi}_{\mathrm{H}}+M\,b_{\mathrm{ro}}+a_{\mathrm{ro}}\,b_{\mathrm{ro}}$$

Power done against external force

$$M\,\dot{\varphi}_{\mathrm{H}} \tag{57}$$

Power into friction

$$a_{\mathrm{ro}}\,\dot{\varphi}_{\mathrm{H}} \tag{58}$$

Power into muscle's operational ability

$$M\,b_{\mathrm{ro}}+a_{\mathrm{ro}}\,b_{\mathrm{ro}} \tag{59}$$

Maximum constant power consumption into motion in Hill's equation

$$M\,\dot{\varphi}_{\mathrm{H}}+a_{\mathrm{ro}}\dot{\varphi}_{\mathrm{H}} \tag{60}$$

Arm rotation experiment presented in Fig. 8 was recorded by the camera system of Rahikainen [16]. The measuring technique has been described in papers [6,7,17,18]. The theoretical angular velocity curve of Eq.(34) is marked with broken-line and it coincides with the measured angular velocity points between C – D, where movement proceeds at constant power according to Eq. (34).

Theoretical angular velocity of Eq. (50), which is Hill's equation in decelerated motion, is marked with open circles and it coincides with the measured angular velocity curves between C – D, where movement proceeds at constant power according to Eq. (50).

Three Maximum Power values for testing Eq. (50) and Eq. (34)

Moment of deceleration $I \cdot d\,\dot{\varphi}/dt$

Moment of muscle's internal friction a_{ro}

Moment of gravitation resisting movement M

Maximum constant moment $M_0 = I\,\dfrac{d\,\dot{\varphi}}{dT} + a_{ro} + M$

1) Maximum constant muscle power (Fig. 8) at the point of intersection (~ 8.7 rad/s, 26 ms) in which maximum muscle power changes into maximum muscle moment (M_0), Eq. (56)

$M_0\,\dot{\varphi}$ $\qquad\qquad$ $\dot{\varphi}$ = 8.7 1/s

This is the only point where the moment has its maximum value, M_0, because in the constant power phase (C - D) moment decreases as $\dot{\varphi}$ increases (>8.7 rad/s).

2) Maximum constant muscle power of Hill's equation, Eq.(36), Eq.(60)

$M\dot{\varphi}_{\mathrm{H}} + a_{\mathrm{ro}}\dot{\varphi}_{\mathrm{H}}$

3) Maximum constant muscle power of Eq.(34), Eq.(52)

$P + M\,\dot{\varphi}_C$

If these **three maximum power** values are same, this analysis of the braking phase of arm rotation movement has been proved true.

Maximum constant power in fluid friction model of muscle contraction Eq.(34)

$$\dot{\varphi} = \sqrt{\frac{P}{C}\left(1 - e^{+\frac{2C}{I}T}\right)}$$

P/C = 287 1/s

Eq.(52) $P + M\,\dot{\varphi}_C = 287 \cdot 3.67 + 0 \cdot \dot{\varphi}_C = 1053\,\mathrm{W}$

In Fig. 8 during braking phase ~150 – 200 ms movement turns to horizontal and therefore, the effect of gravitational force is zero and the moment of gravitational force is also zero $M = 0$.

Average moment of fluid friction (inner friction of muscle) at the velocity 10.6 rad/s $C \cdot \dot{\varphi} = 3.67 \cdot 10.6 = 39.0\ \mathrm{Nm}$

Maximum constant moment, which here is maximum braking force moment, was calculated at point D (Fig. 8). Slope of the velocity curve is equal to the deceleration of movement. The value for the slope of the velocity curve ($\Delta\,\dot{\varphi}/\,\Delta T$) at the point (8.7 rad/s, 26 ms) was approximated 155.4 rad/s^2. The force of deceleration is then

$$I\,\frac{d\dot{\varphi}}{dT} = 0.551 \cdot 155.4 = 85.6\ \mathrm{Nm}$$

The total moment generated by the muscles is maximum constant muscle moment M_0, and its counter moment is comprised of three moment components:

Moment of acceleration $I \cdot d\,\dot{\varphi}/dt$
Moment of muscle's internal friction a_{ro}
Moment of gravitation resisting movement M
Maximum constant muscle moment M_0 = 85.6 + 39.0 + 0.0 = 124.6 Nm

Iteration process

Iteration of the value of Hill's velocity $\dot{\varphi}$ has been done in Fig. 8. The time is from right to left. The time corresponding velocity $\dot{\varphi}$ = 12.5 rad/s has been calculated and by using the starting point for iteration $\dot{\varphi}_{\mathrm{H}} = 30\ \mathrm{rad/s}$.

$$t\,(12.5) = -\frac{0.551}{38.5}\,\left[12.5 + 30\ \ln\,(1 - 12.5\,/30)\right] = 0.0525\ \mathrm{s}$$

The point (12.5 rad/s, 0.0525 s) is above the measured velocity curve, and therefore Hill's velocity must be less than $\dot{\varphi}_{\mathrm{H}}$ = 30 rad/s. After some iterations the right Hill's velocity will be found $\dot{\varphi}_{\mathrm{H}}$ = 26.5 rad/s.

$$t\,(8.7) = -\frac{0.551}{38.5}\,\left[8.7 + 26.5\ln\,(1 - 8.7\,/26.5)\right] = 0.026\,\mathrm{s}$$
$$t\,(12.5) = -\frac{0.551}{38.5}\,\left[12.5 + 26.5\ln(1 - 12.5/26.5)\right] = 0.063\,\mathrm{s}$$
$$t\,(10.6) = -\frac{0.551}{38.5}\,\left[10.6 + 26.5\ln(1 - 10.6/26.5)\right] = 0.042\,\mathrm{s}$$

4. CHECKING THE POWER CALCULATIONS

4.1 Braking Phase of the Arm Rotation Movement

Calculation of the angular velocity $\dot{\varphi}$ at point D (Fig. 8) where the measured velocity points and the theoretical velocity curve (Eq. 50) coincide is checked first below.

$$I = 0.551\,\mathrm{kg\,m^2} \quad M_0 = 124.6\ \mathrm{N\,m} \quad M = 0$$
$$\dot{\varphi}_{\mathrm{H}} = 26.5\,\mathrm{rad/s} \quad a_{\mathrm{ro}} = 39\,\mathrm{N\,m}$$
$$P = 1053\ \mathrm{W} \quad \mathrm{C} = 3.67\ \mathrm{kg\,m^2/s}$$

At the point D (Fig. 8), the estimation of the velocity $\dot{\varphi}$ from the measured data is 8.7 rad/s.

Assuming Eq.(56) = Eq. (60)

$$M_0\,\dot{\varphi} = M\,\dot{\varphi}_{\mathrm{H}} + a_{\mathrm{ro}}\,\dot{\varphi}_{\mathrm{H}}$$

$$\dot{\varphi} = \frac{M\,\dot{\varphi}_{\mathrm{H}} + a_{\mathrm{ro}}\,\dot{\varphi}_{\mathrm{H}}}{M_0}$$

$$= \frac{0 \cdot 26.5 + 39 \cdot 26.5}{124.6} = 8.3\ \mathrm{rad/s}$$

In Fig. 8 it can be seen that Hill's equation Eq.(50) functions right in the decelerated motion when velocity is higher than $\dot{\varphi}$ = 8.3 rad/s (approximately point D).

The calculation of **maximum power into motion in Hill's equation** Eq.(60) and **maximum power in fluid friction model** of muscle contraction, *P* in Eq.(34) is checked below.

The calculation maximum constant power into motion in Hill's equation yields

$$M\,\dot{\varphi}_{\mathrm{H}} + a_{\mathrm{ro}}\,\dot{\varphi}_{\mathrm{H}} = 0 \cdot \dot{\varphi}_{\mathrm{H}} + 39 \cdot 26.5 = 1034\ \mathrm{W}$$

Maximum constant power in fluid friction model of muscle contraction Eq.(34) can be found by utilizing Eq. (52)

$$\dot{\varphi} = \sqrt{\frac{P}{C}\left(1 - e^{+\frac{2C}{I}T}\right)}$$

P/C = 287 1/s (Fig. 8)

Eq.(52) $P + M\,\dot{\varphi}_C = 287 \cdot 3.67 + 0 \cdot \dot{\varphi}_C = 1053\ \mathrm{W}$

Maximum power consumption in decelerated motion calculated at the point of intersection of maximum power and maximum moment at velocity 8.3 rad /s in Fig. 8 can be obtained by Eq.(56):

$M_0\,\dot{\varphi}$ = 124.6 Nm × 8.3 rad/s = 1034 W

In that case the coefficient of friction is

$$a_{\mathrm{ro}} / M_0 = 39/124.6 = 0.31$$

Hill's velocity value 26.5 rad/s must be very near the right one. The intersection point of the phases of Maximum constant power and Maximum constant moment may be lower than 8.7 rad/s, because in the phase of movement D – E there is no phase of Maximum constant moment. Therefore it may be that the value of Maximum constant power 1084 W is too high. However, all the values of Hill's velocity are close each other, and therefore it can be concluded that this analysis of Hill's equation functions right.

4.2 Checking of Shot Puts, 19.47 m and 20.90 m

As in the arm rotation before (4.1), the checking of the calculations for the **shot put of 19.47 m** (Fig. 6) is presented here:

$$m = 10.8\ \mathrm{kg} \quad F_0 = 1260\ \mathrm{N} \quad F = 69.6\ \mathrm{N}$$
$$v_\mathrm{H} = 13.2\ \mathrm{m/s} \quad a = 302\ \mathrm{N} \quad P = 4750\ \mathrm{W}$$

The equations Eq. (56) and Eq.(60) used in the rotational movement (4.1) are changed to the translational movement:

$$F_0\, v = F\, v_\mathrm{H} + a\, v_\mathrm{H} \tag{61}$$
$$v = \frac{F\, v_\mathrm{H} + a\, v_\mathrm{H}}{F_0}$$
$$= \frac{69.6 \cdot 13.2 + 302 \cdot 13.2}{1260} = 3.9\ \mathrm{m/s}$$

After the velocity v = 3.9 m/s Hill's equation acts in the motion. In Fig. 6 it can be seen that the measured velocity curve begins to bend approximately at that point and therefore, it can be inferred that Hill's equation functions right in accelerated motion.

Checking the calculations of **maximum power into motion in Hill's equation**, right side of Eq.(61), $F\, v_\mathrm{H} + a\, v_\mathrm{H}$ and **maximum power in fluid friction model** of muscle contraction P, Eq.(21) in Fig. 6.

Thus maximum constant power into motion in Hill's equation yields

$$F\, v_\mathrm{H} + a\, v_\mathrm{H} = 69.6 \cdot 13.2 + 302 \cdot 13.2 = 4900\ \mathrm{W}$$

Maximum constant power in fluid friction model of muscle contraction yields

Eq.(21) $$V = \sqrt{\frac{P}{C}\left(1 - e^{-\frac{2C}{m}T}\right)}$$

P = 4750 W,

The constant power decelerating the movement is (Eq. 52) $P + M\,\dot{\varphi}_C$ and for corresponding translational movement

$$P + FV_\mathrm{C} = 4750 + 5 \times 69.6 = 5100\ \mathrm{W} \tag{62}$$

Maximum power consumption in accelerated motion calculated at the point of beginning of the phase of maximum constant power at velocity 3.9 m/s in Fig. 6 can be obtained as follows:

left side of Eq. (61)

$F_0 v$ = 1260 N × 3.9 m/s = 4914 W

In this case the friction coefficient is a / F_0 = 0.24
The check for the **shot put of 20.90 m** (Fig. 7) follow the same principle as the previous check for the shot put of 19.47 m.

$$m = 10.8\ \mathrm{kg} \quad F_0 = 1200\ \mathrm{N} \quad F = 69.6\ \mathrm{N}$$
$$v_\mathrm{H} = 12.7\ \mathrm{m/s} \quad a = 288\ \mathrm{N} \quad P = 4520\ \mathrm{W}$$

$$F_0\, v = F\, v_{\mathrm{H}} + a\, v_{\mathrm{H}}$$

$$v = \frac{F\, v_{\mathrm{H}} + a\, v_{\mathrm{H}}}{F_0} =$$

$$\frac{69.6 \cdot 12.7 + 288 \cdot 12.7}{1200} = 3.8 \text{ m/s}$$

In Fig. 7 it can be seen that this point is not a match to the point at which the measured velocity curve begins to bend.

Maximum constant power into motion in Hill's equation, right side of Eq. (61) yields

$$F\, v_{\mathrm{H}} + a\, v_{\mathrm{H}} = 69.6 \cdot 12.7 + 288 \cdot 12.7 = 4540 \text{ W}$$

Maximum constant power (Eq.21) in fluid friction model of muscle contraction yields

P = 4690 W,

The constant power decelerating the movement is

$P + FV_C$ = 4520 W + 69.6 × 5 W = 4870 W

Maximum power consumption in accelerated motion calculated at the point of the beginning of the phase of maximum constant power at velocity 4.0 m/s in Fig. 7:

left side of Eq.(61)

$F_0 v$ = 1200 N × 4.0 m/s = 4800 W

Coefficient of friction is a / F_0 = 288 / 1200 = 24

5. DISCUSSION AND CONCLUSIONS

Rahikainen et al. [7] showed that in the first part of the arm rotation movement acceleration was constant and during the second part of movement equation of constant power (Eq. 34) fitted the measured velocity curve. The following study [6] continued the arm rotation experiments and a new approach to Hill's equation was presented.

5.1 Shot Put

The shot put experiments of the present study have the similar progress of movement with the previous ones, and the above mentioned constant acceleration and constant power phases have been proved to be true as well. The correct functioning of Eq. (21) is also proven as the two forces F_{01} = 1260 N and F_{02} = 1200 N are close to each other.

In Figs. 6 and 7 the measured velocity curve has a phase of constant acceleration between velocity values ~1.5 - 3.5 m/s and it can be inferred that within this range of velocity constant maximum muscle force is produced presuming that the force of friction is constant value glide friction. The constant friction a in the phase of constant acceleration and in the phase of constant power in Eq. (11) is assumed to be the same and mechanics in the equation seems to function properly with this assumption. In MacIntosh and Holash [8] the values of friction coefficient a/P_0 for human elbow flexors are given, 0.45, 0.4 and 0.39, in which P_0 is maximum moment for values 0.4 and 0.45 and maximum force for value 0.39 and a corresponds to the force of friction. Values of friction coefficient a/P_0 0.2-0.3 are also used. Proper functioning of Hill's equation implies the action of counter force F and if the counter force is weak, velocity increases, and the friction coefficient a/P_0 may be bigger 0.6 – 0.7.

The original purpose of this study was to find out how well the models of Eq. (21) and the further development of Hill's equation, Eq. (11), match the measured velocity in the arm pushes of two shot put performances. The fittings of these two constant power equations succeeded and they did function very much the same manner. Both of power equations fitted the measured velocity curve between velocity values 4 - 6 m/s and thus the constant power model proved to be true within this range of velocity. However, these two models of constant power, Eq. (11) and Eq. (21), are different and they differ at higher velocities significantly. The highest velocity value of Hill's equation in accelerated movement (Eq. 11) in Fig. 6 is v_H = 13.2 m/s and the highest velocity of Eq. (21) is $V = \sqrt{P/C}$ = 8.56 m/s.

It was verified that in the phase of constant acceleration three different constant forces are acting: the force of acceleration, the force of friction and the force of gravity and added together they represent the maximum muscle force. The range of correspondence between the measured and theoretical velocities of the shot put experiments was long enough to confirm the existence of constant power models. Kinetic friction was assumed to be directly proportional to velocity at the beginning of the movement. It is possible that kinetic friction at small velocities is constant and at high velocities is directly proportional to velocity. This leads to a constant torque accelerating the movement at the beginning of movement. It is also possible that the constant acceleration phase of movement is rather a matter of human ability to learn effective modes of motion than a direct cause of natural laws. It may be that human nervous system controls the rotational moment accelerating the arm movement.

The present application of Hill's equation in accelerated motion could be worth to apply in the accelerated rotational movement as well. The analysis of additional subjects with different performance level would also help to understand better the function of Hill's equation in accelerated movement.

After checking the shot put of 20.90 m experiment, it can be concluded that further analysis is needed. The friction term of Hill's equation a = 288 N must be bigger.

5.2 Arm Rotation Movement

The braking phase of whole arm rotation of this study was initially analysed by Rahikainen and Virmavirta [6]. However, in that analysis an error was made, so that the whole arm rotation was analysed as downwards rotation, but actually it was upwards rotation. Therefore the time had wrong direction. The error was found, and there was no doubt about the direction of the movement, because maximum velocity of downward movement is much higher than upward movement. Because of this error the very important finding was made that the muscle's braking movement, which is called eccentric movement, is much the same as accelerating movement, which is called concentric movement. The actual Hill's equation at maximum constant velocity values is valid only within concentric movement, but not within eccentric movements. However, the arm rotation movement in Fig. 8 shows that Hill's equation is valid both in accelerated and decelerated movements. Another important finding was that as the equation that fitted the decelerated arm rotation movement was known, the corresponding equation of motion, Eq. (21), was obtained by deriving backwards the solution process:

$$V=\sqrt{\frac{P}{C}\left(1-e^{+\frac{2C}{m}T}\right)}\Rightarrow \text{Eq.(33)}\Rightarrow \text{Eq.(32)}\ldots\Rightarrow \text{Eq.(25)}\Rightarrow I\,\frac{d\,\dot{\varphi}}{d\,T}=-\frac{P}{\dot{\varphi}}+C\,\dot{\varphi}$$

Comparing the two equations of motion in accelerating and decelerating movement, it can be seen that the term of friction $C\,\dot{\varphi}$ is always against the muscle force $P\,/\,\dot{\varphi}$, not against the direction of motion as it usually is. Therefore, it must be related to muscle's force and power generation process decreasing muscle's force generation.

$$I\,\frac{d\,\dot{\varphi}}{d\,T}=\frac{P}{\dot{\varphi}}-C\,\dot{\varphi}$$

$$I\,\frac{d\,\dot{\varphi}}{d\,T}=-\frac{P}{\dot{\varphi}}+C\,\dot{\varphi}$$

COMPETING INTERESTS

Authors have declared that no competing interests exist.

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He received his M.Sc. degree (shipbuilding engineering) from Helsinki University of Technology in 1976. However, his line of work was directed elsewhere. From 1983 – 1985, he did skiing research at the University of Helsinki, Institute of Physics. In this project, he studied the effect of ski elasticity on skier's leg push. After the ski research project he developed a photographic device for measuring advancing human movements. This device generates a series of subject images with light marker on one camera frame and this way the light-lines indicate the progress of movement. He got a patent on that device in 1990. During 1997 to 2008 he worked at Helsinki University of Technology, in Laboratory of Mechanics during 1997 – 2002 and in the Institute of Mathematics during 2002 – 2008. During that time he developed four studies of shot put technique. Important study for the solution of Hill's equation was the study of arm push in shot put. He received the Licentiate of Science Degree (Lic.Sc) in theoretical and applied mechanics from the Helsinki University of Technology in 2008. He continued the postgraduate studies in biomechanics at Jyväskylä University, Department of Biology of Physical Activity. He received his Ph.D. in 2015, and the name of his doctoral thesis is "Modeling Muscle Mechanics of Arm and Leg Movement, A new approach to Hill's equation". This thesis comprises two studies of arm rotation movement and one study of leg jumping movement. The paper "Hill's equation in the arm push of shot put" which solved Hill's equation in accelerated motion, was published in "British Journal of Applied Science & Technology" in 2016. The monograph "Constant Power Solution of Hill's Equation" was published by Book Publisher International in 2019.

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