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## Fascinating puzzle called double beta decay

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# Fascinating Puzzle Called Double Beta Decay 

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#### Abstract

The question of whether neutrinos are Majorana or Dirac particles and what are their average masses remains one of the most fundamental problems in physics today. Observation of neutrinoless double beta decay ( $0 v \beta \beta$ ) would verify the Majorana nature of the neutrino and constrain the absolute scale of the neutrino mass spectrum. The inverse half-life for $0 \nu \beta \beta$-decay is given by the product of a phase space factor (PSF), a nuclear matrix element (NME), which both rely on theoretical description, and a function $f$ containing the physics beyond the standard model. Recent calculations of PSF and NME will be reviewed together with comparison to other available results. These calculations serve the purpose of extracting the average neutrino mass if $0 v \beta \beta$ decay is observed, and of guiding searches if $0 \nu \beta \beta$-decay is not observed. The current situation is then discussed by combining the theoretical results with experimental limits on the half-life of neutrinoless double beta decay. The extracted limits on the average light neutrino mass will be addressed, complemented with a discussion of other possible $0 \nu \beta \beta$-decay mechanisms and scenarios.


## INTRODUCTION

Even though double beta decay was proposed already at 1930's to establish the nature of neutrinos [1], neutrinoless $\beta \beta$-decay has not yet been observed and it remains the most sensitive probe to following open questions: What is the absolute neutrino mass? What is the nature of neutrinos, are they Dirac or Majorana particles? Are there more neutrino species than we know so far? How many neutrino species are there?

A direct measurement of the average mass can be obtained from the observation of the neutrinoless double- $\beta$ decay $(0 \nu \beta \beta)$

$$
\begin{equation*}
{ }_{Z}^{A} X^{N} \rightarrow{ }_{Z \pm 2}^{A} Y_{N \mp 2}+2 e^{\mp} . \tag{1}
\end{equation*}
$$

Several experiments are underway to detect this decay, and others are in the planning stage (for review see e.g. [2, 3]). The half-life for this decay can be written as

$$
\begin{equation*}
\left[\tau_{1 / 2}^{0 v}\right]^{-1}=G_{0 v}\left|M_{0 v}\right|^{2}\left|f\left(m_{i}, U_{e i}\right)\right|^{2}, \tag{2}
\end{equation*}
$$

where $G_{0 v}$ is a phase space factor (PSF), $M_{0 v}$ the nuclear matrix element (NME) and $f\left(m_{i}, U_{e i}\right)$ contains physics beyond the standard model through the masses $m_{i}$ and mixing matrix elements $U_{e i}$ of neutrino species.

Concomitant with the neutrinoless modes, there is also the process allowed by the standard model, two neutrino double beta decay $(2 \nu \beta \beta)$. For this process, the half-life can be, to a good approximation, factorized in the form

$$
\begin{equation*}
\left[\tau_{1 / 2}^{2 v}\right]^{-1}=G_{2 v}\left|M_{2 v}\right|^{2} . \tag{3}
\end{equation*}
$$

[The factorization here is not exact and conditions under which it can be done are discussed in Ref. [4].]
The processes that have attracted the most attention are of the type $\left(\beta^{-} \beta^{-}\right)$

$$
\begin{equation*}
(A, Z) \rightarrow(A, Z+2)+2 e^{-}+\text {anything. } \tag{4}
\end{equation*}
$$

In recent years, interest in the processes $\left(\beta^{+} \beta^{+}\right)$

$$
\begin{equation*}
(A, Z) \rightarrow(A, Z-2)+2 e^{+}+\text {anything } \tag{5}
\end{equation*}
$$

has also arisen. In this case there are also the competing modes in which either one or two electrons are captured from the electron cloud $\left(0 v E C \beta^{+}, 2 v E C \beta^{+}, R 0 v E C E C, 2 v E C E C\right)$. Also for these modes, the half-life can be factorized similarly to Eqs. (2) and (3) (either exactly or approximately) into the product of a phase space factor and a nuclear matrix element which then are the crucial ingredients of any double- $\beta$ decay calculation.

In order to extract physics beyond the standard model, contained in the function $f$ in Eq. (2), we need an accurate calculation of both $G_{0 v}$ and $M_{0 v}$. These calculations will serve the purpose of extracting the neutrino mass $\left\langle m_{v}\right\rangle$ if $0 \nu \beta \beta$ is observed, and of guiding searches if $0 v \beta \beta$ is not observed. PSFs and NMEs have been evaluated, or are under evaluation, systematically for all processes of interest. The nuclear matrix elements have been calculated within the framework of the microscopic interacting boson model (IBM-2) [5, 6, 7, 8, 9, 10, 11], and phase space factors have been evaluated using exact Dirac electron wave functions as reported in $[4,9,12,13,14]$.

## PHASE SPACE FACTORS

A general theory of phase space factors in double- $\beta$ decay was developed years ago by Doi et al. [15, 16] and it was reformulated by Tomoda [17]. In these calculations an approximate expression for the electron wave functions at the nucleus was used. PSF were recalculated as described in detail in Ref. [4] taking advantage of recent developments in the numerical evaluation of Dirac wave functions and in the solution of the Thomas-Fermi equation in order to have more accurate phase space factors for double- $\beta$ decay in all nuclei of interest.

In Fig. 1 the effect that using exact Dirac wave functions has on phase space factors is demonstrated. As shown in the figure the difference between approximate and current calculations is few percent for calcium and comes larger as the mass number increases being already roughly twenty percent for tellurium.


FIGURE 1. (Color online) Comparison of phase space factors $G_{0 v}$ in units of $10^{-15} \mathrm{yr}^{-1}$. The label "approximate" refers to the results obtained by the use of approximate electron wave functions and "this work" refers to current results. The figure is in semilogarithmic scale.

Current calculations including lifetimes, single and summed electron spectra, and angular electron correlations, are available for download on the webpage nucleartheory.yale.edu.

## NUCLEAR MATRIX ELEMENTS

Nuclear matrix elements have been evaluated in a variety of models, traditionally using the quasiparticle random phase approximation (QRPA) and the interacting shell model (ISM), and more recently within energy density functional theory (EDF) and microscopic interacting boson model (IBM-2). The calculation of $0 \nu \beta \beta$ NMEs is challenging task, since neutrinoless double beta decay is a unique process an there is no direct probe which connects the initial and final
states other than the process itself. Thus other relevant data has to be employed, such as single particle occupation probabilities [18], to test the feasibility of the wave functions, and eventually the $0 v \beta \beta$ NMEs.

The double beta decay nuclear matrix elements are calculated by connecting the initial and final state wave functions with proper transition operator depending on the scenario and mechanism of the decay. For light neutrino exchange the "neutrino potential" in closure approximation is defined as $v(p)=2 \pi^{-1}[p(p+\tilde{A})]^{-1}$, where $\tilde{A}$ is the closure energy. The full matrix element is a combination of Fermi (F), Gamow-Teller (GT) and Tensor (T) contributions as

$$
\begin{align*}
M^{(0 v)} & =M_{G T}^{(0 v)}-\left(\frac{g_{V}}{g_{A}}\right)^{2} M_{F}^{(0 v)}+M_{T}^{(0 v)}  \tag{6}\\
M_{0 v} & =g_{A}^{2} M^{(0 v)} .
\end{align*}
$$

In Fig. 2 the comparison of nuclear matrix elements calculated in different models, IBM-2, QRPA, ISM and EDF, is shown. The variation between different calculations is rather large, but the trend is still similar. All listed calculations give larger values at the middle of the shell than at closed shells. The spreading of the results in different models might give some hints on the effective value of the axial vector coupling constant, $g_{A}$, that multiplies the total NME in Eq. (6) and is discussed further later.


FIGURE 2. (Color online) Comparison of IBM-2 [10, 18], QRPA-Jy [19], ISM [20], and EDF [21] 0 $\nu \beta \beta$ NMEs for light neutrino exchange.

## LIMITS ON NEUTRINO MASS

For light neutrino exchange

$$
\begin{equation*}
f=\frac{\left\langle m_{\nu}\right\rangle}{m_{e}}, \quad\left\langle m_{v}\right\rangle=\sum_{k=l i g h t}\left(U_{e k}\right)^{2} m_{k} \tag{7}
\end{equation*}
$$

where the effective neutrino mass, $\left\langle m_{\nu}\right\rangle$, is the quantity of interest to be extracted from experiments. For the extraction of $\left\langle m_{\nu}\right\rangle$ nuclear matrix elements in IBM-2 are combined with the calculated phase space factors, and for now, the free value of $g_{A}=1.269$ is used. Current experimental half-life limits along with the extracted limits to effective neutrino mass are presented in Table 1.

The average light neutrino mass is constrained by atmospheric, solar, reactor and accelerator neutrino oscillation

TABLE 1. Current lower half-life limits for light neutrino exchange coming from different experiments: Majorana [24], GERDA [25], NEMO-3 [26], CUORE [27], EXO-200 [28], KamLAND-Zen [29].

| Experiment | nucleus | $\tau_{1 / 2}$ | $\left\langle m_{v}\right\rangle$ |
| :--- | :---: | :---: | :---: |
| Majorana | ${ }^{76} \mathrm{Ge}$ | $>1.9 \times 10^{25} \mathrm{yr}$ | $<0.27 \mathrm{eV}$ |
| GERDA | ${ }^{76} \mathrm{Ge}$ | $>8 \times 10^{25} \mathrm{yr}$ | $<0.13 \mathrm{eV}$ |
| NEMO-3 | ${ }^{100} \mathrm{Mo}$ | $>1.1 \times 10^{24} \mathrm{yr}$ | $<0.44 \mathrm{eV}$ |
| CUORE | ${ }^{130} \mathrm{Te}$ | $>1.5 \times 10^{25} \mathrm{yr}$ | $<0.19 \mathrm{eV}$ |
| EXO-200 | ${ }^{136} \mathrm{Xe}$ | $>1.8 \times 10^{25} \mathrm{yr}$ | $<0.21 \mathrm{eV}$ |
| KamLAND-Zen | ${ }^{136} \mathrm{Xe}$ | $>1.07 \times 10^{26} \mathrm{yr}$ | $<0.09 \mathrm{eV}$ |

experiments to be [22]

$$
\begin{align*}
&\left\langle m_{\nu}\right\rangle=\left|c_{13}^{2} c_{12}^{2} m_{1}+c_{13}^{2} s_{12}^{2} m_{2} e^{i \varphi_{2}}+s_{13}^{2} m_{3} e^{i \varphi_{3}}\right|, \\
& c_{i j}=\cos \vartheta_{i j}, \quad s_{i j}=\sin \vartheta_{i j}, \quad \varphi_{2,3}=[0,2 \pi],  \tag{8}\\
&\left(m_{1}^{2}, m_{2}^{2}, m_{3}^{2}\right)=\frac{m_{1}^{2}+m_{2}^{2}}{2}+\left(-\frac{\delta m^{2}}{2},+\frac{\delta m^{2}}{2}, \pm \Delta m^{2}\right) .
\end{align*}
$$



FIGURE 3. (Color online) Current limits to $\left\langle m_{\nu}\right\rangle$ from Majorana [24], GERDA [25], NEMO-3 [26], CUORE [27], EXO-200 [28],
KamLAND-Zen [29] experiments, and IBM-2 Argonne SRC nuclear matrix elements. Red shows the normal hierarchy and green the inverted hierarchy. The value of Ref. [33] is shown by $X$. It is consistent only with nearly degenerate neutrino masses. The figure is in logarithmic scale.

Using the best fit values [23]

$$
\begin{align*}
\sin ^{2} \vartheta_{12} & =0.297, \quad \sin ^{2} \vartheta_{13}=0.0215 \\
\sin ^{2} \vartheta_{23} & =0.425, \quad \delta m^{2}=7.37 \times 10^{-5} \mathrm{eV}^{2}  \tag{9}\\
\Delta m^{2} & =2.525 \times 10^{-3} \mathrm{eV}^{2}
\end{align*}
$$

the plot given in Fig. 3 is obtained. Figure also shows the current limits, for $g_{A}=1.269$, coming from Majorana [24], GERDA [25], NEMO-3 [26], CUORE [27], EXO-200 [28], KamLAND-Zen [29] experiments..

## QUENCHING OF $g_{A}$

It is well known from single beta decay and electron capture that $g_{A}$ is renormalized in models of nuclei to $g_{A, e f f}$. Two reasons for this include: The limited model space in which the calculations are done gives rise to a quenching factor $q_{N^{e x}}$, and secondly the omission of non-nucleonic degrees of freedom which gives rise to a quenching factor $q_{\Delta}$. Related to the first reason, an (model-dependent) estimate of $g_{A, e f f}$ can be obtained from the experimental knowledge of single- $\beta$ decay and/or of $2 \nu \beta \beta$-decay.

In case of neutrinoless double beta decay the question of effective value of $g_{A}$ is very much still open due several reasons: It is not known if the renormalization is the same for $0 v \beta \beta$ as it is for $2 v \beta \beta$. In $2 v \beta \beta$ only the $1^{+}$multipole contribute but in $0 v \beta \beta$ all the multipoles and both parities contribute. Also the quenching may be different for different contributing multipoles. In addition, the two processes differ by momentum transfer: in $2 v \beta \beta$ the momentum transfer is about few MeV while in $0 v \beta \beta$ it is of the order 100 MeV .

At the moment the three suggested values for effective value of $g_{A}$ are: The free value 1.269 ; the quark value 1 ; model dependent, maximal quenching $1.269 A^{-0.18}$ (for IBM-2).


FIGURE 4. (Color online) Schematic presentation of three suggested effective $g_{A}$ values. If $g_{A}$ is renormalized to $\sim 1-0.5$, all estimates for limits on the average neutrino mass should be increased by a factor $\sim 1.6-6$, making it very difficult to reach in the foreseeable future even the inverted region.

This is a critical issue, since $g_{A}$ enters the half-life equation to the power of 4 thus giving up 6 times larger limits for the average neutrino mass with maximally quenched value as shown schematically in Fig. 4. Various studies are currently addressing this issue: Theoretical studies using effective field theory [30], experimental and theoretical studies of single beta decay and single charge exchange reactions involving the intermediate odd-odd nuclei, comparison of the shapes of the calculated and measured $\beta$-electron spectra of forbidden nonunique $\beta$-decays [31], and experimental NUMEN program, that aims measuring both single and double charge exchange reaction intensities with heavy ions [32].

## OTHER MODES AND MECHANISMS OF DOUBLE BETA DECAY

In case of $0 \nu \beta^{+} \beta^{+}$and $0 v E C \beta^{+}$the predicted half-lives are $10^{2-6} \mathrm{yr}$ times longer compared to $0 v \beta^{-} \beta^{-}$, and thus these decay modes are hardly detectable in near future. More details about predictions for these decays can be found at [10].

The neutrinoless double electron capture $0 v E C E C$, in general, cannot conserve energy and momentum in the process

$$
\begin{equation*}
(A, Z)+2 e^{-} \rightarrow(A, Z-2) \tag{10}
\end{equation*}
$$

However, conservation of energy and momentum can occur in the special case in which the energy of the initial state matches precisely the energy of the final state. The precise matching condition is an exceptional circumstance which may or may not occur in practice. A slightly less stringent condition is that the decay occurs through the tail of the width of the atomic initial state. For this process the inverse half-life can be to a good approximation factorized as

$$
\begin{equation*}
\left[\tau_{1 / 2}^{E C E C}\left(0^{+}\right)\right]^{-1}=G_{0 v}^{E C E C}\left|M_{0 v} E C E C\right|^{2}\left|f\left(m_{i}, U_{e i}\right)\right|^{2} \frac{\left(m_{e} c^{2}\right) \Gamma}{\Delta^{2}+\Gamma^{2} / 4} \tag{11}
\end{equation*}
$$

where $G_{0 v}^{E C E C}$ is a prefactor depending on the probability that a bound electron is found at the nucleus and the last factor, resonance enhancement factor, is the figure of merit for this process. $\Delta=\left|Q-B_{2 h}-E\right|$ is called the degeneracy parameter, $\Gamma=\Gamma_{e_{1}}+\Gamma_{e_{2}}$ is the two-hole width and $B_{2 h}$ is the energy of the double-electron hole in the atomic shell of the daughter nuclide including binding energies and Coulomb interaction energy. Five most interesting candidates, namely, ${ }^{124} \mathrm{Xe},{ }^{152} \mathrm{Gd},{ }^{156} \mathrm{Dy},{ }^{164} \mathrm{Er}$, and ${ }^{180} \mathrm{~W}$ were analyzed in detail in Ref. [9]. In case of ${ }^{152} \mathrm{Gd} \sim 14$ resonance enhancement was obtained but even in this case the predicted half-life is $\sim 10^{4} \mathrm{yr}$ times longer than lowest prediction for $0 \nu \beta^{-} \beta^{-}$.

## Heavy neutrino exchange

For heavy neutrino exchange

$$
\begin{equation*}
f \equiv \eta=m_{p}\left\langle m_{v_{h}}^{-1}\right\rangle, \quad\left\langle m_{v_{h}}^{-1}\right\rangle=\sum_{k_{h}=\text { heavy }}\left(U_{e k_{h}}\right)^{2} \frac{1}{m_{k_{h}}}, \tag{12}
\end{equation*}
$$

and the matrix elements for heavy neutrino exchange can simply be calculated by replacing the "neutrino potential" by the potential $v_{h}(p)=2 \pi^{-1}\left(m_{e} m_{p}\right)^{-1}$. The phase space factors are the same as in light neutrino exchange.

There are no direct experimental bounds on $\eta$. Recently, Tello et al. [34] have argued that from lepton flavor violating processes and from large hadron collider (LHC) experiments one can put some bounds on the right-handed leptonic mixing matrix $U_{e k_{h}}$ and thus on $\eta$. In the model of Ref. [34], when converted to current notation, $\eta$ can be written as

$$
\begin{equation*}
\eta=\frac{M_{W}^{4}}{M_{W R}^{4}} \sum_{k=\text { heavy }}\left(V_{e k_{h}}\right)^{2} \frac{m_{p}}{m_{k_{h}}} \tag{13}
\end{equation*}
$$

where $M_{W}$ is the mass of the $W$-boson, $M_{W R}$ is the mass of $W R$-boson, and $V=\left(M_{W R} / M_{W}\right)^{2} U$. By comparing the calculated half-lives with their current experimental limits, limits on the lepton nonconserving parameter $|\eta|$ can be set as shown in Table XV of Ref. [10].

## Majoron emission

Even though most current experimental efforts have been focused to the detection of the mass mode where $f\left(m_{i}, U_{e i}\right) \propto$ $\left\langle m_{\nu}\right\rangle$, interest on the mechanism predicting $0 \nu \beta \beta$-decays through the emission of additional bosons called Majorons has also renewed lately. These bosons couple to the Majorana neutrinos and give rise to neutrinoless double beta decay, accompanied by Majoron emission $0 \nu \beta \beta M$. In this case the inverse half-life is given by

$$
\begin{equation*}
\left[\tau_{1 / 2}^{0 v \beta \beta \phi}\right]^{-1}=G_{0 v \phi}\left|M_{0 v}\right|^{2}\left|\left\langle g_{e e}^{M}\right\rangle\right|^{2} \tag{14}
\end{equation*}
$$

where $g$ is the effective Majoron coupling constant. The NME for this scenario are the same as for light neutrino exchange and the PSFs have been recalculated along with limits on ordinary Majoron decay in Ref. [13].

## Sterile neutrinos

Sterile neutrinos, if they exist, will contribute to neutrinoless double beta decay. It is therefore of interest to estimate the expected half-life for Majorana neutrinos of arbitrary mass. When the mass $m_{N}$ is intermediate, and especially, when it is of the order of magnitude of $p_{F}$, the factorization of Eq. (2) is not possible, and physics beyond the standard model is entangled with nuclear physics. In this case, the half-life can be written as [11]

$$
\begin{equation*}
\left[\tau_{1 / 2}^{0 v}\right]^{-1}=G_{0 v}\left|\sum_{N}\left(U_{e N}\right)^{2} M_{0 v}\left(m_{N}\right) \frac{m_{N}}{m_{e}}\right|^{2} \tag{15}
\end{equation*}
$$

In this case $f=m_{N} / m_{e}$ and the "neutrino potential" is written as

$$
\begin{equation*}
v(p)=\frac{2}{\pi} \frac{1}{\sqrt{p^{2}+m_{N}^{2}}\left(\sqrt{p^{2}+m_{N}^{2}}+\tilde{A}\right)} \tag{16}
\end{equation*}
$$

Using Eq. (15) the expected half-life for a single neutrino of mass $m_{N}$ with coupling $U_{e N}$ can be calculated. The exclusion plot [35] in Fig. 5 summarizes current limits from GERDA [25], CUORE [27], KamLAND-Zen [29], and EXO-200 [28] experiments.


FIGURE 5. (Color online) Excluded values of $\left|U_{e N}\right|^{2}$ and $m_{N}$ in the $m_{N}-\left|U_{e N}\right|^{2}$ plane, for $g_{A}=1.269$, by different experiments, GERDA [25], CUORE [27], KamLAND-Zen [29], and EXO-200 [28].

Several types of sterile neutrinos have been suggested: a family of neutrinos at the eV scale [36,37], a family of neutrinos at the keV scale [38], one at the $\mathrm{MeV}-\mathrm{GeV}$ scale [39, 40], and one at the TeV scale [34]. The total
contribution to the half-life can be approximated as [11]

$$
\begin{align*}
{\left[\tau_{1 / 2}^{0 v}\right]^{-1}=G_{0 v} } & \left\lvert\,\left[\frac{1}{m_{e}} \sum_{k=1}^{3} U_{e k}^{2} m_{k}+\frac{1}{m_{e}} \sum_{i} U_{e i}^{2} m_{i}+\frac{1}{m_{e}} \sum_{j} U_{e j}^{2} m_{j}\right] M_{0 v}\right. \\
& +\left.\left[m_{p} \sum_{N} U_{e N}^{2} \frac{m_{N}}{\left\langle p^{2}\right\rangle+m_{N}^{2}}+m_{p} \sum_{k_{h}=1}^{3} U_{e k_{h}}^{2} \frac{1}{m_{k_{h}}}\right] M_{0 v_{h}}\right|^{2} \tag{17}
\end{align*}
$$

separating the contribution of the light, $m_{N} \ll p_{F}$, neutrinos, into known $k=1,2,3$, unknown at eV scale, $i$, unknown at keV scale, $j$, and using the expression appropriate for them in terms of $M_{0 v}$. The contribution of intermediate mass, $m_{N} \sim p_{F}$, is also explicitly written for neutrinos at $\mathrm{MeV}-\mathrm{GeV}$ scale, and finally the the contribution of heavy, $m_{N} \gg p_{F}$, neutrinos at the TeV scale is added, using the form appropriate for them in terms of $M_{0 v_{h}}$.

The presence of sterile neutrinos changes completely the picture of limits in average neutrino mass, as shown in Fig. 6. Considering, for example, the case suggested in [36] of a $4^{\text {th }}$ neutrino with mass $m_{4}=1 \mathrm{eV}$ and $\left|U_{e 4}\right|^{2}=0.03$, we get

$$
\begin{equation*}
\left\langle m_{N, l i g h t}\right\rangle=\sum_{k=1}^{3} U_{e k}^{2} m_{k}+U_{e 4}^{2} e^{i \alpha_{4}} m_{4} \tag{18}
\end{equation*}
$$

where the unknown phase is $0 \leq \alpha_{4} \leq 2 \pi$. The effect of the $4^{\text {th }}$ neutrino is to add to the average mass a contribution of 30 meV , making the spread of the allowed values in Fig. 6 larger than without $4^{\text {th }}$ neutrino and thus improving the possibility to detect it in the next generation experiments.


FIGURE 6. (Color online) Current limits for $\left\langle m_{N, l i g h t}\right\rangle$ from Majorana [24], GERDA [25], NEMO-3 [26], CUORE [27], EXO-200 [28], KamLAND-Zen [29] experiments, and IBM-2 Argonne SRC NMEs and $g_{A}=1.269$. The value of Ref. [33] is shown by $X$. The figure is in semilogarithmic scale. Red shows the normal hierarchy and green the inverted hierarchy. In this figure the scenario suggested in [36], relevant to LSND and reactor anomaly, is considered.

## Non-standard mechanisms of double beta decay

General Lagrangian of neutrinoless double beta decay can be divided into long range and short range parts:

$$
\begin{equation*}
\mathcal{L}_{0 \nu \beta \beta}=\mathcal{L}_{\mathrm{LR}}+\mathcal{L}_{\mathrm{SR}}, \tag{19}
\end{equation*}
$$

where the long range part is written as [41]

$$
\begin{equation*}
\mathcal{L}_{\mathrm{LR}}=\frac{G_{F}^{2}}{\sqrt{2}}\left[J_{V-A}^{\dagger} j_{V-A}^{\mu}+\sum_{\alpha, \beta} \epsilon_{\alpha, \beta} J_{\alpha}^{\dagger} j_{\beta}\right] \tag{20}
\end{equation*}
$$

with $\alpha, \beta=S \pm V, V \pm A, T \pm T_{5}$. The short range part reads [14]

$$
\begin{equation*}
\mathcal{L}_{\mathrm{SR}}=\frac{G_{F}^{2}}{\sqrt{2}}\left[\epsilon_{1} J J j+\epsilon_{2} J^{\mu \nu} J_{\mu \nu} j+\epsilon_{3} J^{\mu} J_{\mu} j+\epsilon_{4} J^{\mu} J_{\mu \nu} j^{\nu}+\epsilon_{5} J^{\mu} J j_{\mu}\right], \tag{21}
\end{equation*}
$$

where the hadronic and leptonic currents are

$$
\begin{gather*}
J_{R / L}=\bar{u}\left(1 \pm \gamma_{5}\right) d, \quad J_{R / L}^{\mu}=\bar{u} \gamma^{\mu}\left(1 \pm \gamma_{5}\right) d, \quad J_{R / L}^{\mu v}=\bar{u} \sigma_{\mu \nu}\left(1 \pm \gamma_{5}\right) d,  \tag{22}\\
j_{R / L}=\bar{e}\left(1 \pm \gamma_{5}\right) e^{c}, \quad j^{\mu}=\bar{e} \gamma^{\mu} \gamma_{5} e^{c}, \tag{23}
\end{gather*}
$$

with $\sigma_{\mu \nu}=\frac{i}{2}\left[\gamma_{\mu}, \gamma_{\nu}\right]$. The fields $u, d$ and $e$ are 4-component Dirac spinor operators representing the up-quark, downquark and electron, respectively. The field $e^{c}=C e$ denotes the charge conjugate, corresponding to the fact that all lepton currents violate the electron lepton number by two units.

In both long range and short range cases, the half-life triggered by a single mechanism can be expressed similarly to Eq. (2),

$$
\begin{equation*}
T_{1 / 2}^{-1}=G_{I}\left|M_{I}\right|^{2}\left|\epsilon_{I}\right|^{2} \tag{24}
\end{equation*}
$$

where $G_{I}$ is the PSF and $M_{I}$ the NME, both generally depending on the Lorentz structure of the effective operator in question. The coupling constant $\epsilon_{I}$ parametrizes the underlying particle physics dynamics. In Ref. [14] a general formalism for short-range mechanisms contributing to neutrinoless double beta decay in an effective operator approach was developed and limits on the effective couplings $\epsilon_{I}$ were derived assuming one contribution is different from zero at a time. Such contributions arise when lepton number is broken at a new physics scale much larger than the typical energy scale of $0 v \beta \beta$-decay $p \approx 100 \mathrm{MeV}$.

In Ref. [14] new phase space factors originating from the electron currents, including interference effects of different short-range contributions were evaluated and for now the nuclear matrix elements were taken to be those of [10] to leading order in $\mathbf{p} / m_{p}$. Using experimental bounds on half-lives and estimating the novel matrix elements arising in short-range contributions, the numerical limits on the effective new physics parameters $\epsilon_{I}$ were calculated. To leading order, only the standard Fermi, Gamow-Teller matrix elements appear, but especially the enhanced values of the exotic and induced pseudo-scalar couplings in the form factor approach necessitate the inclusion of higher order terms in $\mathbf{p} / m_{p}$. This requires the determination of different nuclear matrix elements $M_{F}^{\prime}, M_{G T}^{\prime}, M_{T}^{\prime}, M_{F}^{\prime \prime}, M_{G T}^{\prime \prime}, M_{T}^{\prime \prime}$, the calculation of which is currently in progress. These NMEs are crucially important to accurately determine dominant contributions to short-range $0 \nu \beta \beta$-decay and to verify the strong limits obtained in Ref. [14] on the effective new physics parameters ranging between $\epsilon_{I} \approx 10^{-10}$ to $10^{-7}$, which correspond to new physics scales in the multi- TeV region. Accurate calculations of the limits and sensitivities are crucially important as they probe the phenomenologically interesting TeV scale.

## CONCLUSIONS

In order to extract physics beyond the standard model from experimental $0 v \beta \beta$-decay half-life accurate calculations of both PSF and NME are needed. These quantities have been evaluated, or are under evaluation, systematically for all processes of interest. However, the mechanisms of $0 v \beta \beta$-decay is not yet known and number of different mechanisms can trigger neutrinoless double beta decay. Consequently, if $0 \nu \beta \beta$-decay is observed it may also provide evidence for physics beyond the standard model other than the standard mass mechanism. On the other hand, if $0 v \beta \beta$ is not observed, strict limits on other scenarios and non-standard mechanisms can be set. Thus neutrinoless double beta decay remains to be a fascinating puzzle with great potential to test lepton number, to determine the nature of neutrino mass and to probe its values.

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