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IRA-EMO: Interactive Method using Reservation and Aspiration Levels for Evolutionary Multiobjective Optimization

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Abstract. We propose a new interactive evolutionary multiobjective optimization method, IRA-EMO. At each iteration, the decision maker (DM) expresses her/his preferences as an interesting interval for objective function values. The DM also specifies the number of representative Pareto optimal solutions in these intervals referred to as regions of interest one wants to study. Finally, a real-life engineering three-objective optimization problem is used to demonstrate how IRA-EMO works in practice for finding the most preferred solution.

Keywords: Evolutionary multi-objective optimization · Reference point · Region of interest · Interactive methods · Preferences.

1 Introduction

Many real-world problems involve dealing with several conflicting criteria, which must be optimized simultaneously. These problems, called *multiobjective optimization problems*, are defined by objective functions which model the criteria, and by constraints and bounds for variables which define the feasible set. In order to solve multiobjective optimization problems and to decide which solution is the final one, a person, called decision maker (DM), is usually involved in the solution process in order to choose the solution which best suits her/his preferences (*the most preferred solution*).

Recently, *Evolutionary Multiobjective Optimization* (EMO) methods that include preferences and interactive EMO algorithms have received attention due to the reduction of the computational load and applicability [1]. Within interactive algorithms, the elicitation of preferences can be done in different ways [12]. Many methods use a so-called *reference point*, which is formed by desirable aspiration levels for the objective functions that the DM would like to reach. A

key feature in interactive methods is that only one or few solutions are shown at each iteration in order not to overwhelm the DM with too much information.

We have earlier proposed an interactive version of the preference-based EMO algorithm WASF-GA [15], called Interactive WASF-GA [13], where the DM expresses preferences as aspiration levels (i.e. a reference point) and the number of solutions one wants to see. Then, the algorithm generates exactly the number of nondominated solutions the DM desires, reflecting the preferences in the reference point given.

In this paper, we propose a new interactive EMO algorithm called *Interactive Reservation and Aspiration points-based EMO (IRA-EMO)* method, which uses two kinds of reference points to generate nondominated solutions. In addition to aspiration levels, also reservation levels given by the DM are used. For a minimization problem, reservation levels are values above which the objective function values are not admissible. Thus, we consider preferences expressed as lower and upper bounds for the objective functions defining a region of interest in the Pareto optimal front and generate solutions within it. It is important to properly represent all the possible trade-offs among the objectives in the region of interest to let the DM have an idea of which nondominated solutions can be achieved based on preferences. To this aim, a variant of WASF-GA called *Modified WASF-GA* is proposed to be used within IRA-EMO to approximate the region of interest.

The rest of the paper is organized as follows. In Section 2, we introduce the main concepts and notations used, including a brief overview of interactive EMO algorithms. In Section 3, the IRA-EMO method and Modified WASF-GA are described. The usefulness of IRA-EMO in practice is demonstrated in Section 4, with a real-world problem. Finally, conclusions are drawn in Section 5.

2 Background

We consider *multiobjective optimization problems* of the form:

$$\begin{aligned} & \text{minimize} \quad \{f_1(\mathbf{x}), \dots, f_k(\mathbf{x})\} \\ & \text{subject to} \quad \mathbf{x} \in S, \end{aligned} \tag{1}$$

where $f_i : S \rightarrow \mathbf{R}$, for $i = 1, \dots, k$ ($k \geq 2$), are *objective functions* to be minimized simultaneously over the *feasible set* S in the decision space \mathbf{R}^n , which is formed by *solutions* or *decision vectors* $\mathbf{x} = (x_1, \dots, x_n)^T$. In the objective space \mathbf{R}^k , the solutions are *objective vectors* $\mathbf{f}(\mathbf{x}) = (f_1(\mathbf{x}), \dots, f_k(\mathbf{x}))^T$, for $\mathbf{x} \in S$, belonging to the *feasible objective region* $Z = \mathbf{f}(S)$.

Because of the degree of conflict among the objective functions, it is very unlikely to find a single solution where all of them can reach their individual optima. Therefore, we consider so-called *Pareto optimal solutions*, at which no objective function can be improved without deteriorating, at least, one of the others. A solution $\mathbf{x} \in S$ (and its objective vector $\mathbf{f}(\mathbf{x})$) is said to be *Pareto optimal* if there is no other $\mathbf{x}' \in S$ such that $f_i(\mathbf{x}') \leq f_i(\mathbf{x})$ for all $i = 1, \dots, k$ and $f_j(\mathbf{x}') < f_j(\mathbf{x})$ for, at least, one index j . The set of all Pareto optimal

solutions is called the *Pareto optimal set* E , and its image in the objective space is referred to as the *Pareto optimal front* $\mathbf{f}(E)$. The *nadir objective vector* $\mathbf{z}^{\text{nad}} = (z_1^{\text{nad}}, \dots, z_k^{\text{nad}})^T$ and the *ideal objective vector* $\mathbf{z}^* = (z_1^*, \dots, z_k^*)^T$ provide upper and lower bounds for the objective functions in E , respectively. Their components are $z_i^{\text{nad}} = \max_{\mathbf{x} \in E} f_i(\mathbf{x})$ and $z_i^* = \min_{\mathbf{x} \in E} f_i(\mathbf{x})$ ($i = 1, \dots, k$). While \mathbf{z}^* can be easily obtained, \mathbf{z}^{nad} can usually only be approximated [3].

Many preference-based EMO algorithms and interactive methods are based on the use of reference points [18]. A *reference point* is a vector $\mathbf{q} = (q_1, \dots, q_k)^T$ consisting of desirable objective function values q_i for the DM (*aspiration levels*). We say that \mathbf{q} is *achievable* if $\mathbf{q} \in Z + \mathbf{R}_+^k$ (where $\mathbf{R}_+^k = \{\mathbf{y} \in \mathbf{R}^k \mid y_i \geq 0 \text{ for } i = 1, \dots, k\}$), that is, if either $\mathbf{q} \in Z$ or if \mathbf{q} is dominated by a Pareto optimal objective vector; otherwise, \mathbf{q} is said to be *unachievable*. Using a reference point, an *achievement scalarizing function* (ASF) can be built and minimized over the feasible set to find the Pareto optimal solution that best satisfies the DM's expectations. For a reference point \mathbf{q} and a vector of weights $\boldsymbol{\mu} = (\mu_1, \dots, \mu_k)^T$, with $\mu_i > 0$ ($i = 1, \dots, k$), we consider the following ASF proposed in [18]:

$$s(\mathbf{q}, \mathbf{f}(\mathbf{x}), \boldsymbol{\mu}) = \max_{i=1, \dots, k} \{ \mu_i (f_i(\mathbf{x}) - q_i) \} + \rho \sum_{i=1}^k \mu_i (f_i(\mathbf{x}) - q_i). \quad (2)$$

The parameter ρ has a real positive value which ensures that the solution which minimizes (2) over S is a Pareto optimal solution to the original problem (1). Actually, any Pareto optimal solution of (1) can be obtained by minimizing (2) over S and varying the reference point and/or the weight vector [12].

We use two types of reference points: in addition to those consisting of aspiration levels, we also use reference points formed by *reservation levels* acceptable for the DM, i.e. values above which the objective functions are not admissible.

As explained later, our proposal borrows some ideas from (a) the preference-based EMO algorithm WASF-GA [15], which approximates a *region of interest of the Pareto optimal front defined by a reference point* \mathbf{q} as defined in [15], and from (b) the EMO algorithm Global WASF-GA [16], which approximates the whole Pareto optimal front by using both a utopian (a vector slightly better than the ideal objective vector) and a nadir objective vector as reference points.

In the literature, many preference-based and interactive EMO methods have been proposed. In R-NSGA-II [5], the DM gives one or several reference points and the crowding distance used in NSGA-II [4] is replaced by a preference distance. PBEA [17] considers a reference point to modify the binary quality indicator of IBEA [20]. In [8], an interactive version of MOEA/D [19] is suggested, where a DM selects one among a set of solutions shown at intermediate generations. The algorithm with a controllable accuracy proposed in [10] is also based on reference points. Regarding the use of both aspiration and reservation levels in the EMO field, to the best of our knowledge, few references can be found. In [7], a preference-based EMO method is proposed for the selection of direct load control actions in electrical distribution networks, in which preferences are elicited in a similar way. The nondominated solutions violating some reservation level are penalized and those closer to the aspiration levels according to

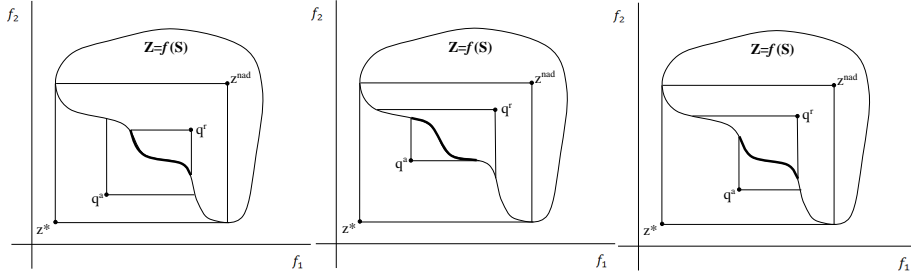


Fig. 1. Region of interest R when \mathbf{q}^a is unachievable and \mathbf{q}^r is achievable.

the Euclidean distance are rewarded. Furthermore, [9] suggests an interactive EMO method based on RVEA [2], where the DM can specify, if desired, preferred ranges for the objectives (i.e. aspiration and reservation levels), which are used to adjust the set of reference vectors in RVEA. These two methods follow a solution process which is different from our proposal, as described next.

3 IRA-EMO for decision making

In this section, we describe the *Interactive Reservation and Aspiration points-based EMO* (IRA-EMO) method. At each iteration it of IRA-EMO, the DM indicates her/his preferences by specifying desirable bounds for the objective functions in the form of aspiration and reservation levels, denoted by $q_i^{a,it}$ and $q_i^{r,it}$, respectively, with $z_i^* \leq q_i^{a,it} < q_i^{r,it} \leq z_i^{\text{nad}}$, for $i = 1, \dots, k$. Thus, an aspiration point $\mathbf{q}^{a,it} = (q_1^{a,it}, \dots, q_k^{a,it})^T$ and a reservation point $\mathbf{q}^{r,it} = (q_1^{r,it}, \dots, q_k^{r,it})^T$ can be formed. The DM also sets the number of solutions (s)he wants to analyze at each iteration, denoted by N_S^{it} . Then, N_S^{it} nondominated solutions are shown to the DM, whose objective values are between the levels.

For an aspiration and a reservation points, \mathbf{q}^a and \mathbf{q}^r , respectively, with $z_i^* \leq q_i^a < q_i^r \leq z_i^{\text{nad}}$ for $i = 1, \dots, k$, we denote by R^a and R^r the regions of interest they define, respectively. For this preference information, the DM is interested in Pareto optimal solutions which are in $R = R^a \cap R^r$. Figure 1 represents examples of different situations for a biobjective minimization problem, where the subset R is highlighted in bold. We have assumed that \mathbf{q}^a is unachievable and \mathbf{q}^r is achievable, which is the most logical situation when the DM gives both points. In case \mathbf{q}^a and \mathbf{q}^r are both unachievable, $R = R^a \cap R^r = R^r$, and if both are achievable, $R = R^a \cap R^r = R^a$.

The ideal and the nadir objective vectors can be estimated if needed, and the aspiration and the reservation levels are supposed to satisfy $z_i^* \leq q_i^a < q_i^r \leq z_i^{\text{nad}}$ for $i = 1, \dots, k$. It may occur that \mathbf{z}^* does not dominate \mathbf{q}^a and/or \mathbf{z}^{nad} is not dominated by \mathbf{q}^r . If \mathbf{q}^a dominates \mathbf{z}^* and \mathbf{q}^r dominates \mathbf{z}^{nad} , we have $R = R^r$. In case \mathbf{z}^{nad} dominates \mathbf{q}^r and \mathbf{z}^* dominates \mathbf{q}^a , then $R = R^a$. If \mathbf{q}^a dominates \mathbf{z}^* and \mathbf{z}^{nad} dominates \mathbf{q}^r , then R is the whole Pareto optimal front. When \mathbf{q}^a

and \mathbf{z}^* do not dominate each other, and also \mathbf{q}^r and \mathbf{z}^{nad} do not dominate each other, R is constituted by a part of the Pareto optimal front.

We want to emphasize that IRA-EMO does not depend on the availability of estimations for \mathbf{z}^* and \mathbf{z}^{nad} . If no estimations are known, in practice, the DM may need to take a few iterations of IRA-EMO at the beginning of the solution process just to gain an idea of the possible objective function ranges and to fine-tune her/his preferences. Also, \mathbf{z}^* and \mathbf{z}^{nad} may be used for normalizing the objective function values in (2) if they are in different scales. In case they are not available, any other normalization approach can be used.

To generate nondominated solutions in the subset R defined by the aspiration and the reservation points at each iteration it , we propose *Modified WASF-GA* as an internal part of IRA-EMO. It is based on the working procedure of Global WASF-GA for using two reference points, instead of only one as in WASF-GA.

3.1 Modified WASF-GA

Let us consider an aspiration and a reservation points, \mathbf{q}^a and \mathbf{q}^r , and N_μ vectors of weights representing the weight vector space $(0,1)^k$. Let us denote by N the population size, P^{final} the final set of nondominated solutions generated by Modified WASF-GA and h the generation counter. At each generation h , P^h is the population of individuals, Q^h is the offspring population, Z^h is the population of parents and offspring, and F_n^h is the n -th front. The number of elements in a set A is denoted by $\#(A)$.

At each generation h , Modified WASF-GA selects solutions which best match with \mathbf{q}^a and \mathbf{q}^r . The population of parents and offspring Z^h is divided into several fronts according to the values they take on the ASF (2) for both \mathbf{q}^a and \mathbf{q}^r at the same time. The lower the values of (2) reached by a solution for one of these two reference points, the more this solution is highlighted. To be more precise, once Z^h is formed (of size $2N$), the division of the individuals into different fronts is performed as follows. The first front is formed by the solutions in Z^h with the lowest values of (2) for \mathbf{q}^a taking into account a half of the N_μ weight vectors (the odd order ones), and by the solutions in Z^h with the lowest values of (2) for \mathbf{q}^r using the other half of the N_μ weight vectors (the even order ones). These solutions are removed from Z^h . Similarly, the second front is formed by the solutions in Z^h with the next lowest values of (2) for \mathbf{q}^a and a half of the N_μ weight vectors, and by the solutions in Z^h with the next lowest values of (2) for \mathbf{q}^r and the other half of the N_μ weight vectors. This process continues until every individual in Z^h has been classified. The set of nondominated solutions generated by Modified WASF-GA, P_{final} , consists of the N_μ solutions in the first front of the last generation. So far, these solutions are the best ones with respect to the weight vectors and the aspiration and the reservation points used. Note that P_{final} approximates the regions of interest defined by \mathbf{q}^a and \mathbf{q}^r , that is, $R^a \cup R^r$. Since the DM wants to see solutions in $R = R^a \cap R^r$ at each iteration of IRA-EMO, P_{final} is later filtered to select the solutions belonging to this set.

Modified WASF-GA minimizes, at each generation, the ASF (2) for both the aspiration and the reservation points. Therefore, in practice, Modified WASF-GA

projects the aspiration and the reservation points onto the Pareto optimal front simultaneously, using the set of projection directions defined by the weight vectors. Thus, it is important that the N_μ weight vectors used define a well-spread set of projection directions in order to preserve diversity.

3.2 Algorithm of IRA-EMO

At each iteration it with the DM, we denote by $\boldsymbol{\mu}^{it,j}$ the weight vectors used in Modified WASF-GA ($j = 1, \dots, N_\mu$), \bar{P}^{it} the outcome of Modified WASF-GA, and P^{it} the subset of solutions of \bar{P}^{it} whose objective function values are within the given aspiration and reservation levels. The main steps of IRA-EMO are:

- Step 1. Initialization.** Set $it = 1$. Show \mathbf{z}^{nad} and \mathbf{z}^* to the DM (if available). Ask the DM how many solutions (s)he would like to see, denoted by N_S^{it} .
- Step 2. Preference information I.** If $it = 1$, ask the DM to specify aspiration and reservation levels for the objective functions, which define $\mathbf{q}^{a,it}$ and $\mathbf{q}^{r,it}$, respectively. If $it > 1$, $\mathbf{q}^{a,it}$ and $\mathbf{q}^{r,it}$ are set according to the preference information the DM wants to give as follows:
- The DM is asked if the current reservation point is to be updated. If so, the DM specifies a new reservation point, $\mathbf{q}^{r,it}$. If not, let $\mathbf{q}^{r,it} = \mathbf{q}^{r,it-1}$.
 - The DM is asked if the current aspiration point is to be updated. If so, the DM specifies a new aspiration point, $\mathbf{q}^{a,it}$. If not, let $\mathbf{q}^{a,it} = \mathbf{q}^{a,it-1}$.
- Step 3. Preference information II.** Ask if the DM wants to change the number of solutions to be obtained, update N_S^{it} accordingly. Next, define $N_\mu = 2N_S^{it}$. If $it > 1$ and $N_S^{it} = N_S^{it-1}$, set $\boldsymbol{\mu}^{it,j} = \boldsymbol{\mu}^{it-1,j}$ for all $j = 1, \dots, N_\mu$ and go to Step 5. Otherwise, continue.
- Step 4. Generation of the weight vectors.** Following the procedure described in [15], generate N_μ weight vectors $\boldsymbol{\mu}^{it,j}$, with $j = 1, \dots, N_\mu$.
- Step 5. Generation of solutions.** Generate N_μ nondominated solutions by applying Modified WASF-GA using $\mathbf{q}^{a,it}$, $\mathbf{q}^{r,it}$, and the set of weight vectors $\boldsymbol{\mu}^{it,j}$, with $j = 1, \dots, N_\mu$. Let \bar{P}^{it} be the set formed by these N_μ solutions.
- Step 6.** Set $P^{it} = \{\mathbf{x} \in \bar{P}^{it} | q_i^{a,it} \leq f_i(\mathbf{x}) \leq q_i^{r,it} \text{ for all } i = 1, \dots, k\}$. If $\#(P^{it}) > N_S^{it}$, show N_S^{it} representative solutions in P^{it} to the DM. If $\#(P^{it}) = N_S^{it}$, then show all the solutions in P^{it} to the DM. Otherwise, if $\#(P^{it}) < N_S^{it}$, complete P^{it} with individuals $\mathbf{x} \in \bar{P}^{it} \setminus P^{it}$ which satisfy $f_i(\mathbf{x}) \leq q_i^{r,it}$ for every $i = 1, \dots, k$ and whose objective vectors are closer to $\mathbf{q}^{a,it}$ regarding the Euclidean distance, until $\#(P^{it}) = N_S^{it}$. Then, show all the solutions in P^{it} to the DM. Let $\{\bar{\mathbf{x}}_1^{it}, \dots, \bar{\mathbf{x}}_{N_S^{it}}^{it}\}$ be the set of solutions shown to the DM.
- Step 7. Solutions closer to the aspiration point (or to the reservation point).** Ask the DM if (s)he desires to see solutions with objective function values closer to their aspiration levels (respectively, to their reservation levels). If no, go to Step 8. If yes, ask which of the N_S^{it} solutions (s)he would like to replace. Let $\{\bar{\mathbf{x}}_1^{it}, \dots, \bar{\mathbf{x}}_t^{it}\}$ (with $t < N_S^{it}$) be the set of solutions to replace and $\{\bar{\mathbf{x}}_{t+1}^{it}, \dots, \bar{\mathbf{x}}_{N_S^{it}}^{it}\}$ the set of solutions to maintain. Remove from \bar{P}^{it} the solutions $\{\bar{\mathbf{x}}_1^{it}, \dots, \bar{\mathbf{x}}_t^{it}\}$. Set $P^{it} = \{\mathbf{x} \in \bar{P}^{it} | q_i^{a,it} \leq f_i(\mathbf{x}) \text{ for all } i = 1, \dots, k\}$ (respectively, $P^{it} = \{\mathbf{x} \in \bar{P}^{it} | f_i(\mathbf{x}) \leq q_i^{r,it} \text{ for all } i = 1, \dots, k\}$) and find t solutions in P^{it} whose objective vectors are the closest ones to $\mathbf{q}^{a,it}$ (respectively, the furthest ones to $\mathbf{q}^{r,it}$) regarding the Euclidean distance. Let us denote these solutions by $\{\bar{\mathbf{x}}_1^{it}, \dots, \bar{\mathbf{x}}_t^{it}\}$. Show $\{\bar{\mathbf{x}}_1^{it}, \dots, \bar{\mathbf{x}}_t^{it}\} \cup \{\bar{\mathbf{x}}_{t+1}^{it}, \dots, \bar{\mathbf{x}}_{N_S^{it}}^{it}\}$ to the DM.
- Step 8. Optional ordering.** Ask if the DM wants to order the solutions:
- According to some of the objective functions f_r , with $r \in \{1, \dots, k\}$: in this case, show the solutions $\{\bar{\mathbf{x}}_1^{it}, \dots, \bar{\mathbf{x}}_{N_S^{it}}^{it}\}$ in a descending order with respect to their values for f_r .
 - According to their ASF values for the aspiration point $\mathbf{q}^{a,it}$: in this case, show the solutions $\{\bar{\mathbf{x}}_1^{it}, \dots, \bar{\mathbf{x}}_{N_S^{it}}^{it}\}$ in an ascending order based on their values for $s(\mathbf{q}^{a,it}, \mathbf{f}(\mathbf{x}), \mathbf{w}^{it})$,
- where $\mathbf{w}^{it} = \left(\frac{1}{f_1^{\text{max}} - f_1^{\text{min}}}, \dots, \frac{1}{f_k^{\text{max}} - f_k^{\text{min}}} \right)$, with $f_j^{\text{min}} = \min_{l=1, \dots, N_S^{it}} f_j(\bar{\mathbf{x}}_l^{it})$ and $f_j^{\text{max}} = \max_{l=1, \dots, N_S^{it}} f_j(\bar{\mathbf{x}}_l^{it})$.

Step 9. Termination rule. Ask the DM to select the most preferred solution from the set $\{\bar{\mathbf{x}}_1^{it}, \dots, \bar{\mathbf{x}}_{N_S^{it}}^{it}\}$ and denote it by \mathbf{x}^{it} . If the DM is satisfied enough with this solution and (s)he wishes to *Stop*, the solution process concludes with \mathbf{x}^{it} as the final solution and $\mathbf{f}(\mathbf{x}^{it})$ as the final objective vector. Otherwise, set $it = it + 1$ and go to Step 2.

In Step 1, estimations of \mathbf{z}^* and \mathbf{z}^{nad} are shown to the DM to give her/him an idea of the objective function ranges for giving the reservation and aspiration levels. If they are not available, (s)he sets the levels based on her/his intuition.

In Steps 2 and 3, the DM expresses her/his initial preferences. Then, a set of solutions is generated and filtered in Steps 4-6. In Step 5, new $N_\mu = 2N_S^{it}$ solutions are generated using Modified WASF-GA and, in Step 6, N_S^{it} solutions are selected from its outcome and shown to the DM. Once the first solutions have been generated, the interaction with the DM starts from Step 7 onwards. Step 7 allows the DM to fine-tune the solutions shown by replacing some of them by others with objective function values closer to either aspiration or reservation levels. The idea is to let her/him freely explore different trade-offs among the objectives given the current preferences. With Step 8, the main aim is to support the DM in analyzing the solutions, but note that it can be skipped if desired.

In Step 6, if needed, we complete P^{it} with solutions not worsening the reservation levels and violating the aspiration ones as little as possible. It is very unlikely, but there may be no solutions that meet the reservation levels. In this case, the DM must be informed, so that (s)he can decide whether to give new reservation levels, analyze just the available solutions satisfying $q_i^{a,it} \leq f_i(\mathbf{x}) \leq q_i^{r,it}$ ($i = 1, \dots, k$), or even just solutions with $q_i^{a,it} \leq f_i(\mathbf{x})$ ($i = 1, \dots, k$).

Internally, to accelerate the speed of the solution process, the final population P^{it} generated by Modified WASF-GA at an iteration it can be used as its initial population at the next iteration $it+1$. Also, if desired, the local Pareto optimality of the final solution can be assured by minimizing (2) using its objective function values as a reference point with some local optimization method.

4 Numerical example

Next, we illustrate the performance of IRA-EMO with the three-objective optimization problem proposed in [14]. The aim is to identify the most convenient combination of improvements in the auxiliary services of a 1100 MW thermal power plant, to maximize the energy saving (denoted by f_1 , in MWh), to minimize the economic investment required (denoted by f_2 , in € million) and to maximize the Internal Rate of Return (IRR) of the investment (denoted by f_3 , in %). The problem has 13 continuous and 20 binary decision variables and is modelled using a black-box simulator. We present objective values in their original form (and not using a minimization formulation) to make the interaction with the DM more understandable. The Pareto optimal front of this problem is discontinuous and formed by several disconnected subsets of solutions [14]. We have approximated $\mathbf{z}^* = (47526.37, 0.0, 100.0)$ and $\mathbf{z}^{\text{nad}} = (0.0, 9.28, 0.0)$.

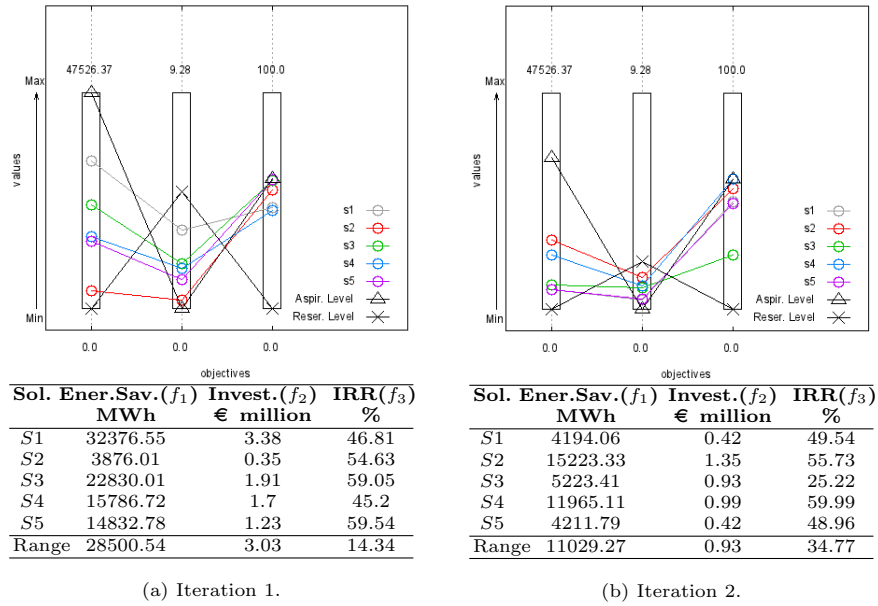
IRA-EMO and Modified WASF-GA have been implemented using jMetal [6], a Java-based framework for multiobjective optimization.⁴ The parameter setting used in Modified WASF-GA is the same used in [14]. We use the simulated binary crossover (SBX) operator and a polynomial distribution mutation operator for continuous variables and the binary crossover and the binary mutation for integer variables. The crossover and mutation distribution indices used are 2 and 25, respectively. For all variables, the crossover and the mutation probabilities are 0.9 and $1/n$, respectively, where n is the number of binary or continuous variables. We have used a population size of 50 individuals and 100 generations.

Next, we describe the interactive solution process, i.e. how the DM used IRA-EMO to identify his most preferred solution. At the first iteration, the DM decided to generate five solutions ($N_S^1 = 5$) using $\mathbf{q}^{a,1} = (47526.37, 0.0, 60.0)$ and $\mathbf{q}^{r,1} = (0.0, 5.0, 0.0)$. Initially, he wished to study which type of trade-offs were possible by setting the aspiration and the reservation levels as their ideal and nadir objective values, except the aspiration level for f_3 (having a 60% IRR was profitable enough for him) and the reservation level for f_2 (the nadir value for f_2 was too much money for him and, although his budget limit was €2 million, initially he wanted to see solutions needing more expensive investments, such as €5 million, at most, to study the “price” to pay for a lower investment). In Figure 2 (a), the solutions generated by IRA-EMO are plotted using a value path and a table shows the objective vectors. This representation enables the DM to analyze the objective values reached within the aspiration and the reservation levels, and to see how wide the given ranges are in comparison to their maximum ranges (i.e., from the ideal to the nadir values), helping him to broaden or to shrink them if he wants to relax or to further limit the objective values.

All the solutions generated at the first iteration reached objective values within the specified ranges. Overall, limiting the investment to €5 million enabled very high energy savings to be obtained, but still far from the desired aspiration level. However, all the solutions achieved IRR values near to 60% (the aspiration level), reflecting their high profitability. Next, the DM wanted to know if it was possible to get the same profitability but limiting the investment to €2 million. At a new iteration, five solutions ($N_S^2 = 5$) were obtained with $\mathbf{q}^{a,2} = (33000.0, 0.0, 60.0)$ and $\mathbf{q}^{r,2} = (0.0, 2.0, 0.0)$, shown in Figure 2 (b). Here he also relaxed the aspiration level for f_1 until a value near the highest energy saving obtained at the first iteration. The solutions found required an investment below €2 million, but the energy savings were not as high as at the first iteration. This highlights the conflict among the two objectives. Solution $S2$ needed the highest investment (still far from the budget limit), but reached the best energy saving value, and the second best IRR value. Observe that nearly all the solutions attained IRR values close to the aspiration level. Furthermore, solution $S4$ reached a 59.99% IRR, having the second best value for the energy saving. Based on this, the DM mainly liked solutions $S2$ and $S4$.

Although his budget was limited by €2 million, he wished to check what happened when relaxing the reservation level for f_2 a bit, with the condition of

⁴ The source code is freely available at <https://github.com/rsain/IRA-EMO>


Fig. 2. Solution process of IRA-EMO.

having a 20-70% IRR and energy savings up to 10000 MWh. A new iteration was carried out, with $\mathbf{q}^{a,3} = (40000.0, 0.0, 70.0)$, $\mathbf{q}^{r,3} = (10000.0, 3.0, 20.0)$, and $N_5^3 = 5$. Figure 3 depicts the five new solutions. Most of them improved the energy saving and the IRR values achieved in the second iteration, requiring to invest less than €2 million. Finally, the DM selected S_2 as his most preferred solution. Although it needed the highest investment, it was below his budget limit and it reached the highest energy saving, with a satisfactory IRR.

To evaluate the performance of IRA-EMO, we compare it with R-NSGA-II [5], which can also use more than one reference point. In R-NSGA-II, a parameter ϵ controls the extent of the distribution of solutions. We set $\epsilon = 0.001$ (the same value used in [5] for two engineering design problems). To perform a fair comparison, we executed R-NSGA-II with the parameter setting used for IRA-EMO and performed three iterations with the same aspiration and reservation points. Thus, for $t = 1, 2, 3$, we generated nondominated solutions with $\mathbf{q}^{a,t}$ and $\mathbf{q}^{r,t}$. From the final population generated at each iteration, we first selected the solutions in the region of interest. Then, we applied the *k-means clustering* [11] to select the five most representative solutions. They are shown in Table 1.

Let us then analyze the dominance relations among the five solutions provided to the DM by each algorithm at each iteration. In other words, we compare objective vectors in the tables of Figures 2 and 3 (solutions of IRA-EMO) with the ones in Table 1 (solutions of R-NSGA-II). We can see that, at any iteration, no solution of R-NSGA-II dominates any solution IRA-EMO. However, at the first iteration, solution S_1 of IRA-EMO dominates solutions S_3 and S_5 of

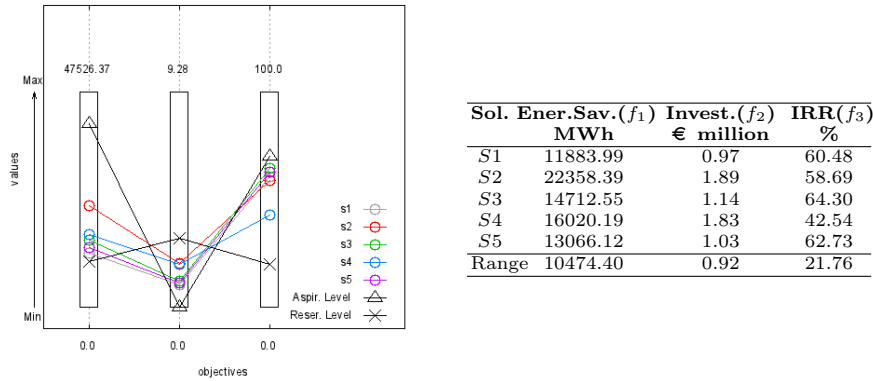


Fig. 3. Solution process of IRA-EMO (iteration 3).

Table 1. Solutions generated by R-NSGA-II.

Sol.	f_1			f_2			f_3		
	MWh	€ million	%	MWh	€ million	%	MWh	€ million	%
	Iteration 1			Iteration 2			Iteration 3		
S1	5601.56	1.07	22.85	5407.63	0.99	24.15	16330.36	1.99	39.62
S2	25101.18	2.11	58.87	15439.39	1.54	49.11	24979.27	2.04	60.71
S3	29555.99	3.41	42.08	4167.04	0.59	33.49	24670.84	2.01	60.74
S4	16123.46	1.93	40.47	4828.34	0.76	29.15	16136.67	1.89	41.37
S5	28021.10	3.40	39.70	15136.85	1.44	51.85	24844.48	2.03	60.76
Range	23954.44	2.34	36.02	11272.34	0.96	27.70	8842.60	0.15	21.14

R-NSGA-II, and $S3$ of IRA-EMO dominates $S4$ of R-NSGA-II. Furthermore, at the second iteration, both $S1$ and $S5$ of IRA-EMO dominate $S3$ of R-NSGA-II, $S2$ of IRA-EMO dominates $S5$ of R-NSGA-II, and $S4$ of IRA-EMO dominates $S1$ of R-NSGA-II. Finally, at the third iteration, $S2$ of IRA-EMO dominates both $S1$ and $S4$ of R-NSGA-II. This demonstrates that IRA-EMO was able to produce higher quality solutions (regarding the Pareto dominance) than R-NSGA-II. Thus, IRA-EMO generated sets of solutions which better represent the trade-offs existing among the objectives in the region of interest.

Comparing interactive EMO algorithms from a decision-making point of view is a research topic of its own and deserves further research. Because we have a real-world problem, we reported results for a single run, since this is what usually happens when interacting with real DMs. Actually, this constitutes the main difference between using EMO methods to solve real-world applications instead of testing with benchmark problems designed to study performance in general.

5 Conclusions

In this paper, we have proposed the IRA-EMO method for solving multiobjective optimization problems. At each iteration of IRA-EMO, very easy to understand preference information is asked from the DM: (a) aspiration and reservation

levels for the objective functions and (b) the number of solutions (s)he wishes to analyze. According to this, the desired number of solutions is generated to represent the part of the Pareto optimal front bounded by the aspiration and the reservation levels given. Such solutions are internally generated at each iteration with the DM by means of Modified WASF-GA. The applicability of IRA-EMO has been described with a real three-objective optimization problem. The DM could analyze the solutions obtained with objective values within the aspiration and the reservation levels given at each iteration until finding a suitable final solution.

The region of interest could be approximated by methods such as R-NSGA-II using appropriate preference information. However, the interactive solution process of our method is different, including specific steps for decision making purposes. In addition, IRA-EMO assures that the region of interest bounded by the aspiration and the reservation points is approximated in just one run, generating the number of solutions the DM desires to see. These two features are not so easily controllable by other methods. We found that IRO-EMO was able to produce better solutions than R-NSGA-II for a real three-objective optimization problem. Actually, IRA-EMO generated solutions that better represent the trade-off existing among the objectives in the region of interest.

In the future, we plan to investigate how to configure IRA-EMO to explore, in the same iteration, solutions with objective function values within several regions of interest. We would also like to define and develop a way to compare the performance of interactive EMO methods from a decision-making perspective.

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