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Quantum and Classical Dynamics of Heavy Quarks in a Quark-Gluon Plasma

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Abstract

Heavy quarkonium related observables are very useful to obtain information about the medium created in relativistic heavy ion collisions. In recent years the theoretical description of quarkonium in a medium has moved towards a more dynamical picture in which decay and recombination processes are very important. In this talk we will discuss the equations that describe the evolution of the heavy quark reduced density matrix in different approximations, highlighting the color dynamics that is absent in the Abelian case, and we will study their semi-classical limit. This will allow us to obtain stochastic equations (similar to Langevin or Boltzmann equations) that can be useful to obtain phenomenological predictions. We will observe that the region of validity of the Langevin-like or Boltzmann-like equations in QCD is much smaller than the corresponding QED case. The reason for this can be understood by studying how differently the free energy evolves in these two theories, and this observation will allow us to propose an equation with a small computational cost that captures many of the essential features of quarkonium evolution in a QCD plasma. These results are based on [1, 2].

Keywords: Quarkonium suppression, quark-gluon plasma

1. Introduction

Quarkonium suppression is believed to be one of the signals of the formation of a quark-gluon plasma. The original idea was proposed in [3], color screening in a quark-gluon plasma dissociates quarkonium and this is a signal of the formation of a deconfined plasma. However, this is not the only mechanism that can dissociate quarkonium. It can also collide with a gluon or a light quark from the thermal medium and this can induce a transition from a color singlet to a color octet state that will dissociate quarkonium. This mechanism can be as important or more than screening. Another idea that changed the view of quarkonium suppression is recombination [4, 5]. Two initially uncorrelated heavy quarks have a probability to be close to each other when they are in the thermal medium and form a new bound state.

On a practical level, to get information about the nuclear modification factor we need to know the probability to find a specific bound state of quarkonium given some initial conditions. This information is encoded in the density matrix and its evolution. A convenient framework to treat this kind of problems is

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that of open quantum systems [6]. We want to study a number of heavy quarks interacting with a thermal medium of gluons and light quarks. This can be divided in two parts. The heavy quarks, that we are going to call the system, and the thermal medium, that we are going to call the environment. In this sense the heavy quarks are an open quantum system interacting with another one, the environment. The object obtained after tracing over the environment degrees of freedom in the density matrix is the reduced density matrix $\mathcal{D}_Q$. The equation that rules the time evolution of $\mathcal{D}_Q$ is called the master equation.

This approach has been used to study quarkonium suppression in the last years (an exhaustive reference to the literature can be found in [1]). However it has been observed that the computational cost of solving the master equation is extremely high. As a consequence until now the quantum master equation has been explicitly solved only in simple cases and the study of recombination is at the moment out of reach. A possible solution to this is to study cases in which a classical approximation can be justified. The aim of the present discussion is to obtain a well motivated evolution equation that can describe quarkonium in a fireball with a small computational cost.

2. Langevin-like equations in QCD

We begin by deriving the master equation for the evolution of a heavy quark and an antiquark inside of the medium. We do it by using a Hamiltonian H that is the sum of three parts. H_{pl} is the Hamiltonian that describes the evolution of the gluons and light quarks that form the thermal medium. H_Q is the Hamiltonian of quarkonium and has the form

$$H_Q = H_{s,o} = -\tfrac{\Delta_r}{M} - \tfrac{\Delta_R}{4M} + V_{s,o}(r)\,, \tag{1}$$

where the subindex s or o corresponds respectively to color singlet or color octet configuration. Finally, the part of the Hamiltonian that describes the interaction between quarkonium and the medium is

$$H_1 = -g\int_r A_0^a(r)n^a(r)\,, \tag{2}$$

where A_0^a is the temporal component of the gluon field and n^a is the color current. This last part is treated as a perturbation.

Using this Hamiltonian we derive the master equation that describes the evolution of $\mathcal{D}_Q$. The unperturbed time evolution operator $U_{s,o}$ of quarkonium appears in the master equation and this makes its solution difficult. A common approximation is to assume that the gluon exchange is much faster than the evolution of quarkonium's wave function and therefore $U_{s,o} \approx 1$. This approximation is valid when the energy of the gluon is much bigger than the binding energy and we will refer to it as the instantaneous gluon exchange approximation. The next to leading order in this expansion would be $U_{s,o} \approx 1 - iH_{s,o}dt$, however it has been seen in the case of QED that just using $U_{s,o} \approx 1 - iK\,dt$, where K is the kinetic term, leads to a Langevin equation and provides a good description of the phenomenology [7]. Using this approximation we obtain the following master equation

$$\begin{aligned}\tfrac{\mathrm{d}\mathcal{D}_Q}{\mathrm{d}t} \approx -i[H_Q,\mathcal{D}_Q(t)] - \tfrac{i}{2}\int_{\mathbf{x}\mathbf{x}'} V(\mathbf{x}-\mathbf{x}')[n^a_{\mathbf{x}}n^a_{\mathbf{x}'},\mathcal{D}_Q] + \tfrac{1}{2}\int_{\mathbf{x}\mathbf{x}'} W(\mathbf{x}-\mathbf{x}')\left(\{n^a_{\mathbf{x}}n^a_{\mathbf{x}'},\mathcal{D}_Q\} - 2n^a_{\mathbf{x}}\mathcal{D}_Q n^a_{\mathbf{x}'}\right)\\ + \tfrac{i}{4T}\int_{\mathbf{x}\mathbf{x}'} W(\mathbf{x}-\mathbf{x}')\left([n^a_{\mathbf{x}},\dot{n^a_{\mathbf{x}'}}\mathcal{D}_Q] + [n^a_{\mathbf{x}},\mathcal{D}_Q\dot{n^a_{\mathbf{x}'}}]\right).\end{aligned} \tag{3}$$

Let us now discuss the following object

$$\langle x+\frac{y}{2}|\mathcal{D}_Q|x-\frac{y}{2}\rangle = \int\frac{d^3p}{(2\pi)^3}e^{i\mathbf{p}\mathbf{y}}\mathcal{D}_Q(\mathbf{x},\mathbf{p})\,, \tag{4}$$

where $\mathcal{D}_Q(\mathbf{x},\mathbf{p})$ is the Wigner transform. We expect that at large times $\mathcal{D}_Q(\mathbf{x},\mathbf{p}) \to \mathrm{e}^{-\frac{p^2}{MT}}\mathrm{F}(\mathbf{x},\mathbf{p})$ where F only has a mild dependence with p. This implies that $y \sim \frac{1}{\sqrt{MT}}$ and therefore $x \gg y$. In other words, the reduced density matrix is almost diagonal in coordinate space and therefore can be approximately thought

of as a classical variable. A systematic way to obtain semi-classical equations in this situation is to perform a small y expansion in the master equation and then perform a Wigner transform. In the strict $y = 0$ limit in QED we obtain

$$\frac{\partial D}{\partial t} = -\left(\frac{\mathbf{P}\cdot\nabla_{\mathbf{R}}}{4M} + \frac{2\mathbf{p}\cdot\nabla_{\mathbf{r}}}{M}\right) D\,, \tag{5}$$

while in the QCD case we have

$$\begin{aligned}\tfrac{\partial D_s}{\partial t} &= -\left(\tfrac{\mathbf{P}\cdot\nabla_{\mathbf{R}}}{4M} + \tfrac{2\mathbf{p}\cdot\nabla_{\mathbf{r}}}{M}\right) D_s - 2C_F\Gamma(\mathbf{r})(D_s - D_o)\,,\\ \tfrac{\partial D_o}{\partial t} &= -\left(\tfrac{\mathbf{P}\cdot\nabla_{\mathbf{R}}}{4M} + \tfrac{2\mathbf{p}\cdot\nabla_{\mathbf{r}}}{M}\right) D_o + \tfrac{1}{N_c}\Gamma(\mathbf{r})(D_s - D_o)\,,\end{aligned} \tag{6}$$

where $\Gamma(r) = W(r) - W(0)$. Note that in QCD case we have a type of contribution that is not present in QED already at $y = 0$, namely the second terms in the right-hand side. In the following we are going to discuss two strategies to deal with this.

2.1. Random color force

We can choose to diagonalize eq. (6) in such a way that one of the contributions is long lived. This can be done defining $D_0 = \frac{1}{N_c^2}(D_s + (N_c^2 - 1)D_o)$ and $D_8 = \frac{2}{N_c}(D_s - D_o)$. At order $y = 0$ the combination D_0 is long lived and corresponds to what would be the maximum entropy state if color was the only relevant degree of freedom. At order y^2 the long lived combination is $D_0{}'$ which is approximately equal to D_0 and fulfils the following equation

$$\partial_t D_0' + \frac{2\mathbf{p}\cdot\nabla}{M} D_0' - \frac{C_F}{4}\mathcal{H}_{ij}(0)\Delta_p^{ij} D_0' - \frac{2C_F F^i(\mathbf{r})F^j(\mathbf{r})}{N_c^2\Gamma(\mathbf{r})}\Delta_p^{ij} D_0' - \frac{C_F}{2MT}\mathcal{H}_{ij}(0)\nabla_p^i(p^j D_0') = 0\,. \tag{7}$$

Note that the fourth term is singular in the limit $r \to 0$. The reason is that in order to obtain eq. (7) we have assumed that $\Gamma(r) \gg yF(r)$ which is not fulfilled at small distances. This problem can be mitigated by introducing a small distance cut-off in the force.

On the phenomenological level, the virtue of this method is that it can be easily generalized to the case of an arbitrary number of heavy quarks and antiquarks and it allows to study recombination with a very small computational cost. The disadvantage is that, due to the fourth term in eq. (7), it produces unphysically strong kicks that can even produce superluminal heavy quarks. This can only be mitigated by introducing a cut-off at small distance that is quite close to the Bohr radius.

2.2. Boltzmann-Langevin equation

Let us now show the analogous to eq. (6) at linear order in y (Now $P_o = (N_c^2 - 1)D_o$)

$$\begin{aligned}\left[\partial_t + \tfrac{2\mathbf{p}\cdot\nabla_{\mathbf{r}}}{M} + C_F\mathbf{F}(\mathbf{r})\cdot\nabla_{\mathbf{p}}\right] P_s &= -2C_F\Gamma(\mathbf{r})\left(P_s - \tfrac{P_o}{N_c^2-1}\right)\,,\\ \left[\partial_t + \tfrac{2\mathbf{p}\cdot\nabla_{\mathbf{r}}}{M} - \tfrac{1}{2N_c}\mathbf{F}(\mathbf{r})\cdot\nabla_{\mathbf{p}}\right] P_o &= -\tfrac{1}{N_c}\Gamma(\mathbf{r})(P_o - (N_c^2 - 1)P_s)\,.\end{aligned} \tag{8}$$

This is a Boltzmann equation. Unfortunately this structure is not maintained at order y^2. However we can obtain a mixture of a Langevin and a Boltzmann equation if the terms proportional to $y^2((N_c^2 - 1)P_s - P_o)$ are neglected. This color combination is zero in the state of maximum color entropy. Under these approximations we obtain the following equation for the singlet

$$\begin{aligned}&\partial_t P_s + \tfrac{2p^i\nabla_r^i}{M}P_s + C_F F^i(\mathbf{r})\nabla_p^i P_s - \tfrac{C_F}{2MT}(\mathcal{H}_{ij}(0) + \mathcal{H}_{ij}(\mathbf{r}))\nabla_p^i(p^j P_s)\\ &\quad = -2C_F\Gamma(\mathbf{r})\left(P_s - \tfrac{P_o}{N_c^2-1}\right) + \tfrac{C_F}{4}(\mathcal{H}_{ij}(0) + \mathcal{H}_{ij}(\mathbf{r}))\Delta_p^{ij} P_s\,,\end{aligned} \tag{9}$$

if we exclude the collision term (first one in the second line) it coincides exactly with the Langevin equation that was found in QED [7]. A similar formula can be found for the octet. By construction this set of equations will produce more suppression that the corresponding QED case, however qualitative results are similar. Comparing with the random color force approach, this method gives more realistic result with a similar numerical cost but it has the disadvantage that it is not trivial to generalize to a situation with more than one heavy quark and one heavy antiquark.

3. Beyond the instantaneous gluon exchange approximation

In this section we are going to discuss the reason why the validity region of Langevin-like equations in QCD is much smaller than the analogous QED case. We will also propose an alternative.

The approximations applied to the time evolution operator $U_{s,o}$ in the previous section have a consequence on the physical information contained in the master equation. $U_{s,o}$ encodes the fact that transitions that release energy are more probable that those that need to absorb energy from the medium, in other words, it allows the master equation to be consistent with detailed balance. The reason why this is more important in QCD than in QED is that in QCD the entropy increasing part of the evolution (related with collisions) does not vanish in the semiclassical limit (small y). This makes QCD physics more sensitive to the energy gap between the different states.

These observations allow us to propose a semi-classical equation derived in the limit in which the decay width is much smaller than the real part of the potential (details are given in [2]). Here we study a simplified version of this equation in which the octets are treated in the large N_c limit and the only singlet state considered is the $1S$ state.

$$\frac{\mathrm{d}p^{\mathrm{s}}}{\mathrm{d}t} = g^2 C_F \int_{\mathbf{p}} \left(p^{\mathrm{o}}_{\mathbf{p}} - p^{\mathrm{s}} \mathrm{e}^{-\frac{E^{\mathrm{o}}_{\mathbf{p}} - E^{\mathrm{s}}}{T}} \right) \int_{\mathbf{q}} \Delta^{>}(\omega^{\mathrm{o}}_{\mathbf{p}} - E^{\mathrm{s}}, \mathbf{q}) |\langle \mathrm{s}|\mathcal{S}_{\mathbf{q}\cdot\hat{\mathbf{r}}}|\mathrm{o}, \mathbf{p}\rangle|^2 \,, \tag{10}$$

$$\begin{aligned} &\tfrac{\partial p^{\mathrm{o}}_{\mathbf{p}}}{\partial t} - \gamma \nabla(\mathbf{p} p^{\mathrm{o}}_{\mathbf{p}}) - \tfrac{T\gamma M}{2} \Delta^2 p^{\mathrm{o}}_{\mathbf{p}} = \\ &-\tfrac{g^2}{2N_c} \tfrac{1}{\Omega} \left(p^{\mathrm{o}}_{\mathbf{p}} - p^{\mathrm{s}} \mathrm{e}^{-\frac{E^{\mathrm{o}}_{\mathbf{p}} - E^{\mathrm{s}}}{T}} \right) \int_{\mathbf{q}} \Delta^{>}(\omega^{\mathrm{o}}_{\mathbf{p}} - E^{\mathrm{s}}, \mathbf{q}) |\langle \mathrm{s}|\mathcal{S}_{\mathbf{q}\cdot\hat{\mathbf{r}}}|\mathrm{o}, \mathbf{p}\rangle|^2 \,, \end{aligned} \tag{11}$$

In this framework the survival probability of quarkonium is much higher than in Langevin-like equations. This is a consequence of taking into account the gap between singlet and octet states. This gap is very sensitive to the value of the Debye radius, therefore screening and the decay width are correlated in this framework.

4. Conclusions

We have derived a master equation that describes the evolution of heavy quarks in a medium. This equation depends on quarkonium's time evolution operator $U_{s,o}$. If this is approximated by $1 - K\,dt$ we obtain Langevin-like equations which allow to study many of the features of quarkonium in a medium with a small computational cost but have an applicability region much smaller than the corresponding QED case. We have shown that the reason for this is that in QED collisions vanish in the semiclassical limit. Taking this into account we proposed a set of equations in which the decay of singlet bound states is suppressed by this gap. More details and the numerical computations that substantiate our claims can be found in [1, 2].

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