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Student teachers' types of probing questions in inquiry-based mathematics teaching with and without GeoGebra

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Previous studies have produced several typologies of teacher questions in mathematics. Probing questions that ask students to explain are often included in the types of questions. However, only rare studies have created subtypes for probing questions or investigated how questioning differs depending on whether technology is used or not. The aims of this study are to elaborate on different ways of asking students to give explanations in inquiry-based mathematics teaching and to investigate whether questioning in GeoGebra lessons differs from questioning in other lessons. Data was collected by video recording 29 Finnish mathematics student teachers' lessons in secondary and upper secondary schools. The lesson videos were coded for the student teachers' probing questions. After this, categories for the types of probing questions were created, which is elaborated in this paper. It was found that the student teachers who used GeoGebra emphasised conceptual probing questions during the explore phase of a lesson slightly more than the other student teachers.

Key words: inquiry-based teaching; probing; teacher education; teacher questioning; technology

1. Introduction

One of the roles of a teacher is to facilitate and orchestrate classroom discussions.[1]
Teacher questioning is an essential component of classroom interaction. According to

Hattie's synthesis of meta-analyses, teacher questioning has a medium effect on student learning.[2] Traditionally, mathematics teachers ask a lot of questions for which there is one correct answer that they already know whereas the proportion of questions that ask for explanation is relatively small (e.g. [3–6]). For example, Kawanaka and Stigler found that only 9.6% of German, 22% of Japanese, and 1% of U.S. eighth-grade teachers' questions requested an explanation or description instead of a yes/no answer or stating a fact.[3] More emphasis needs to be given to probing questions which ask students to explain their thinking.[7–8] Currently there exist efforts to promote inquiry-based mathematics teaching in schools.[9] In inquiry-based mathematics teaching, students are building mathematical ideas, and these ideas need to be made visible and communicated to others. Thus, the use of probing questions plays an important role in developing one's teaching toward inquiry-based mathematics teaching.[cf. 7]

Teacher questioning in mathematics has been studied intensively and several classifications of questions have been developed (e.g. [3–5,10–11]). However, only rare studies have developed more detailed classifications of probing questions. Often the distinction is made only between requesting a description of how something is done and asking the reasons for something (e.g. [3]).

The use of dynamic mathematics software such as GeoGebra is claimed to aid students' inquiry as they can explore and notice mathematical properties and explain the reasons for their observations (e.g. [12]). The teacher has a crucial role in activating students to reason more mathematically and to build mathematical explanations for their findings. [13–14] This means asking deliberate probing questions. By asking probing questions, the teacher complements the feedback or information provided by the software.[15] Furthermore, the use of technology may affect teacher questioning. According to Hollebrands and Lee, technology adds complexity to teacher questioning.[16] For instance, the role of proving

might change from assuring that a theorem is true to finding reasons why it is true.[17–18] Because the use of technology may have an effect on teacher questioning, Hollebrands and Lee call for more research on this issue. [16] In their own study, Hollebrands and Lee analysed how three pairs of pre-service teachers used questions and statements in a lesson involving the use of dynamic geometry software. They found that the pre-service teachers asked questions focusing purely on technology, on technology to notice mathematics, on mathematics with the use of technology, and purely on mathematics. However, they found that the pre-service teachers did not push students to explain why the observed property might be true. This raises the issue of whether pre-service teachers are prepared to ask probing questions while using technology.

In another study about teacher questioning in technology enriched mathematics lessons, Akkoç provided evidence of an increased number of mathematical and technical questions in pre-service teachers' lesson plans after a workshop on technology use.[19] She found that the number of all kinds of questions increased but especially those that promote reasoning. However, it is not known whether these changes would also happen in a real lesson or whether the questioning in technology-aided lessons differs from other lessons after the same training. More research-based knowledge about teacher questioning in technology-enhanced lessons would help to understand the differences in questioning when using or not using technology and to design support for teachers. In particular, as the probing questions have an essential role when working with dynamic mathematics software, we should know if teachers use these questions more or less frequently when using technology than in other lessons. In addition, the types of probing questions may vary when using technology. This kind of research-based understanding would help teacher educators to support teachers as the differences in questioning could be discussed with teachers.

Most of the studies about teacher questioning have been focusing on in-service teachers. Only a few studies have focused on the questioning approaches of pre-service mathematics teachers.[11,16,19–20] These studies indicate that although pre-service teachers are still novice teachers and need to develop their questioning practices, they do ask varied kinds of questions and are able to develop their questioning skills even during the initial teacher training.[11,19–20] However, the mentioned studies have investigated questioning in interviews [11,20] or mainly from lesson plans [19]. Thus, more research is needed to understand the questioning practices of student teachers, particularly in real classrooms. A better understanding of the ways student teachers ask probing questions would help teacher educators promote better inquiry-based mathematics teaching.

The aim of this study is to elaborate on different ways of asking students to explain in inquiry-based mathematics teaching. This study complements the previous studies by creating a more detailed classification of probing questions, by comparing questioning with and without the use of GeoGebra in inquiry-based mathematics teaching, and by focusing on less-researched student teachers. The following research questions guided the data analysis: (1) What different types of probing questions do the student teachers ask? (2) Are there differences in frequency of the types of probing questions asked in GeoGebra-enriched and in other lessons?

2. Probing questions in mathematics teaching

Sahin and Kulm [5] have built a question classification for mathematics teaching. They consider three types of questions: factual, guiding, and probing. Factual questions request a known fact, guiding questions give hints or scaffold a solution, and probing questions ask for elaboration, explanation, or justification. Sahin and Kulm used the following three criteria for identifying probing questions: (1) ask students to explain or elaborate their thinking, (2) ask students to use prior knowledge and apply it to a current

problem or idea, (3) ask students to justify or prove their ideas. They found that the use of probing questions by two sixth-grade teachers varied from 17% to 42%.

Although teacher questioning has been studied intensively, only a few studies have produced classifications of different types of probing questions. Kawanaka and Stigler [3] divided questions that request explanation into five categories: (1) requesting analysis, synthesis, conjecture, or evaluation; (2) requesting how to proceed in solving a problem; (3) requesting the methods that were used to solve a problem; (4) requesting the reasons why something is true, why something works, or why something is done; and (5) requesting other information. They found that German teachers asked all these kinds of questions evenly, Japanese teachers asked mostly method-used questions, and U.S. teachers asked mostly for reasons in the rare cases when they probed for explanation. Boaler and Brodie developed nine categories of teacher questions.[10] In their typology, question types that request an explanation are (1) exploring mathematical meanings and/or relationships, (2) probing, getting students to explain their thinking, and (3) extending thinking.

Some studies have also investigated the questioning of pre-service mathematics teachers. Moyer and Milewicz created categories for pre-service teacher questioning strategies when interviewing elementary school children.[11] The category of using probing and follow-up questions included the following questioning strategies: (1) questioning of only incorrect responses, (2) non-specific questioning that did not acknowledge an individual child's responses, and (3) competent questioning that attended to a child's responses and probed for more information.[11] These categories were adapted by Weiland et al., who found that pre-service teachers can develop their questioning practice in the interview context but they still have areas to improve in probing students' thinking.[20]

Kazemi and Stipek [8] analysed classroom episodes which had either a high or a low press approach for conceptual thinking. They noticed that in the high press episodes, teachers

asked students to give “reasons for their mathematical actions, focusing their attention on concepts rather than procedures” [8, p. 68]. In the low press episodes, the emphasis was on describing steps that were taken to solve a problem. Similarly, probing questions can be conceptual or procedural depending on whether students are asked to explain reasons or steps. According to Hiebert and Lefevre’s classical definition, conceptual knowledge is knowledge that is connected to other pieces of knowledge and the holder of the knowledge recognizes the connection.[21] In contrast, procedural knowledge includes rules, algorithms, and procedures used to solve mathematical tasks.[21] Conceptual probing questions invite students to articulate their reasoning or thinking, which requires making the connections explicit. On the other hand, procedural probing questions ask students to explain procedures, methods or actions. While previous researchers have found it difficult to interpret and differentiate between conceptual and procedural knowledge, it may be easier to investigate teacher press for conceptual knowledge [cf. 8] through observing conceptual and procedural probing questions.

3. Inquiry-based mathematics teaching

Artigue and Blomhøj [9] conceptualise inquiry-based mathematics teaching by comparing it to other existing frameworks. Inquiry-based teaching has many commonalities with, for example, problem-solving approaches while other frameworks add emphasis on classroom discussion.[9] Artigue and Blomhøj summarize practices included in mathematical inquiry as follows: “elaborating questions; problem solving; modelling and mathematizing; searching for resources and ideas; exploring; analysing documents and data; experimenting; conjecturing; testing, explaining, reasoning, arguing and proving; defining and structuring; connecting, representing and communicating” [9, p. 808].

In this study, inquiry-based mathematics teaching means that students work alone or in small groups to solve non-standard mathematical problems designed to potentially bring

forth mathematical ideas related to the topic at hand while the teacher supports the students in their reasoning and orchestrates classroom discussion. Inquiry-based mathematics teaching consists of launch, explore, and discuss/summarize phases as elaborated by Stein et al.[1] The teacher introduces the problems in the launch phase. Then, students work in small groups during the explore phase. Finally, the students' solutions are discussed in the discuss/summarize phase.

This view of inquiry-based teaching emphasizes the teacher's role in facilitating and orchestrating classroom discussions.[1,7] In particular, it is important to have students explain their ideas.[8] Thus, probing questions have an important role in inquiry-based teaching. There is already evidence that teachers ask more probing questions in reform mathematics teaching, that is similar to inquiry-based teaching, than in traditional classrooms.[10,22–23] However, probing questions do not necessarily indicate successful inquiry-based teaching. As Kazemi and Stipek's study shows, two classroom discussions may seem at a surface level to include similar teacher probing but a closer look may reveal differences in how much conceptual thinking is pressed for through requiring students to explain reasons in addition to procedures.[8] Thus, when investigating probing questions, special attention should be given to what types of probing questions are asked.

3. Methods

3.1. Data collection

The participants in this study are 29 Finnish secondary and upper secondary mathematics student teachers. They were in the final phase of the teacher training program and had taught several school lessons during the program. The student teachers participated in a unit about inquiry-based mathematics teaching. The unit was taught by the author and included nine 90-minute group work sessions.

In the unit, the student teachers were introduced to the basic ideas of inquiry-based mathematics teaching and challenges for implementing it in schools. They participated in solving as well as designing problems with and without GeoGebra, and their experiences and emerging ideas were discussed. The basic functions of GeoGebra and how to use them in designing problems for students were studied. Possible teacher actions in different lesson phases were considered and the student teachers practiced guiding students in hypothetical teaching situations involving GeoGebra (see details in [13]). In connection to the hypothetical teaching situations, the importance of interpreting student ideas and guiding them toward more mathematical reasoning based on their own ideas instead of guiding them to follow pre-designed steps was discussed. Ways to get students to build mathematical explanations for their observations and to justify their complete or incomplete ideas were explored. The discussion about possible teacher actions also included using appropriate probing questions. The unit also included the basics of educational research methods and an analysis of previous research about inquiry-based mathematics teaching. Afterwards, when the data collection for this study was complete, the student teachers conducted their own research by analysing their own lesson videos in pairs.

After the unit, each student teacher implemented one inquiry-based mathematics lesson in grades 7–12. All of the lessons were structured using the launch, explore, and discuss/summarize phases. During the explore phase, students usually worked in pairs or in three-person groups. The number of student groups varied between 7 and 10. Altogether, there were 16 secondary school lessons (grades 7–9) and 13 upper secondary school lessons (grades 10–12). Lesson length was 45 minutes in the secondary school and either 45 or 90 minutes in the upper secondary school. Students used GeoGebra software in 7 secondary school lessons and in 10 upper secondary school lessons.

The lessons were video recorded. The video camera was connected to a wireless microphone attached to the teacher. The hand-held video camera followed the teacher as he or she moved around the classroom. When the teacher was interacting with a student pair, the camera was positioned so that students' notebooks or computer screens could be seen. Although the microphone was attached to the teacher, it also captured the students' comments when the teacher talked with a group of students. Students' written notes were collected after each lesson.

3.2. Data analysis

The data was analysed using Atlas.ti video analysis software. All the teachers' subject-related questions were coded as probing, guiding, or factual questions. The definitions for these codes were constructed on the basis of Sahin and Kulm's [5] definitions. In particular, all teacher utterances that asked students to explain or examine their thinking, solution method, or a mathematical idea were coded as probing questions. A teacher utterance was considered as a question if it invited the students to give an oral response. For example, utterances such as "explain" were considered as questions even though grammatically they are not questions. On the other hand, grammatical questions were not coded as questions if the teacher did not give the students an opportunity to answer the question. In addition to probing, guiding, and factual questions, all other questions such as questions concerning classroom control were coded as other questions. The inter-rater reliability for coding probing, guiding, factual, and other questions for a sample of 150 questions was 89% (Cohen's kappa = .845, 95% CI 0.776–0.914, $p = 0.000$).

Next, all of the probing questions were further analysed through data-driven coding.[24] First, the probing questions and interactions around them were viewed several times to become familiar with them. Then, the probing questions were clustered into

categories. The categories were constructed by interpreting what the teacher asks students to explain. The method of constant comparison [24] was used as each coded question was compared to the other questions coded to the same category. In addition, it was compared how each question would fit to the other categories. After creating the categories, the properties of the categories were examined by repeatedly viewing the questions of a certain category. In addition, the categories were compared to each other and the relationships between them were explored. Through this process, the original subcategories were organised into seven main categories. The inter-rater reliability for coding the subcategories for a sample of 70 probing questions was 87% (Cohen's kappa = .857, 95% CI 0.771–0.943, $p = 0.000$).

Finally, it was quantitatively investigated whether teacher questioning depended on using or not using GeoGebra. Chi-square tests for independence were used to examine whether distributions of questions depended on the use of GeoGebra in different lesson phases. The effect sizes were measured using Cramer's V.

4. Results

The probing questions were divided into seven main categories. In the following sections, examples of these different types of probing questions are given and the situations around the questions are elaborated on. Following that, statistics about the types of probing questions are given.

4.1. Types of probing questions

4.1.1. Probing method

These questions ask students to explain their solution methods. Typically, a teacher asked students to explain how they solved a problem or what they did. For example, in an 8th grade lesson about percentages, students were determining how much juice can be made from 1.5L

of concentrate when 30% of the juice has to be concentrate. A pair of students had solved the problem, as shown in Figure 1, when the teacher came to talk with the students:

Teacher: Explain a little what you have done here.

Student: We took first 10%, which is this 0.5. Then we multiplied it by 7 to get 70%. Then we added the 30% to 70%.

$$\begin{array}{l}
 30\% = 1,5\text{L} \\
 1,5 : 3 = 0,5 = 10\% \\
 70\% = 0,5 \cdot 7 = 3,5\text{L} \\
 3,5\text{L} + 1,5\text{L} = 100\% \\
 \quad \quad \quad = 5\text{L}
 \end{array}$$

Figure 1. The students' solution of how much juice can be made from 1.5L of concentrate when 30% of the juice has to be concentrate.

The teacher's utterance was a question in the sense that it invited an oral response from the students. The question explicitly asked the students to explain what they did, and thus, encouraged the students to explain how they solved the problem.

There were also questions that asked how students reached a solution without explicitly expressing whether students should explain what they did or their reasoning. For example, the teacher discussed the same task as above with another student pair:

Teacher: Where did you get that kind of an equation [$x > 0.30 = 1.5$]?

Student: Well, you need 30% concentrate. So. This is 30%. So, when x is multiplied by it, we get 30% of x , which is 1.5.

In this case, the student actually responded by explaining the reasoning behind the equation. After this, the teacher continued by guiding the students in solving the equation. Sometimes these kinds of probing questions focused on intermediate steps in students' solutions as in the

above case. However, the teachers also asked how students had reached the solution of the whole problem.

4.1.2. Probing reasoning

In this category, a teacher explicitly asked students to explain their reasoning or thinking. This includes questions in which a teacher asked students to explain how they reasoned about, invented, or concluded something. For example, in a 10th grade lesson about the contingency angle of two tangents to a circle, a student claimed that the sum of the central angle and the angle of contingency is 180° (see Figure 2). Then, the teacher asked her to explain her reasoning:

Teacher: From which did you conclude it?

Student: Because the two other angles are 90, it becomes 180 [sum of the angles C and D], and because this is quadrangle, it is 360 [sum of the angles A, B, C, and D].



Figure 2. GeoGebra applet for investigating the sum of the contingency angle and the central angle.

There were also questions in which a teacher asked what students were thinking or what kind of ideas they had for approaching the problem. In these questions, the teacher is not asking for reasoning behind a specific result. Instead, he or she requests an explanation of how students are thinking in general in the situation. For example, in an 8th grade lesson about trigonometry, a pair of students were wondering how to calculate one side of a right triangle

when an angle and a side were given. When the students asked for help, the teacher asked a probing question:

Teacher: What are you thinking here at the second [task] or did you get started?

Student: No, because we were thinking, but because this side is not known, we cannot use Pythagoras [Pythagorean theorem].

In the above situation, the teacher first asked about students' ideas and then continued by guiding the students to think about how to apply the appropriate trigonometric function.

Often, the function of these kinds of probing questions seemed to be finding out what kind of ideas students had before guiding them.

This category included also two questions that asked students to explain what kind of difficulty they had in thinking about the problem. These questions were posed to student pairs in an 11th grade lesson with similar situations. In both cases, the student pairs had constructed a way to define the peak of a certain parabola using the x -intercepts of the parabola but struggled in finding the peak of a parabola that did not have x -intercepts:

Teacher: What troubles you the most here?

Student: The parabola is behaving badly. It does not intersect the x -axis.

Teacher: How could you go over the problem?

After this, the students deduced to draw a horizontal line that intersected the parabola at two points and drew a perpendicular bisector to the intersection points. This question type is close to a guiding question but still invites students to explain their thinking.

4.1.3. Probing cause

These questions ask students to explain reasons for a mathematical property, the cause of

something, or reasons why students did something. For example, in a 9th grade lesson about divisibility rules, a student claimed that a number is divisible by two if the last digit is even.

Then, the following discussion occurred:

Teacher: What is the reason, could you..?

Student: Because they are divisible by two. [...]

Teacher: Why is it enough to look at the last digit?

Student: Because if the last one were odd, then the number would not be divisible by two. [...]

Teacher: What is the reason that you can divide the whole number by two? I can see that you can divide four [by two].

Student: They are round thousands, round hundreds, round tens, to which only the digit in the end is added to. So it is the one digit that matters instead of the whole number. [...] They are complete thousands, hundreds, and tens, which all are divisible by two, and therefore, the whole number is divisible by two if the last one is not odd.

In this episode, the teacher repeatedly asked the student to explain the reason for the divisibility rule noticed by the student. At first, the student seemed not to understand what kind of reason is asked for, but finally, when the teacher kept on asking, the student formulated a mathematical explanation for the divisibility rule.

4.1.4. Probing meaning

These types of questions are concerned with finding meaning, so with these questions, a teacher may ask students to explain what something means. For example, in a 7th grade lesson about the concept of a variable, the teacher asked about the formula that a student pair

had constructed to describe a certain phenomenon:

Teacher: Tell about this. What does this [the students' formula $h \cdot 5 + 2$] mean?

Student 1: Every hour costs 5 euros plus the 2 euros entrance fee.

Student 2: So, hours [points to h] times the fee [points to 5] plus the entrance fee [points to 2].

There were also questions that did not explicitly ask about meanings but encouraged students to explain more, and thus clarify what they meant. For example, in a 10th grade lesson about the contingency angle of two tangents to a circle, a group of students were explaining their conclusion about the sum of the central angle and the angle of contingency (see Figure 2):

Student: Isn't it that these are 180 and these two have to be 180, then these have to be, their sum has to be 180?

Teacher: Yeah. What were 180? Could you show again?

Student: First of all, these [points to angles C and D in Figure 2]. And then because these all together are 360, then these have to be 180 [points to angles A and B in Figure 2].

In the above situation, the teacher's question caused the student to explain more and clarify what she meant. At first, the student did not even mention that the sum of the angles of the quadrangle equals 360° . Typically, in these questions a teacher asks about some detail of the students' solution.

4.1.5. Probing argument

These questions ask students to give arguments. Some of these questions explicitly request a justification or proof of a mathematical idea. For example, an 11th grade lesson about

logarithms included the following whole class discussion about $\log_2 16$:

Student: We got 4.

Teacher: Yeah. What would be the justification?

Student: Because 2 to 4 equals 16. Isn't it? 4 to 2. I don't know.

In this case, the teacher asked the student to justify his answer but the student was not sure about the justification.

There were also questions that asked students to explain how they know that a mathematical idea is true or whether something really is true. For example, in the 9th grade lesson about divisibility rules, a student claimed that a divisibility rule for 9 is true. Then, the teacher asked how the student knew this:

Teacher: How do you know that this is true?

Student: Well, I calculated enough many times. That was enough.

The student had tested that the rule works with several numbers that are divisible by 9 and that it does not work for numbers that are not divisible by 9. Thus, the teacher's question revealed that the student had tested the rule empirically.

4.1.6. Probing extension

In these kinds of probing questions, a teacher asked students to explain how their solution method would work in a slightly different situation or how the problem could be solved differently. These questions invite an explanation of how a solution could be extended in a new direction. For example, a teacher asked this kind of question in an 11th grade lesson about continuity when a group of students argued that a certain piecewise function is continuous because, according to the graphic calculator, the graphs join at the point where the

expression of the function changes:

Teacher: If you calculated it, what would happen? [...] How could you calculate whether the graphs overlap without drawing the graphs?

Student: Is it possible to calculate the intersection points? If you substitute $x = 1$, it will not be possible [one of the expressions is not defined at $x = 1$].

In the above episode, the teacher's questions steered the students towards considering using the equation of the function in addition to the graph of the function. The question also invited students to explain how they could do this. Thus, the question was a probing question that asked students to extend their solution in a new direction. The difference from guiding question is that in extension questions, students are invited to examine their solution in relation to the potential extension suggested by the teacher. In contrast, guiding questions help students to solve the problem that they are already working on.

4.1.7. Unfocused probing

In addition, there were 21 unfocused probing questions. Unfocused probing questions invite students to explain but do not express what should be explained. For example, this category included the following questions: "Would you like to say something?" and "Explain."

4.2. Procedural and conceptual probing questions in GeoGebra and other lessons

Altogether, the student teachers asked 348 probing questions, which was 25% of all the subject-related questions. Taking into account the lengths of the lessons, they asked 15.2 probing questions per hour on average. Despite the large number of questions, student groups also had periods without any teacher questions while the teacher worked with other groups. In the statistical analysis, it was found that the proportion of probing question to all content-related questions was the same in the lessons utilizing GeoGebra (24%) as in the other

lessons (26%), $\chi^2(1) = 0.804$, $p = 0.370$.

In the categories of probing reasoning, probing cause, probing meaning, probing argument, and probing extension, the questions explicitly focus on conceptual issues as opposed to the probing method category, which has questions focusing on procedural issues. According to the focus, these types of questions are called conceptual and procedural probing questions. The frequencies of the types of student teachers' probing questions are given in Table 1. The majority (70%) of these questions focused on conceptual issues.

Procedural and conceptual probing questions	f	%
Procedural probing questions		
Probing method	97	30
Conceptual probing questions		
Probing reasoning	71	22
Probing cause	61	19
Probing meaning	47	14
Probing argument	37	11
Probing extension	14	4
Total	327	100

Table 1. The types of procedural and conceptual probing questions asked by the student teachers.

Table 2 presents the distributions of the student teachers' procedural and conceptual probing questions in the GeoGebra and other lessons according to the phase of the lesson. The student teachers in this study asked a slightly higher proportion of conceptual probing questions in the GeoGebra lessons (73%) than in the other lessons (68%). However, this result is not statistically significant, $\chi^2(1) = 1.227$, $p = 0.268$.

When considering only the explore phase of the lesson, in which GeoGebra is used the most, the differences are bigger (Table 2). In the explore phase, the student teachers asked a larger proportion of conceptual probing questions in the GeoGebra lessons (86%) than in the other lessons (75%), $\chi^2(1) = 3.915$, $p = 0.048$, Cramer's $V = 0.134$. The effect, however, is small.

In both the GeoGebra and the other lessons, the proportion of conceptual probing questions was highest in the explore phase of the lesson (Table 2). For this reason, the distributions of conceptual and procedural probing questions varied according to the lesson phases, $\chi^2(2) = 32.070$, $p = 0.000$, Cramer's $V = 0.313$. Thus, the lesson phase has a medium effect.

		Procedural probing	Conceptual probing	Total
Launch phase	GeoGebra lessons	43	57	100
	Other lessons	67	33	100
	Total	54	46	100
Explore phase	GeoGebra lessons	14	86	100
	Other lessons	25	75	100
	Total	20	80	100
Discuss/summarize phase	GeoGebra lessons	55	45	100
	Other lessons	45	55	100
	Total	49	51	100
Total	GeoGebra lessons	27	73	100
	Other lessons	32	68	100
	Total	30	70	100

Table 2. Percentages of student teachers' procedural and conceptual probing questions ($n = 327$) in the GeoGebra and other lessons according to the phase of the lesson.

When examining the differences between the GeoGebra lessons and the other lessons in terms of the proportions of all six categories of procedural and conceptual probing questions, no difference was found, $\chi^2(5) = 4.465$, $p = 0.485$. Thus, the difference exists only

when comparing all of the conceptual probing questions to the procedural ones. When considering all types of procedural and conceptual probing questions, the distributions were different in the explore and the discuss/summarize phases of the lesson, $\chi^2(6) = 31,348$, $p = 0.000$, Cramer's $V = 0.316$. Thus, the lesson phase has a medium effect. The distributions are given in Table 3. The launch phase was excluded because few probing questions were asked during that phase.

	Probing method	Probing reasoning	Probing cause	Probing meaning	Probing argument	Probing extension	Total
Explore phase	20	26	19	17	13	6	100
Discuss/summarize phase	50	15	18	8	7	2	100

Table 3. Percentage distributions of the types of probing questions ($n = 314$) in the explore and discuss/summarize lesson phases.

According to Table 3, probing reasoning, probing meaning, and probing argument questions were clearly asked more often in the explore phase than in the discuss/summarize phase.

5. Discussion

This study has elaborated on several different types of probing questions. Although all probing questions request an explanation, different things are asked to be explained. Some probing questions focus on steps in solving the problem and some on reasoning. By creating a classification of different kinds of probing questions, this study extends previous studies on probing questions (e.g. [3–5]).

A relatively large proportion (25%) of the student teachers' questions were probing questions when compared to previous studies.[3–5] Thus, it seems that student teachers are prepared to ask probing questions. Furthermore, the student teachers asked varied kinds of probing questions focusing on procedural and conceptual issues.

According to the results, there was no significant difference between GeoGebra lessons and other lessons with regards to asking probing questions or emphasising procedural and conceptual probing questions. When considering only the explore phase of a lesson, slightly more emphasis was placed on conceptual probing questions in the GeoGebra lessons than in the other lessons. These results suggest that when student teachers are implementing their first inquiry-based mathematics teaching, including GeoGebra activities into the lessons does not direct teacher questioning towards emphasising the actions that the students did with the software. It seems that student teachers can still ask many kinds of probing questions although technology adds complexity to teacher questioning.[16] This result is promising as one of the big ideas in working with dynamic mathematics software is to make observations or conjectures and then explain what is happening [12]. The teacher has an important role in guiding students in making the transition from observing to explaining.[17] Some previous studies [19,15] have also given preliminary evidence that student teachers are able to use purposeful questioning when working with technology.

It is known that orchestrating a classroom discussion in the discuss/summarize phase is a challenge to teachers.[1] According to this study, in the GeoGebra as well as in the other lessons, student teachers tended to ask more varied kinds of procedural and conceptual probing questions in the explore phase than in the discuss/summarize phase. Thus, student teachers could be advised to observe their questioning in the different lesson phases and to examine the opportunities to ask different kinds of probing questions in the discuss/summarize phase.

Some of the created categories of probing questions resemble those of previous studies. The category of probing method is similar to Kawanaka and Stigler's [3] question types that ask how to proceed in solving a problem or what methods were used to solve a problem. The other question types of Kawanaka and Stigler do not have such a clear correspondence to the question types in this study. For example, reasons may be asked using the following questions: probing reasoning, probing argument, and probing cause. When compared to Sahin and Kulm's [5] three criteria of probing questions, their justification criteria is similar to probing argument but otherwise, the criteria do not clearly correspond to the types of probing questions found in this study. Probing extension is similar to Boaler and Brodie's [10] question type extending thinking. In addition, probing meaning corresponds to Boaler and Brodie's category of exploring mathematical meanings and/or relationships. The other categories of Boaler and Brodie do not have a one-to-one correspondence with the categories constructed in this study.

Probing questions that extend students' thinking were rare. This is consistent with Boaler and Brodie's [10] findings. One explanation for this might be that this type of probing is demanding for novice teachers. For example, Martino and Maher analysed how an expert teacher/researcher used these kinds of questions after carefully monitoring students' reasoning.[25] Another reason might be the nature of the lesson, because five of these questions were asked in an open problem-solving lesson (see [14]) in which it is more natural to engage students to think about other possibilities for solving a problem. However, extending students' thinking based on their current situation should be more frequent and student teachers need support for when and how to extend students' thinking.

Probing argument and probing cause are similar in that both request an explanation that justifies an idea. However, they request it in a different ways. When asking for argument, a teacher acts as if the truth of the claim is not known. In contrast, when asking for cause, the

truth is not questioned but we are interested in the reason why the claim holds. As Hanna has suggested, justifications have many functions besides assuring the truth of a claim.[18] In particular, when using dynamic geometry software, students can easily empirically verify the claim, but there is still a need for justification in the sense of determining why the claim holds.[17]

This study has proposed categories of probing questions. As an educational implication, these categories offer a way to recognise and conceptualise different ways of asking students to explain in mathematics, which could help teachers ask more varied kinds of probing questions. In future studies, it would be interesting to study teachers' questioning in other conditions and examine how these conditions affect the use of different types of probing questions. In particular, comparing the methods expert and novice teachers use when asking different types of probing questions would contribute to understanding questioning strategies. Furthermore, this study has suggested that student teachers may emphasize slightly more conceptual issues in their questioning when working with GeoGebra. Thus, whether this effect becomes even greater when using GeoGebra regularly in inquiry-based mathematics teaching could be examined.

The main limitation of the study was that each student teacher was observed only once. Thus, it is not known how well the observed questioning represented each student teacher's questioning in general. It is also possible that the topic or some practical issues affected the teacher questioning only in this one lesson. When comparing questioning in the GeoGebra and other lessons, we have to take into account that the student teachers in these lessons were not the same. Thus, it remains unknown whether teacher-related factors in addition to GeoGebra affected the results.

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