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A Thermoelastic Instability Problem for Axially Moving Plates*

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Abstract

Problems of stability and deformation of a moving web, traveling between a system of rollers at a constant velocity are considered. The plate is subjected to a combined thermomechanical loading, including pure mechanical in-plane tension and also centripetal forces. Thermal strains corresponding to thermal tension and bending of the plate are accounted for. The problem of out-of-plane thermomechanical divergence (buckling) is reduced to an eigenvalue problem, which is studied analytically.

Keywords: Axially moving plate, thermoelastic instability, divergence

1 Introduction

The aim of our studies has been to develop mathematical models representing the behavior of the paper making process, simplifying the problems of moving materials sufficiently, while still providing an understanding of the phenomena. A key point is that the productivity of the paper mill is strongly dependent on the efficiency and reliability of the running paper web. In this context it is important to investigate critical aspects of high speed movement of a web subjected to thermomechanical actions.

Note that in many papers an axially moving web have been modeled as traveling flexible strings, membranes, beams and plates. Classical articles in this field are, for example, Archibald & Emslie (1958), Mote (1972), Simpson (1973), Wang et al. (2005), Parker (1998), Wickert (1992) and Wickert & Mote (1990).

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Webs interacting with external media are considered in, e.g., Chang & Moretti (1991), Banichuk et al. (2010b, 2011). As for orthotropic moving materials, many important results can be found in the books by Marynowski (2008) and Banichuk et al. (2014). An extensive literature review about mechanics of axially moving continua can also be found in these books; see also Marynowski & Kapitaniak (2014).

The focus of this article is the deformation and static stability of a simply supported axially moving elastic rectangular plate with two opposite edges simply supported and two opposite edges free. The equations of dynamics and equilibrium of the plate are derived taking into account local acceleration, acting in the out-of-plane (transverse) direction, and also thermomechanical forces.

We study thermoelastic behavior of the moving plate, focusing on steady-state stability analysis. To this purpose we formulate an eigenvalue problem and solve it analytically. As a result, we determine the critical stability parameters and divergence modes.

2 Basic relations of a traveling plate

Let us consider an elastic plate, which occupies the region

$$\Omega = \{\ (x, y, z) :\ 0 < x < \ell,\ -b < y < b,\ -h/2 < z < h/2\ \}$$

of the rectangular coordinate system xyz, and moves axially at a constant transport velocity V_0 in the x-direction. Here ℓ, b, h and V_0 are given positive numbers. The transverse (out-of-plane) displacement is described by the function $w = w(x, y, t)$. By first setting up the problem in the co-moving coordinate system, and transforming into Euler (laboratory) coordinates, we have the equation for small transverse vibrations of the moving plate in the following form:

$$m\frac{\mathrm{d}^2 w}{\mathrm{d}t^2} = \mathcal{L}^M(w) - \mathcal{L}^B(w), \tag{1}$$

where the second material derivative is

$$\frac{\mathrm{d}^2 w}{\mathrm{d}t^2} = \left(\frac{\partial}{\partial t} + V_0\frac{\partial}{\partial x}\right)\left(\frac{\partial w}{\partial t} + V_0\frac{\partial w}{\partial x}\right) = \frac{\partial^2 w}{\partial t^2} + 2V_0\frac{\partial^2 w}{\partial x \partial t} + V_0^2\frac{\partial^2 w}{\partial x^2}. \tag{2}$$

Here m is the mass per unit area of the plate. Thus the physical dimension of each term in (1) is $(\mathrm{kg/m^2}) \cdot \mathrm{m/s^2} = (\mathrm{kg\,m/s^2}) \cdot 1/\mathrm{m^2} = \mathrm{N/m^2} = \mathrm{Pa}$; from a dimensional analysis viewpoint, the equation represents pressures.

Note that the last form of (2) assumes that the axial velocity V_0 is constant. The right-hand side in (2) contains three terms, representing local acceleration, Coriolis acceleration, and centripetal acceleration, respectively. The operators $\mathcal{L}^B$ and $\mathcal{L}^M$ on the right-hand side of (1) are, respectively, the bending and membrane operators.

To find the explicit expression for the membrane operator

$$\mathcal{L}^M(w) = T_x\frac{\partial^2 w}{\partial x^2} + 2T_{xy}\frac{\partial^2 w}{\partial x \partial y} + T_y\frac{\partial^2 w}{\partial y^2},$$

we will use the relations for the total components of in-plane tensions T_x, T_{xy} and T_y:

$$T_x = T_{0x} - \frac{Eh}{1-\nu}\varepsilon_\theta, \qquad T_y = T_{0y} - \frac{Eh}{1-\nu}\varepsilon_\theta, \qquad T_{xy} = T_{0xy},$$

where T_{0x}, T_{0y} and T_{0xy} are the mechanical tensions and $Eh\varepsilon_\theta/(1-\nu)$ is the thermal part.

Let us introduce the modelling assumptions

$$T_{0x}(x,y) = T_0 = \text{const}, \quad T_{0y}(x,y) = T_{0xy}(x,y) = 0.$$

We can represent the membrane operator $\mathcal{L}^M$ in the following form:

$$\mathcal{L}^M(w) = T_0 \frac{\partial^2 w}{\partial x^2} - \frac{Eh}{1-\nu}\varepsilon_\theta \Delta w, \quad \text{where } \Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}. \tag{3}$$

Here and in what follows E is the Young modulus, ν is the Poisson ratio, h is the thickness of the plate. The thermal strain ε_θ, corresponding to the thermal tension, is defined as

$$\varepsilon_\theta = \frac{1}{h}\int_{-h/2}^{h/2} \alpha_\theta\, \theta \, \mathrm{d}z, \tag{4}$$

where α_θ is the coefficient of linear thermal expansion. The function $\theta = \theta(z)$ is the temperature difference

$$\theta(z) = \theta_{\text{absolute}}(z) - \theta_0,$$

where θ_{absolute} and θ_0 are given in kelvins. The constant θ_0 is a reference temperature, at which the system is considered to experience no thermal strain. If the plate is at a uniform temperature of θ_0, then $\theta(z) = 0$, and by (4), consequently, $\varepsilon_\theta = 0$.

To determine an explicit expression for the bending operator

$$\mathcal{L}^B(w) = -\left(\frac{\partial^2 M_x}{\partial x^2} + 2\frac{\partial^2 M_{xy}}{\partial x \partial y} + \frac{\partial^2 M_y}{\partial y^2}\right), \tag{5}$$

we apply the following relations for the total moments M_x, M_{xy}, and M_y, and cylindrical rigidity D:

$$\begin{aligned} M_x &= D\left[\varkappa_x + \nu\varkappa_y - (1+\nu)\,\varkappa_\theta\right], \\ M_y &= D\left[\nu\varkappa_x + \varkappa_y - (1+\nu)\,\varkappa_\theta\right], \\ M_{xy} &= (1-\nu)\,D\varkappa_{xy}, \quad D = \frac{Eh^3}{12(1-\nu^2)}. \end{aligned} \tag{6}$$

We will also use the following representations for the curvatures $\varkappa_x$, $\varkappa_y$ and $\varkappa_{xy}$:

$$\varkappa_x = -\frac{\partial^2 w}{\partial x^2}, \quad \varkappa_{xy} = -\frac{\partial^2 w}{\partial x \partial y}, \quad \varkappa_y = -\frac{\partial^2 w}{\partial y^2}. \tag{7}$$

For thermal bending deformation $\varkappa_\theta$,

$$\varkappa_\theta = \frac{12}{h^3} \int\limits_{-h/2}^{h/2} \alpha_\theta \, \theta \, z \, \mathrm{d}z.$$

Applying (5)–(7) and performing the necessary transformations and substitutions, we will have

$$\mathcal{L}^B(w) = D\left[\Delta^2 w + (1+\nu)\Delta\varkappa_\theta\right] = D\Delta\left[\Delta w + (1+\nu)\varkappa_\theta\right], \tag{8}$$

where Δ is the Laplace operator (Laplacian) and Δ^2 is the biharmonic operator, $\Delta^2(\dots) = \Delta(\Delta(\dots))$. Using (1)–(2), (3), and (8), we obtain the equation of small transverse vibrations of the travelling elastic plate subjected to thermomechanical loading:

$$m\left(\frac{\partial^2 w}{\partial t^2} + 2V_0\frac{\partial^2 w}{\partial x \partial t} + V_0^2\frac{\partial^2 w}{\partial x^2}\right) - T_0\frac{\partial^2 w}{\partial x^2} - \frac{Eh}{1-\nu}\varepsilon_\theta \Delta w$$

$$+ D\Delta\left[\Delta w + (1+\nu)\,\varkappa_\theta\right] = 0. \tag{9}$$

In what follows we will consider the steady-state case, where $\partial/\partial t \to 0$, and w is a function of x and y only, $w = w(x,y)$. Equation (9) takes the form

$$\left(mV_0^2 - T_0\right)\frac{\partial^2 w}{\partial x^2} + D\Delta^2 w + \frac{Eh}{(1-\nu)}\varepsilon_\theta \Delta w + (1+\nu)D\Delta\varkappa_\theta = 0. \tag{10}$$

The steady-state equation (10) of transverse plate deformation is a fourth-order partial differential equation. Therefore two boundary conditions at each edge of the considered rectangular plate are needed.

We consider the case where two opposite edges at $x = 0$ and $x = \ell$ are simply supported, and the other two opposite edges at $y = -b$ and $y = b$ are free. To formulate the boundary conditions, we introduce the unit normal to the plate boundary, n, and the coordinate s, measured counterclockwise. The symbol α denotes the angle from the $+x$ axis to the vector n, taken counterclockwise. See Figure 1.

The boundary Γ consists of the following parts. At the right edge Γ_r, we have $x = \ell,\ \alpha = 0$; at the left edge Γ_ℓ, $x = 0,\ \alpha = \pi$; at the upper edge Γ_+, $y = b,\ \alpha = \pi/2$; and finally at the lower edge Γ_-, $y = -b,\ \alpha = 3\pi/2$.

At the simply supported boundaries Γ_r and Γ_ℓ,

$$w = 0, \quad M_n = M_x \cos^2\alpha + M_y \sin^2\alpha + M_{xy}\sin 2\alpha = 0. \tag{11}$$

Inserting (6)–(7) into (11), we will have

$$w = 0, \quad \frac{\partial^2 w}{\partial x^2} + (1+\nu)\,\varkappa_\theta = 0, \quad x = 0, \ell. \tag{12}$$

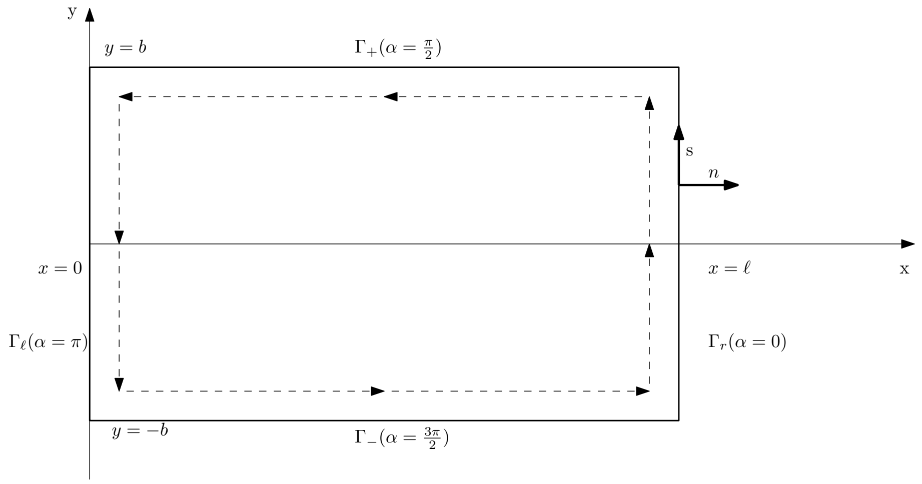

Figure 1: Integration path and definition of the boundaries.

At the free edges of the plate Γ_+ and Γ_-,

$$M_n = M_x \cos^2 \alpha + M_y \sin^2 \alpha + M_{xy} \sin 2\alpha = 0, \quad Q_n + \frac{\partial M_{ns}}{\partial s} = 0, \tag{13}$$

where the shear force Q_n and moment M_{ns} are expressed as (e.g., Timoshenko & Woinowsky-Krieger, 1959)

$$Q_n = Q_x \cos \alpha + Q_y \sin \alpha = \left(\frac{\partial M_x}{\partial x} + \frac{\partial M_{xy}}{\partial y} \right) \cos \alpha + \left(\frac{\partial M_{xy}}{\partial x} + \frac{\partial M_y}{\partial y} \right) \sin \alpha, \tag{14}$$

$$M_{ns} = -\frac{M_x - M_y}{2} \sin 2\alpha + M_{xy} \cos 2\alpha.$$

As for the partial derivative $\partial/\partial s$ in (13), we note that

$$\frac{\partial}{\partial s} = -\frac{\partial}{\partial x} \quad \text{at } \Gamma_+ \, (y = b) \, , \quad \frac{\partial}{\partial s} = \frac{\partial}{\partial x} \text{ at } \Gamma_- \, (y = -b) \, . \tag{15}$$

As a result of (6)–(7) and (13)–(15), we have the following boundary conditions at Γ_+ and Γ_-:

$$\frac{\partial^2 w}{\partial y^2} + \nu \frac{\partial^2 w}{\partial x^2} + (1 + \nu) \varkappa_\theta = 0, \quad y = \pm b, \tag{16}$$

$$(2 - \nu) \frac{\partial^3 w}{\partial x^2 \partial y} + \frac{\partial^3 w}{\partial y^3} = 0, \quad y = \pm b. \tag{17}$$

3 Eigenvalue problem of plate divergence

In this section we study the divergence (static instability) of a travelling isotropic plate, subjected to homogeneous thermomechanical in-plane tension, $\varkappa_\theta = 0$.

The problem is formulated as an eigenvalue problem of the partial differential equation

$$\left(mV_0^2 - T\right)\frac{\partial^2 w}{\partial x^2} + \frac{Eh}{(1-\nu)}\varepsilon_\theta \Delta w + D\Delta^2 w = 0 \tag{18}$$

with the boundary conditions (12), (16) and (17). In order to determine the corresponding eigenfunction $w = w(x, y)$, we apply the representation

$$w = w(x, y) = f\left(\frac{y}{b}\right)\sin\left(\frac{\pi x}{\ell}\right), \tag{19}$$

where $f\,(y/b)$ is an unknown function to be determined. It follows from (19) that the desired buckling mode (steady-state solution) w satisfies the boundary conditions (12) (with $\varkappa_\theta = 0$). The half-sine shape of the solution in the longitudinal direction is well-known (see, e.g., Banichuk et al. (2014)).

Introducing the dimensionless quantities

$$\eta = \frac{y}{b}, \quad \mu = \frac{\ell}{\pi b}, \tag{20}$$

and using the relations (17)–(20) and (16) (with $\varkappa_\theta = 0$), we obtain the eigenvalue problem for the unknown function $f(\eta)$:

$$\mu^4 \frac{\mathrm{d}^4 f}{\mathrm{d}\eta^4} - \mu^2 (2-\beta)\frac{\mathrm{d}^2 f}{\mathrm{d}\eta^2} + (1-\lambda) f = 0, \quad -1 < \eta < 1, \tag{21}$$

$$\mu^2 \frac{\mathrm{d}^2 f}{\mathrm{d}\eta^2} - \nu f = 0, \quad \eta = \pm 1, \tag{22}$$

$$\mu^2 \frac{\mathrm{d}^3 f}{\mathrm{d}\eta} - (2-\nu)\frac{\mathrm{d} f}{\mathrm{d}\eta} = 0, \quad \eta = \pm 1, \tag{23}$$

where we have denoted

$$\lambda = \lambda_0 + \beta, \quad \lambda_0 = \frac{\ell^2}{\pi^2 D}\left[mV_0^2 - T_0\right], \tag{24}$$

$$\beta = \frac{\ell^2}{\pi^2 D}\left(\frac{Eh}{1-\nu}\,\varepsilon_\theta\right). \tag{25}$$

The boundary conditions (24)–(25) correspond to the free-of-traction plate edges.

In what follows we will solve the eigenvalue problem (21)–(23). We consider this problem as a spectral boundary value problem. The problem is invariant with respect to the symmetry operation $\eta \to -\eta$, and, consequently, all its eigenfunctions can be classified as

$$f^{\mathrm{s}}(\eta) = f^{\mathrm{s}}(-\eta), \quad f^{\mathrm{a}}(\eta) = -f^{\mathrm{a}}(-\eta), \quad 0 \le \eta \le 1.$$

Here f^{s} and f^{a} are symmetric and antisymmetric (skew-symmetric) with respect to the x axis ($\eta = 0$). A divergence mode symmetric with respect to the x axis can be represented in the form

$$w = f^{\mathrm{s}}(\eta)\sin\left(\frac{\pi x}{\ell}\right). \tag{26}$$

The algebraic characteristic equation

$$\mu^4 r^4 - \mu^2(2-\beta)r^2 + (1-\lambda_0-\beta) = 0,$$

corresponding to the differential equation (21), has the following roots:

$$r_{1,2} = \pm\frac{k_+}{\mu}, \quad r_{3,4} = \pm\frac{k_-}{\mu},$$

where

$$k_\pm(\lambda_0,\beta) = \sqrt{\left(1-\frac{\beta}{2}\right) \pm \sqrt{\lambda_0 + \left(\frac{\beta}{2}\right)^2}}. \tag{27}$$

Consequently, a symmetric solution of the differential equation can be written as

$$f^{\mathrm{s}}(\eta) = A^{\mathrm{s}}\cosh\left(\frac{k_+\eta}{\mu}\right) + B^{\mathrm{s}}\cosh\left(\frac{k_-\eta}{\mu}\right), \tag{28}$$

where A^{s} and B^{s} are arbitrary constants.

At first, we concentrate on the symmetric case and return to the antisymmetric case later. Using the boundary conditions (22) and (23) and the representations (26) and (28), we obtain linear algebraic equations for determining the constants A^{s} and B^{s} :

$$A^{\mathrm{s}}\left(k_+^2-\nu\right)\cosh\left(\frac{k_+}{\mu}\right) + B^{\mathrm{s}}\left(k_-^2-\nu\right)\cosh\left(\frac{k_-}{\mu}\right) = 0, \tag{29}$$

$$A^{\mathrm{s}}k_+\left(k_+^2+\nu-2\right)\sinh\left(\frac{k_+}{\mu}\right) + B^{\mathrm{s}}k_-\left(k_-^2+\nu-2\right)\sinh\left(\frac{k_-}{\mu}\right) = 0. \tag{30}$$

The condition for a non-trivial solution to exist in the form (26), (27)–(28) is that the determinant of the system (29)–(30) must vanish, since (29)–(30) is a homogeneous system of linear equations in A^{s} and B^{s}. From linear algebra, it is well known that a non-trivial solution satisfying (29)–(30) can only exist if the corresponding matrix is singular. Hence its determinant must be zero. This zero determinant condition leads to the transcendental equation

$$\begin{aligned} &k_-\left(k_+^2-\nu\right)\left(k_-^2+\nu-2\right)\cosh\left(\frac{k_+}{\mu}\right)\sinh\left(\frac{k_-}{\mu}\right) - \\ &\qquad k_+\left(k_-^2-\nu\right)\left(k_+^2+\nu-2\right)\sinh\left(\frac{k_+}{\mu}\right)\cosh\left(\frac{k_-}{\mu}\right) = 0, \end{aligned} \tag{31}$$

which determines the value λ_0 implicitly.

The left-hand side of (31) (with the help of (27) defining $k_\pm(\lambda_0,\beta)$) can be used in a numerical root finder to find the critical eigenvalue λ_0 for any given values of β (thermal parameter), μ (scaled aspect ratio) and ν (Poisson ratio). At $\beta = 0$, the

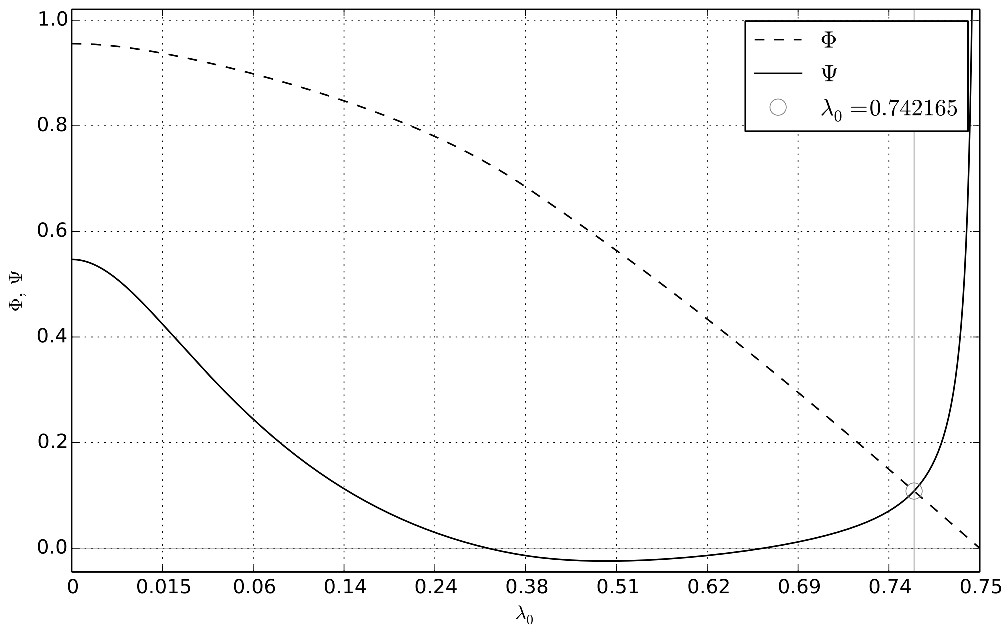

Figure 2: Functions Φ and Ψ for $\beta = 1/4$, $\nu = 0.3$ and $\ell/2b = 1$ ($\mu = 2/\pi$). Note the nonlinear horizontal axis, used to show the ends more clearly.

present consideration reduces to the purely mechanical case considered in Banichuk et al. (2010a).

For analytical considerations, (31) can be transformed into the form

$$\Phi\left(\lambda_0, \beta, \mu\right) - \Psi\left(\lambda_0, \beta, \nu\right) = 0,$$

where we have defined the transcendental function

$$\Phi\left(\lambda_0, \beta, \mu\right) = \tanh\left(\frac{k_-(\lambda_0, \beta)}{\mu}\right) \coth\left(\frac{k_+(\lambda_0, \beta)}{\mu}\right) \tag{32}$$

and the rational function

$$\Psi\left(\lambda_0, \beta, \nu\right) = \frac{k_+\left(\lambda_0, \beta\right)\left(k_+^2\left(\lambda_0, \beta\right) + \nu - 2\right)\left(k_-^2\left(\lambda_0, \beta\right) - \nu\right)}{k_-\left(\lambda_0, \beta\right)\left(k_-^2\left(\lambda_0, \beta\right) + \nu - 2\right)\left(k_+^2\left(\lambda_0, \beta\right) - \nu\right)}. \tag{33}$$

Using the expressions (27), we note that, in order for $k_\pm$ to remain real-valued, we must have $\lambda_0 < 1 - \beta$, and, consequently,

$$\lambda = \lambda_0 + \beta < 1.$$

For an illustration of a typical case, see Figure 2.

To obtain the corresponding buckling mode, we need to determine A^{s} and B^{s}. At the point where the determinant (31) vanishes, one of equations (29) and (30) is

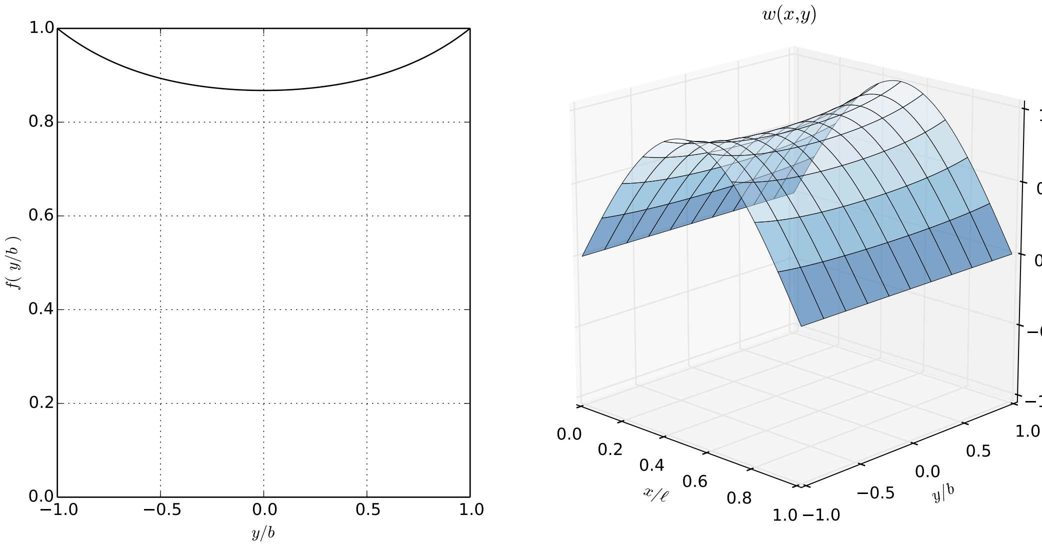

Figure 3: Symmetric buckling shape for $\beta = 1/4$, $\nu = 0.3$ and $\ell/2b = 1$ ($\mu = 2/\pi$).

redundant. We choose to use (29), obtaining the expression

$$A^{\mathrm{s}} = -B^{\mathrm{s}} \frac{\left(k_-^2 - \nu\right) \cosh\left(\dfrac{k_-}{\mu}\right)}{\left(k_+^2 - \nu\right) \cosh\left(\dfrac{k_+}{\mu}\right)}. \tag{34}$$

Because $f(\eta)$ is an eigenfunction, B^{s} is arbitrary. For purposes of illustration, it is convenient to choose B^{s} such that $\|f(\eta)\|_\infty = 1$ on $-1 \le \eta \le 1$. To do this, we may tentatively set $B^{\mathrm{s}} = 1$, and determine the corresponding A^{s} from (34). Then, using these values for A^{s} and B^{s}, we evaluate $f(\eta)$ from (28), on the visualization grid $\{\eta_j \,:\, j = 1, 2, \ldots, N\}$. Finally, we divide both A^{s} and B^{s} by $\max_j |f(\eta_j)|$, obtaining the desired normalization. Figure 3 shows the buckling shape corresponding to the same parameter values as used in Figure 2.

Let us now consider modes of buckling that are antisymmetric about the x axis:

$$w = f^{\mathrm{a}}(\eta) \sin\left(\frac{\pi x}{\ell}\right),$$

where

$$f^{\mathrm{a}}(\eta) = A^{\mathrm{a}} \sinh\left(\frac{k_+ \eta}{\mu}\right) + B^{\mathrm{a}} \sinh\left(\frac{k_- \eta}{\mu}\right). \tag{35}$$

The values k_+ and k_- are given by (27). Using (35) for f^{a}, and the boundary conditions (22)–(23) for the free edges of the plate, we obtain the following system of linear homogeneous algebraic equations for determining the constants A^{a} and B^{a}, corresponding to the antisymmetric case:

$$A^{\mathrm{a}}\left(k_{+}^{2}-\nu\right)\sinh\left(\frac{k_{+}}{\mu}\right)+B^{\mathrm{a}}\left(k_{-}^{2}-\nu\right)\sinh\left(\frac{k_{-}}{\mu}\right)=0, \tag{36}$$

$$A^{\mathrm{a}}k_{+}\left(k_{+}^{2}+\nu-2\right)\cosh\left(\frac{k_{+}}{\mu}\right)+B^{\mathrm{a}}k_{-}\left(k_{-}^{2}+\nu+2\right)\cosh\left(\frac{k_{-}}{\mu}\right)=0. \tag{37}$$

The condition that the determinant of the system (36)–(37) must vanish can be transformed into a transcendental equation for determining the quantity λ_0:

$$\Phi\left(\lambda_0,\beta,\mu\right)-\frac{1}{\Psi\left(\lambda_0,\beta,\nu\right)}=0.$$

The functions $\Phi\left(\lambda_0,\beta,\mu\right)$ and $\Psi\left(\lambda_0,\beta,\nu\right)$ are, again, given by (32) and (33).

4 Conclusion

This paper focused on the static stability (divergence) problem of a moving elastic web, traveling through a system of rollers at a constant velocity. The plate was subjected to combined thermomechanical loading, including pure mechanical in-plane tension and also centripetal forces. Thermal strains corresponding to thermal tension and bending of the plate were accounted for. The problem of out-of-plane thermomechanical divergence (buckling) was reduced to an eigenvalue problem, which was solved analytically, leading to a transcendental equation.

The presented results can be used for fundamental analysis of, for example, the behaviour of a paper web in a cylinder dryer section, where thermal effects may be significant.

References

Archibald, F. R. & Emslie, A. G. (1958). The vibration of a string having a uniform motion along its length. *ASME Journal of Applied Mechanics*, 25, 347–348.

Banichuk, N., Jeronen, J., Neittaanmäki, P., Saksa, T., & Tuovinen, T. (2014). *Mechanics of moving materials*, volume 207 of *Solid mechanics and its applications*. Switzerland: Springer.

Banichuk, N., Jeronen, J., Neittaanmäki, P., & Tuovinen, T. (2010a). On the instability of an axially moving elastic plate. *International Journal of Solids and Structures*, 47(1), 91–99.

Banichuk, N., Jeronen, J., Neittaanmäki, P., & Tuovinen, T. (2010b). Static instability analysis for travelling membranes and plates interacting with axially moving ideal fluid. *Journal of Fluids and Structures*, 26(2), 274–291.

Banichuk, N., Jeronen, J., Neittaanmäki, P., & Tuovinen, T. (2011). Dynamic behaviour of an axially moving plate undergoing small cylindrical deformation submerged in axially flowing ideal fluid. *Journal of Fluids and Structures*, 27(7), 986–1005.

Chang, Y. B. & Moretti, P. M. (1991). Interaction of fluttering webs with surrounding air. *TAPPI Journal*, 74(3), 231–236.

Marynowski, K. (2008). *Dynamics of the Axially Moving Orthotropic Web*, volume 38 of *Lecture Notes in Applied and Computational Mechanics*. Berlin: Springer.

Marynowski, K. & Kapitaniak, T. (2014). Dynamics of axially moving continua. *International Journal of Mechanical Sciences*, 81, 26–41.

Mote, C. D. (1972). Dynamic stability of axially moving materials. *Shock and Vibration Digest*, 4(4), 2–11.

Parker, R. G. (1998). On the eigenvalues and critical speed stability of gyroscopic continua. *ASME Journal of Applied Mechanics*, 65, 134–140.

Simpson, A. (1973). Transverse modes and frequencies of beams translating between fixed end supports. *Journal of Mechanical Engineering Science*, 15, 159–164.

Timoshenko, S. P. & Woinowsky-Krieger, S. (1959). *Theory of plates and shells*. New York: McGraw-Hill, 2nd edition.

Wang, Y., Huang, L., & Liu, X. (2005). Eigenvalue and stability analysis for transverse vibrations of axially moving strings based on Hamiltonian dynamics. *Acta Mechanica Sinica*, 21, 485–494.

Wickert, J. A. (1992). Non-linear vibration of a traveling tensioned beam. *International Journal of Non-Linear Mechanics*, 27(3), 503–517.

Wickert, J. A. & Mote, C. D. (1990). Classical vibration analysis of axially moving continua. *ASME Journal of Applied Mechanics*, 57, 738–744.