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1 **Atomism revisited**

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6

7 **Abstract:** The ancient atomism inspires us to reconsider everything as being composed of indivisible
8 entities, known today as quanta of actions. The quantum of light is the familiar single quantum in its
9 open waveform. Likewise, any other physical action is a geometric notion in terms of energy and time.
10 The quantized systems, e.g., elementary particles take forms of geodesics, i.e., paths of least action in
11 quest for energetic balance with surrounding quanta. The fine-structure constant, as the ratio of two
12 actions corresponding to the electron and neutrino, allows us to deduce unambiguously the
13 characteristic symmetries of leptons, mesons and baryons. We exemplify the quantized structures of
14 photon, neutrino, electron, proton and neutron as well as those of weak bosons and the Higgs boson.
15 This universal description can be related to quantum field theory so that the quantized entity, e.g., an
16 elementary particle, is the source for surrounding field quanta, expressly those that embody vacuum.
17 Moreover, we model some nuclei, among them chemically important ¹²C, as high-symmetry complexes
18 of nucleons. The elementary ingredients can be assembled to models of atoms to illustrate notions of
19 quantum mechanics. Finally, we discuss the four fundamental forces and their relative strengths in
20 terms of actions.

21

22 **Résumé:** L'ancien atomisme nous inspire à reconsidérer tout comme étant composé d'entités
23 indivisibles, connues aujourd'hui comme quanta d'actions. Le quantum de lumière est le quantum
24 unique familier dans sa forme d'onde ouverte. De même, toute autre action physique est une notion

25 géométrique en termes d'énergie et de temps. Les systèmes quantiques, par exemple les particules
26 élémentaires, prennent des formes de géodésiques, c'est à dire, les chemins de moindre action en quête
27 d'équilibre énergétique avec le quanta environnant. La constante de structure fine comme rapport de
28 deux actions, correspondant à l'électron et au neutrino, permet de déduire sans ambiguïté les symétries
29 caractéristiques de leptons, mésons et baryons. Nous illustrons les structures quantiques du photon,
30 neutrino, électron, proton et du neutron, ainsi que ceux des bosons faibles et du boson de Higgs. Cette
31 description universelle peut être liée à la théorie quantique des champs afin que l'entité quantifiée, par
32 exemple une particule élémentaire, soit la source pour le champ de quanta environnant, précisément
33 ceux qui incarnent le vide. De plus, nous modélisons certains noyaux, parmi lesquels le ^{12}C
34 chimiquement important, comme les complexes de nucléons à haute symétrie. Les ingrédients
35 élémentaires peuvent être assemblés à des modèles d'atomes pour illustrer les notions de mécanique
36 quantique. Enfin, nous discutons des quatre interactions élémentaires et de leurs forces relatives en
37 termes d'actions.

38

39 Key words: Elementary Particles; Free Energy; Fundamental Forces; Geodesic; Quantum of Action; The
40 Principle of Least Action.

41

42 I. INTRODUCTION

43 It is well-known that the notion of atom, literally 'uncuttable', emerged in ancient Greek natural
44 philosophy to account for everything as being composed ultimately of indivisible entities.^{1,2,3,4} However,
45 it is less well-known what exactly follows from this universality⁵ in terms of physics.^{6,7} The early
46 philosophers assigned the atom with only a few intrinsic properties, such as size and shape, as well as
47 recognized only a few modes of interactions, such as atoms striking against one another, rebounding
48 and interlocking in the universal void. Today physicists describe particles, i.e., constituents of elements,

49 in terms of field quanta with energy attributes as well as recognize four fundamental interactions
50 mediated by force carriers.^{8,9} In contrast to the ancient atomism with its eternal elementary entities,
51 modern physics portrays the force carriers as virtual or ephemeral particles that exist only for a short
52 time dictated by the uncertainty principle.¹⁰ Parallels and disparities between the old tenet and new
53 theory inspire us to examine the elements of existence by regarding the quantum of action as the modern
54 embodiment of the ancient notion of a-tomos.

55 At first sight it is perhaps not so obvious that there is need for revival of the ancient atomism, or any
56 other alternative perspective for that matter. Calculations of quantum chemistry comply with
57 measurements, although in cases correspondence is limited by precision and computational
58 requirements.¹¹ Further down toward the fundamental description of existence the Standard Model
59 (SM) of particle physics classifies elegantly wealth of subatomic particles and accounts for
60 electromagnetic, weak and strong nuclear interactions. Moreover, the standard theory appears
61 self-consistent¹² and its calculations match well with measurements. Yet, SM falls short of being the
62 complete theory of fundamental interactions since it does not include gravitation. By the same token the
63 large difference between the weak force and gravity, known as the hierarchy problem, remains a puzzle.
64 Although the elementary particles are grouped to three generations of quarks and leptons as well as the
65 force carriers are tabulated to gauge bosons and the Higgs boson, the origin of rule itself is somewhat of
66 a mystery. In the same sense, the characteristic quark and antiquark composition of mesons and baryons
67 as well as those of exotic hadrons is some kind of a riddle. Also neutrino oscillations and finite masses
68 are at variance with SM.¹³

69 The quantum field theory (QFT) describes, in turn, interactions between particles in terms of
70 interacting quantum fields. Feynman diagrams are familiar illustrations of various interaction processes
71 mediated by virtual particles.¹⁴ Despite its success the virtual particle model is worth reconsidering
72 because it follows from perturbation theory, and hence interactions are assumed to be weak. This is

73 typical of scattering processes, but the weak-limit approximation is not valid for bound states such as
74 atoms.¹⁵ Specifically the perturbation theory has not provided results compatible with experiments for
75 the strong force that binds quarks into nucleons. Moreover, we are motivated by the debate whether
76 particles or fields is a fundamental notion that calls for a resolution.

77 We are in no position to solve contemporary theoretical problems. Yet, we argue that it is both
78 insightful and new to consider what can be deduced when considering the quantum of action as an
79 indivisible basic building block of everything. This modern correspondence of the old atomistic can be
80 elaborated with logic to describe elementary particles and their interactions as well as their complexes as
81 atomic nuclei and finally the whole atom, all in agreement with observations and measurements. In this
82 way our study completes earlier accounts on thermodynamics, evolution and emergence that follow
83 from the same stance where the quantum of action is regarded as the modern embodiment of the ancient
84 atom.^{16,17}

85 It turns out that our exercise will not expose anything fundamentally new, but it still provides a
86 tangible viewpoint by rendering quarks and gluons as concrete constituents of elementary particles in
87 the same manner as atoms are building blocks of molecules. Perhaps some specialists will find this
88 portrayal somewhat astounding and as if it were falling short of mathematical rigor. However, we
89 emphasize, results themselves comply with observations. It is only the universal premise and ensuing
90 logic as well as the geometric character of an action distinguish from contemporary expectations. Of
91 course it is not easy to examine a new form from an established mindset. On the other hand, we believe
92 many will welcome this revelation of seemingly abstract and technical notions of elementary particle
93 physics as inspirational in search for ways to comprehend constituents of existence. In a sense our paper
94 also exemplifies the role of themata in science.¹⁸ It is well understood but still often ignored that science
95 is not solely guided by empirical information but in its effort to explain observations in a comprehensive
96 manner also general principles and universal tenets are invariably invoked.¹⁹

97

98

99 II. THE QUANTUM OF ACTION

100 Much of physics entertains with the notion of energy. However, energy does not exist as such. It is
101 the attribute of an action, e.g., that of the quantum of light.^{16,20,21} The action identifies by its unit Js, or
102 equivalently kgm/s-m, to a physical entity with geometric character having energy E on its period of
103 time t , or equivalently momentum \mathbf{p} on its wavelength \mathbf{x} . In other words, the action is an integral over
104 time, or equivalently over wavelength

105

$$106 \quad A = \int E dt = \int \mathbf{p} \cdot d\mathbf{x} = nh \quad (1)$$

107

108 where n denotes the number of quanta in terms of Planck's constant h . The equation says that everything
109 is in motion either along open paths of evolution or along closed stationary trajectories.²² The familiar
110 invariance of steady-state motion is given by Noether's theorem.²³ In quantum theory the constancy of
111 energy, i.e., invariance of observables under certain transformations, is expressed so that there is a
112 unitary operator on the Hilbert space of states.⁸ Conversely, symmetry will be broken when the system
113 evolves from one state to another either by acquiring quanta of actions from its surroundings or losing
114 them to its surroundings in quest of consuming free energy. The least-time free energy consumption, in
115 turn, manifests itself as time asymmetry.^{22,24,25}

116 Hereafter we merely follow the old atomistic idea by considering everything in terms of actions and
117 constructing everything from multiples of the elementary action ($n = 1$). We present actions that comply
118 with observations and measurements, but there could be also alternative actions that comply equally
119 well with data. Then again uniqueness is no end itself either, since resonances and oscillations are
120 natural phenomena.

121

122

123 A. Photon

124 The photon is the most familiar form of the quantum of action.²⁶ We see light and we sense heat.

125 The photon has energy E on its period of time t as well as momentum \mathbf{p} on its wavelength λ , so that the

126 invariant product of these pair attributes is a geometric notion (Fig. 1). The elementary action integrates

127 momentum over the action's path to Planck's constant

128

$$129 \quad h = \int E dt = \int \mathbf{p} \cdot d\mathbf{x} \quad (2)$$

130

131 The lowest ($n = 1$) invariance implies that the photon is a manifestation of the basic building block of

132 nature. The textbook form $E = hf$, where $f = 1/t$ is the photon's frequency, is of course mathematically

133 equivalent to Eq. 2, but it puts emphasis on the photon's energy attribute. This conventional view makes

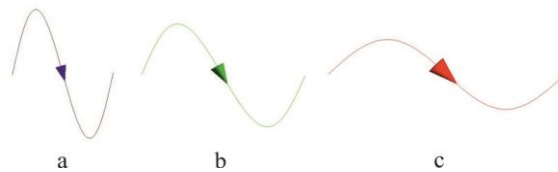
134 sense because it is the photon energy in interactions that determines what will happen, if anything. In

135 contrast, our view by recognizing Planck's constant as the photon's measure, makes sense because it is

136 the photon with its pair attributes that must either arrive to the system or depart from the system for

137 something to happen (Figure 1).

138



139

140

141 FIG. 1. The freely propagating photon is an open quantum of action whose pair-attribute product equals Planck's

142 constant. Thus, the quantum of light when propagating from a higher (a) to lower surrounding energy densities

143 (b,c), will lengthen its period of time, which appears as red shift (b,c), to maintain energetic balance with its
144 surrounding quanta. (Illustration by Mathematica)

145

146 It is easy to understand that the freely propagating photon is massless when the mass is defined as
147 Leonhard Euler did. The Euler characteristic, also familiar from the theorem of Gauss and Bonnet, is
148 obtained by summing up geodesic curvature

149

$$150 \quad k_g = \frac{\mathbf{n} \cdot \gamma'' \times \gamma'}{|\gamma'|^3} = \mathbf{n} \cdot \frac{\gamma''}{\gamma'^2} \times \frac{\gamma'}{|\gamma'|} \quad (3)$$

151

152 over the whole curve γ , i.e., along the quantized action at each point by calculating the cross product of
153 acceleration γ'' and velocity γ' and projecting it on the surrounding curvature with normal \mathbf{n} .^{27,28} The
154 provided factorization helps to recognize the familiar curvature $\kappa = 1/r = \mathbf{a}/v^2$ as given in physical terms
155 of acceleration \mathbf{a} and velocity \mathbf{v} multiplied with unit velocity vector $\mathbf{v}/|\mathbf{v}|$.

156 The universal surroundings is characterized by the tiny curvature of the Universe, i.e., by $1/R$ where
157 the huge radius $R = cT$ at the current age of $T = 13.8$ billion years. The geometric notion of mass m in
158 terms of curvature when given in relation to the curvature of the whole Universe, complies with
159 renowned $E = mc^2$. It allows us to recognize that the squared speed of light c^2 is the (least) L^2 norm of the
160 vacuum.²⁹ For the path of a wave the sum of k_g vanishes, and hence the photon is massless. Physically
161 speaking k_g expresses how much surrounding quanta will depart from the energy density of the vacuum
162 when at energetic balance in the vicinity of a particle.^{30,31}

163 The action characterized at any point by its three Cartesian components, (Eq. 3), implicitly identifies
164 dimensionality of space as three. Time as the fourth dimension associates with changes in energy due to
165 absorption or emission of quanta, when the stationary system opens up for evolution from one state to
166 another.

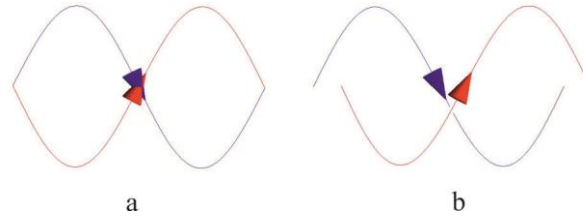
167 The photon's open path entails that any system, ultimately the whole Universe, housing freely
168 propagating photons is invariably an open system. In other words, one may imagine of enclosing a
169 system accommodating also free photons, but such a thought would be fictitious. Moreover, due to the
170 photon propagation from the system to its surroundings or vice versa, energy of the system will change
171 concomitantly with change in energy of its surroundings. Only at a dynamic stationary state the
172 to-and-fro flows balance exactly. Therefore at the thermodynamic balance the photon exchange may
173 well be modelled by virtual photons, because actually nothing happens. Otherwise the approximation,
174 albeit convenient, does not hold.

175

176 **B. Vacuum**

177 We acknowledge insightful studies on ontological elusiveness of space³², and at the same time we
178 admit our incompetence to operate with concepts other than concrete. Thus, we remark simply that
179 when two photons of equal energy, and hence having equal periods of time, co-propagate but
180 out-of-phase, their electromagnetic fields cancel each other out (Figure 2). At the complete destructive
181 interference we see no light, but according to the atomistic tenet the photons do not vanish for
182 nothing.^{16,26,33,34} This conclusion is consistent with observations.^{35,36,37,38} The sky is dark. The vacuum is
183 black, but still full of paired photons giving rise to the energy density of space about nJ/m^3 , which is, as
184 known, approximately at balance with the energy density that is bound in the total amount of matter in
185 the Universe.^{30,31,39,40} Likewise, the gravitational field about a body comprises of photons that are
186 out-of-phase in pairs and at energetic balance with the energy density bound in the body. Moreover,
187 variations in the freely propagating paired-photon density above the universal background density are
188 recognized as gravitational waves.⁴¹

189



190

191

192 FIG. 2. The co-propagating pair of photons is an open compound quantum of action. When the phase configuration
 193 is exactly out-of-phase, the net electromagnetic field vanishes (a). However, the photons themselves do not vanish
 194 for nothing but continue in propagation and carrying energy density. Conversely, when the phase configuration
 195 deviates from the complete destructive interference, electromagnetic fields manifest themselves (b). (Illustrations by
 196 Mathematica)

197

198 When the energy density of the vacuum and its local variations are understood to embody the
 199 paired photons, gravity can be comprehended simply as an energy density difference, i.e., a force just
 200 like any other. It is attractive for two bodies, when the surrounding energy density is lower than that
 201 bound in the system of the two bodies.^{30,31,34,42} Then the surroundings will accept quanta from the
 202 system. The dissipative character of gravity is also obvious from the argument of reversibility. Namely,
 203 it takes work to restore a fallen object back on its initial height. Also the recently detected gravitational
 204 waves from the binary black hole spiraling to a merger, revealed an energy loss that corresponded to
 205 three solar masses.⁴¹

206 Conversely, gravity manifests itself as a repulsive force, when the surrounding energy density is
 207 higher than that bound in the system of the two bodies. This too is obvious, since some work has to be
 208 done to pull two bodies apart. Here on Earth insolation is the common source of photons with sufficient
 209 energy to do the work on the system of bodies. On the universal scale, a distant galaxy moves away from
 210 us, because the greater Universe supplies quanta, though mostly in pairs of the out-of-phase photons,
 211 between us and the distant galaxy from its numerous sources, most notably from stars and black holes.

212 From this perspective space is understood to expand because the quanta bound in matter are converted
213 to freely propagating photons constituting the vacuum.³⁶

214 The out-of-phase configuration for the two co-propagating photons is the free-energy minimum
215 state where their electromagnetic fields balance exactly, and hence it is the natural form of the
216 background energy density. Conversely, it will require some force to move the two photons apart from
217 their out-of-phase relation. This manifests itself as an electromagnetic field (Figure 2). When reasoning
218 in this way, it is no mystery where the photons will emerge all of a sudden when an atom becomes
219 ionized. They have been around all the time, but in the out-of-phase configuration, and hence
220 manifesting only as surrounding energy density.

221 Our view of the vacuum as the paired-photon substance, makes it easy to understand that both
222 gravity and electromagnetic forces follow the inverse square law, because the force carrier is the same
223 for both forces, and hence also coupling effects are anticipated.⁴³ By the same token the norm of vacuum
224 $c^2 = 1/\epsilon_0\mu_0$ depends on the electromagnetic properties, denoted by permittivity ϵ_0 and permeability μ_0 .
225 Also earlier it has been understood that c , ϵ_0 and μ_0 are not fundamental constants but observable
226 density-dependent parameters of the quantum vacuum, although the vacuum has been pictured to have
227 a different embodiment than proposed here.^{44,45}

228 Of course, the vacuum's photon embodiment has been suspected for a long time, but one has not
229 been quite able to put one's finger on it. To recognize the photon as an indivisible and indestructible
230 quantum of action is a decisive conclusion.^{46,47,48} Moreover, the tangible photon-embodied vacuum
231 makes it easy to understand the two-slit experiment and its variants, where the interference involves
232 also the photons embodying the vacuum.^{49,50} The observed interference, even in the case when no more
233 than a single particle is propagating at a time, is no different from the passage when a boat enters to a
234 harbor through one opening of a breakwater while its backwash enters also through another opening, so
235 that at the quay the boat rocks, i.e., interferes with waves that its own propagation generated. This

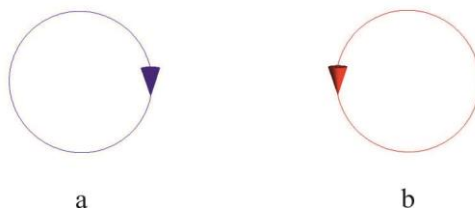
236 reasoning is familiar from the old pilot wave theory.^{51,52} Moreover, the physical vacuum explains Unruh
237 effect⁵³ in tangible terms. An accelerating observer is no longer at thermodynamic balance in its
238 surrounding vacuum, and hence will observe increase in black-body radiation.

239

240 C. Neutrino

241 Geometry of the photon is open, but it is easy to envisage how the quantum will turn back to its
242 beginning and closes to a loop. We associate the closed quantum ring to neutrino, to be precise, the
243 perfectly planar geodesic is the electron neutrino. This conversion from the photon to neutrino
244 exemplifies what Newton conceived about matter being ultimately made of photons.⁵⁴ The invariant
245 measure of the quantum loop is $h/2\pi = \hbar$ and the unitary group $U(1)$ is neutrino's characteristic
246 symmetry. The high-symmetry geometry reveals that the neutrino is its own antiparticle (Figure 3), just
247 as the photon and antiphoton are the one and same particle. The quantized circulation manifests itself as
248 spin.

249



250

251

252 FIG. 3. Neutrino is the elementary action in its closed form (a). Specifically the electron neutrino is portrayed as a
253 perfectly planar loop. The symmetric loop reveals that neutrino is its own antiparticle (b). (Illustrations by
254 Mathematica)

255

256 The tiny mass of electron neutrino is understood via Euler's formula (Eq. 3) to arise from the minute
257 difference between the perfectly planar ring and the almost flat Universe of curvature $1/R$. Conversely,

258 muon neutrino is expected to be a bent ring to account for its higher mass. Likewise, tau neutrino is
259 expected to be still a more curved geodesic found in surroundings where energy density is much higher
260 than in the vacuum. From this geometric perspective neutrino oscillations are means for the neutrinos to
261 seek and maintain balance with surrounding energy density. Put differently, high flavor portions
262 increase when energy density increases. This is indeed observed when comparing neutrinos that arrive
263 directly to the detector from the Sun and those that pass through the Earth before the detection.^{55,56}

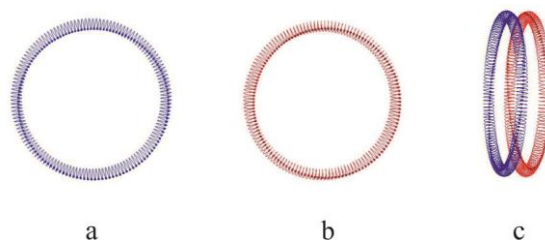
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265 **D. Electron and Positron**

266 When resolving other particles as quantized actions, i.e., geometric entities, the fine structure
267 constant α is a revealing starting point. This number can be understood as a ratio of two actions.^{10,31}
268 When the charge of an electron e is given by Gauss law $\alpha = e^2/4\pi\epsilon_0\hbar c \approx 1/137.036$ yields the action of an
269 electron $e^2/4\pi\epsilon_0\alpha$ relative to the neutrino action \hbar . The numerical value of α implies that the electron
270 comprises of 138 quanta in a toroid form that already Andre-Marie Amperé proposed (Figure 4).⁵⁷ Due
271 to the helical pitch, one quantum does not quite close one full loop, and hence a lag accrues along the
272 torus and one extra quantum will be needed to close the curve of 137 loops.

273 This conclusion about the electron's quantized structure matches observations. The net number of
274 loops amounts to the total charge. The spin characterizes the quantized circulation whose frequency
275 could be associated with Zitterbewegung.⁵⁸ The magnetic moment amasses primarily from the large
276 circle and its anomalous part $\alpha/2\pi$ from the small loops, because due to the rising helix the small loops
277 are not exactly perpendicular to the large circle. The ratio of the electron mass m_e and the mass M of the
278 Universe can be computed from the toroid curve as the vector sum of signed curvature, just the way
279 Euler did. The electron mass is minute, because the vector sum of signed curvature of any two quanta at
280 the opposite faces of the torus is almost zero, departing from nil only due to the pitch.

281 The positron is just like the electron, but its charge is the opposite because its handedness is the
282 opposite (Figure 4). This revelation of the elements of existence in terms of chiral quantized actions
283 resolves the matter vs. antimatter asymmetry problem by regarding antimatter merely as the opposite
284 standard of handedness.^{30,31,59}



285
286
287 FIG. 4. Electron comprises of 138 quanta in a toroid ring of 137 loops (a). The positron is a toroid just as the electron
288 but its handedness is the opposite (b). The net number of windings relate to the charge. The electron magnetic
289 moment sums up to μ_e along the curve from the differential magnetic moment μ . The electron mass m_e is minute
290 because the quanta at the opposite sides of the torus have opposite orientation apart from the toroid pitch, and
291 hence the overall geodesic curvature is minute. When the electron e^- and positron e^+ tori pack face-to-face (c), the
292 two tori may open up and consume each other in annihilation for pairs of photons that co-propagate in the
293 out-of-phase relation. In addition two easily detectable photons propagating in the opposite directions emerge
294 from the annihilation to balance the opposite handedness. (Illustrations by Mathematica)

295
296 The Euler characteristic associates with the notion of mass so that the signed curvatures are
297 summed along the action and projected onto the curvature of surroundings. The obtained quantity
298 means physically speaking how much the photons in the surrounding vacuum have to curve, that is, to
299 become denser, when near the particle. In the vicinity of the electron not that much, because the torus,
300 apart from its pitch, is symmetrical. However, the toroidal winding forces the paired photons away from
301 the minimum-energy out-of-phase relation, which manifests itself as an electromagnetic field about the
302 electron (Figure 2b).

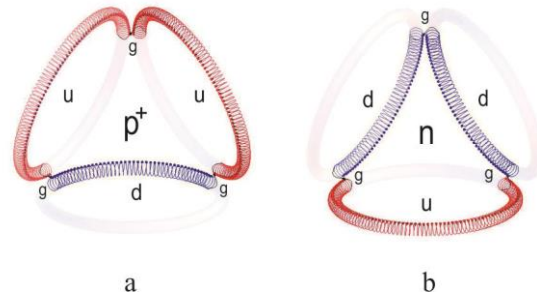
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304 **E. Nucleons**

305 The electron torus structure suggests that the down-quark with charge $1/3e^-$ comprises $1/3$ of the
306 electron torus, i.e., 46 quanta and accordingly that the up-quark with charge $2/3e^+$ comprises $2/3$ of the
307 positron torus, i.e., 92 quanta.^{30,31} When the quarks are connected to each other by gluons, i.e., short
308 wavelength photons, structures of the proton and neutron are obtained unambiguously without
309 alternatives (Figure 5). The closed directional loop of gluon-connected quarks in tetrahedral symmetry
310 displays the familiar SU(3) characteristic of baryons in accordance with quantum chromodynamics.⁶⁰
311 The notion of color means, for instance, that the two up-quarks are distinct from each other in the signed
312 geodesic of proton, because one of them precedes and the other succeeds the down quark. Moreover, the
313 quark composition of baryons is easy to understand, because only a three-quark geodesic will close just
314 as only three antiquarks will form a closed loop.

315 The models of proton and neutron comply with measurements. It is easy to calculate from the
316 displayed structures their approximate magnetic moments $\mu_{p^+} = 2.667\mu_N$ and $\mu_n = -1.889\mu_N$.³¹ When
317 angles between the quarks are slightly adjusted, e.g., due to Coulomb forces, we expect the calculated
318 moments to converge toward the experimental values $\mu_{p^+} = 2.793\mu_N$ and $\mu_n = -1.913\mu_N$.

319 Also the plain mass of a nucleon $937.54 \text{ MeV}/c^2$ can be calculated when knowing the electron mass,
320 to yield elementary estimates 938.82 and $938.22 \text{ MeV}/c^2$ that agree well with measured values $m_{p^+} =$
321 $938.27 \text{ MeV}/c^2$ and $m_n = 939.57 \text{ MeV}/c^2$.³¹ Again we expect these values to home in to the measured values
322 when the structures are slightly adjusted to account for electrostatic effects between the quarks. The
323 proton and the neutron are much heavier than the electron, because there is no curvature at the opposite
324 side of the arcs of quarks to balance the vector sum.



325

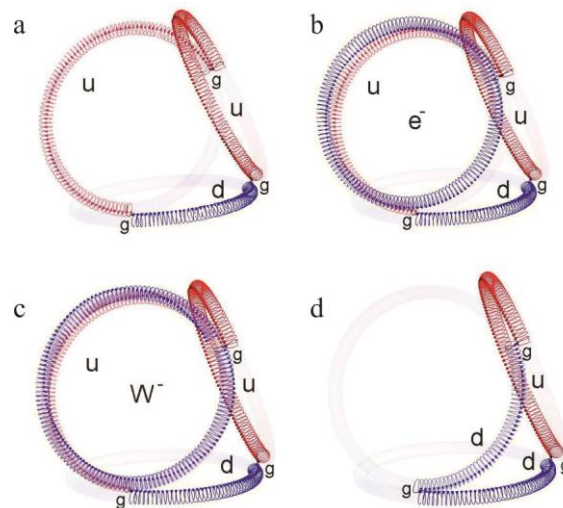
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327 FIG. 5. Models of the proton p^+ (uud) (a) and neutron n (udd) (b) having their up quarks (u) and down quarks (d) in
 328 tetrahedral symmetry follow unambiguously from the electron torus structure when the down quark is identified as
 329 the $1/3$ -arc of the electron torus and the up quark is identified as the $2/3$ -arc of the positron torus as well as gluons (g)
 330 are recognized as short wavelength photons. (Illustrations by Mathematica)

331

332 F. Electron Capture

333 The quantized models of particles make it easy to illustrate how an atomic nucleus captures an
 334 electron so that a proton transmutes to a neutron.^{31,61} When the electron comes close to the up-quark, it
 335 will open up to become W^- boson by losing one of its loops with neutrino. The commencing annihilation
 336 consumes the up-quark altogether, so that $1/3$ of an arc is left from W^- that subsequently closes to a
 337 down-quark, and thereby closing the action as the neutron (Figure 6).



338

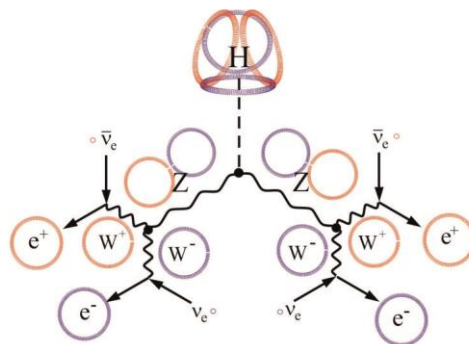
339 FIG. 6. The proton in a nucleus is the starting point of electron capture (a). Next an electron (blue torus) comes
 340 face-to-face to the up-quark (red $\frac{2}{3}$ arc of torus) of a proton (b). When the electron opens up by losing one of its
 341 loops with neutrino, it will become a reactive W^- boson (blue open torus) (c). Subsequently $\frac{2}{3}$ of W^- will annihilate
 342 with the up-quark (red $\frac{2}{3}$ arc of torus). The remaining $\frac{1}{3}$ of W^- (blue $\frac{1}{3}$ arc of torus) will close to a down-quark that
 343 completes the transmutation from proton to neutron (d). (Illustrations by Mathematica)

344

345 G. The Higgs Boson

346 The quantized structure of many a particle can be inferred from its decay scheme. The decay of
 347 Higgs boson to a pair of Z-bosons⁶² suggests to us that it is a perfect tetrahedron with four open tori
 348 (Figure 7), and hence the Higgs particle its own antiparticle. The mass of Higgs, as well as those of weak
 349 bosons, W^- , W^+ and Z is big because each torus ring is open by being short of one small neutrino loop.
 350 These tiny slots will accommodate only very high-frequency photons of the vacuum, i.e., high energy
 351 density, and hence the particle energy is high compared to the vacuum and it is also short-lived.
 352 According to the quantized atomism the Higgs particle, apart from being a highly symmetric particle, is
 353 not special from other particles when considering the concept of mass as geodesic curvature (Eq. 3).⁶³

354



355

356 FIG. 7. The decay scheme of Higgs boson (H) to two Z bosons, and further to W bosons and to electrons (e^-) and
 357 positrons (e^+) (or to muons and anti-muons) via neutrino (ν_e) absorption, implies that the Higgs boson is a highly

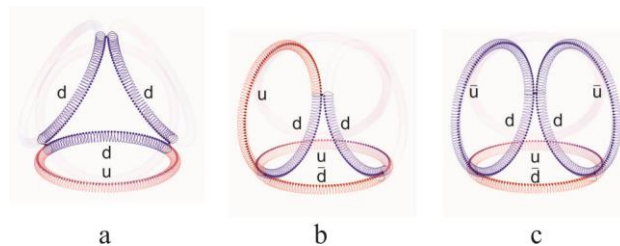
358 symmetric tetrahedron of quanta (at top). Most of its mass, i.e., the particle's high energy attributes to tiny slots in
359 the open rings that are matched by high-frequency photons of the vacuum. (Illustration by Mathematica)

360

361 H. Exotic Hadrons

362 A quark and antiquark combine to a meson. It is straight-forward to realize from meson masses that
363 when the two-quarks are in a plane the configuration is known as the pseudoscalar meson and when
364 they are coordinated along the tetrahedron faces the diquark is referred to as the vector meson. The
365 triquarks are baryons. Four quarks, such as two quarks and two antiquarks, are easily envisioned to
366 form the quantized path of a tetraquark on the tetrahedron's faces (Figure 8a), i.e., a dimeson with
367 compositions such as $u\bar{d}-u\bar{d}$ or $ud-\bar{d}\bar{d}$. Likewise, one may imagine five quarks, such as four quarks and
368 one antiquark, to form a pentaquark (Figure 8b), i.e., the molecule comprising of a baryon, e.g., udd and
369 a meson, e.g., $u\bar{u}$. Finally it is easy to picture a hexaquark, i.e., a dibaryon, e.g., as a combination the
370 neutron udd and antineutron $\bar{u}\bar{d}\bar{d}$ (Figure 8c). Moreover, two neutrons could pack tightly on the four
371 tetrahedron faces, for instance, in the core structure of a compact star. Thus the 'exotic' hadrons^{64,65,66}
372 when modeled as quantized actions, are not that exotic after all.

373



374

375

376 FIG. 8. Examples of quantized models for a tetraquark comprising a dimeson, e.g., $ud-\bar{d}\bar{d}$ (a), a pentaquark
377 encompassing a baryon and a meson, e.g., $udd-u\bar{d}$ (b) and a hexaquark containing a dibaryon, e.g., $udd-\bar{u}\bar{d}\bar{d}$ (c).

378 (Illustrations by Mathematica)

379

380 The mass of an exotic hadron, similarly to the Higgs particle, we expect to stem primarily from
381 geometric details to which the photons of the vacuum must adapt. Most notably any tiny slot relates to
382 high energy. Conversely, the particle's lifetime, despite its high energy, may be long enough to allow
383 detection, when the reactive open end of a quark is not immediately accessible to a breakdown reaction.

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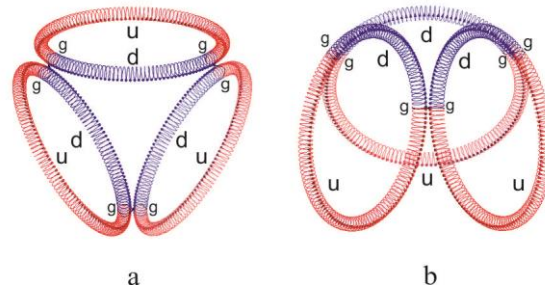
385 **I. Nuclei**

386 The quantized structures of proton and neutron are the building blocks to assemble models of
387 nuclei. The modeling is guided by measured quantities and observed nuclear processes as well as by
388 symmetry arguments that typically relate to free energy minimum structures. However, the models that
389 are presented here mainly serve to illustrate and exemplify the insight to nuclear structure provided by
390 the atomism that considers everything to be composed of quanta.

391 The models of proton and neutron can be assembled to an isospin doublet model of deuterium
392 (Figure 9). In the compact high-symmetry configuration the proton and neutron are intertwined. In the
393 model their magnetic moments, as calculated separately above, add up the total moment $0.886 \mu_N$. This
394 value, by being rather close to the measured value $0.857 \mu_N$, implies to us that the model makes sense.
395 Also it is apparent from the high-symmetry configuration that ${}^2\text{H}$ has only a small electric quadrupole
396 moment. Moreover, when the proton and neutron magnetic moments, as measured separately, are
397 added together, the total moment is $0.879 \mu_N$. The deviation from the experimental value implies that
398 deuterium is an admixture of the ${}^2\text{H}$ compact low-energy state, indexed with spin $s = 1$ and orbital
399 angular momentum $l = 0$, as well as another extended high-symmetry state, indexed with $s = 1$ and $l = 2$.

400 Considering the intertwined structure and elaborated model for electron capture (Fig. 6) it is of
401 interest to note that the deuterium can be dissociated to proton and neutron by neutral current
402 interactions with neutrinos. The cross section for this interaction is comparatively large, and hence ${}^2\text{H}_2\text{O}$
403 is a very good neutrino target.⁶⁷

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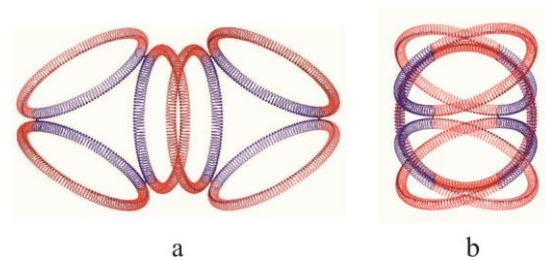
406

407 FIG. 9. The measured electric quadrupole as well as the magnetic moment of ${}^2\text{H}$ and those of proton (p^+) comprising
408 of quarks (uud) linked by three gluons (g) and neutron (n) comprising of quarks (udd) linked by three gluons (g)
409 imply a high-symmetry compact configuration where p^+ and n are intertwined when viewed above (a) and from
410 side (b). (Illustrations by Mathematica)

411

412 The model of ${}^2\text{H}$, when duplicated, assembles to a high-symmetry model of ${}^4\text{He}$ (Figure 10) in
413 agreement with measurements that reveal no magnetic and electric moments. Here we have made no
414 effort to quantify the distance between two ${}^2\text{H}$ units.

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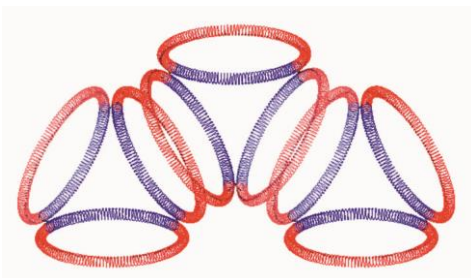
418 FIG. 10. The model of ${}^4\text{He}$ comprising two models of H^2 in the high-symmetry compact configuration is viewed
419 from side (a) and above (b). (Illustrations by Mathematica)

420

421 In a similar manner we propose a model for ${}^6\text{Li}$ to account for its low electric quadrupole moment
422 and nuclear magnetic moment. (Figure 11). Comparing the magnetic moment of ${}^6\text{Li}$ $0.82 \mu_N$ with the

423 value $0.857 \mu_N$ of ^2H , it seems to us there is a slight reorientation of the central unit away from the
424 high-symmetry configuration. Such a spontaneous symmetry breaking to attain the free energy
425 minimum state is the characteristic of many a system.

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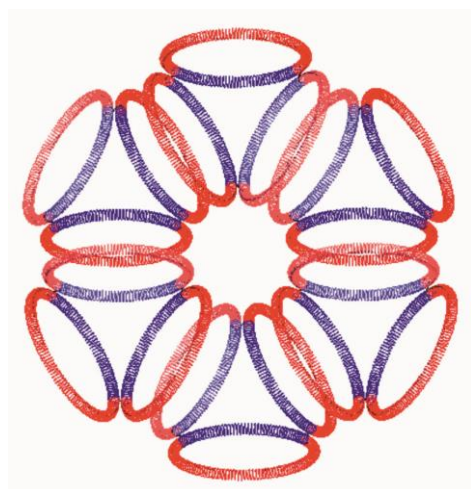
428

429 FIG. 11. A model of ^6Li comprises three models of ^2H in a symmetry configuration. (Illustrations by Mathematica)

430 Analogously ^{12}C can be constructed from two models of ^6Li to comply with its electromagnetic characteristics

431 (Figure 12).

432



433

434

435 FIG. 12. The model of ^{12}C comprising two models of ^6Li in the high-symmetry hexagonal configuration. (Illustrations

436 by Mathematica)

437

438 We expect studies of tetrahedron and truncated tetrahedron packing^{68,69} to be useful when
439 modeling isotopes with increasing number of nucleons. Among various arrangements the correct ones
440 can be identified by comparing their properties with observables, most notably, magnetic moments,
441 electric quadrupole moments, scattering cross sections, binding energy for the last neutron and
442 excitation energy as well as stability. For example, it is well known that 20 tetrahedrons pack tightly to
443 an icosahedron. The high-symmetry model complies with inert characteristics of ²⁰Ne. This and other
444 magic as well as double magic numbers are, of course, contained in the nuclear shell model.⁷⁰ Thus, the
445 nucleons with quarks coordinated at tetrahedron faces illustrated here merely serve to give tangible
446 insight to nuclear complexes in accordance with the established theory of nuclei.

447

448 **III. THE MODEL OF AN ATOM**

449 The scale-free formalism describes the atom likewise by its constituent actions of protons and
450 neutrons forming the nucleus as well as by encircling electrons and quantized interactions (photons)
451 that embody the enfolding space. However, to infer the exact number of quanta of an atom, as denoted
452 in Eq. 1, is troubled by difficulties in detecting the paired photons without electromagnetic fields that
453 constitute the vacuum and hence also embody the open imprecise realm referred to as the atom. In other
454 words, since the constituents of an atom are interacting with the quanta of surrounding vacuum, there is
455 no exact boundary for an atom. Instead the atom is an open system like another at thermodynamic
456 balance with its surroundings actions. Accordingly the atom will respond to changes in surroundings,
457 e.g., in electromagnetic fields, and changes in energy density on the whole.

458 In quantum mechanics⁷¹ the notion of a wave function models this indeterminacy due to
459 interactions with the surrounding actions. For example, the wave function $\Psi(x,t)$ extends in space x and
460 time t to account for the electron's influence on the vacuum. The elementary equation for a stationary
461 motion is the renowned Schrödinger's equation

462

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$$i\hbar \frac{\partial \Psi}{\partial t} = H\Psi \quad (4)$$

465

466 where energy in motion is at balance with potential energy denoted by Hamiltonian H . The formal
467 solution of Eq. 4 $\Psi = \Psi_0 \exp(-iHt/\hbar)$ indeed complies with a closed orbit, i.e., a stationary action where the
468 exponent's ratio of action Ht to the elementary action \hbar relates to the rate of precession. Conversely the
469 complex conjugate Ψ^* denotes motion with the opposite sense of direction. Obviously, despite of the
470 spatial and temporal spread of interactions, the electron of an atom will be found for sure, and hence the
471 probability $P = \int \Psi^* \Psi dx = 1$ sums up to unity.

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The change from one state to another entails either influx or efflux of quanta, and hence
concomitant change in energy. The open path from an initial state can be expressed by an evolutionary
equation of motion for Ψ as well as by the concurrent change $d_t P$ in the probability.²² Since energy is not
conserved along the open path, the evolving probability does not integrate to unity either. This means
for instance, that when the atom absorbs a high-energy photon, there is no guarantee that the electron
will remain bound. Eventually if the electron ends up on an excited orbital, the associated change in
mass can be understood by Eq. 3 as a change in curvature, most notably in the paths of photons that
embody the energy density within the atom.

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When the wave function is understood to model the electron's influence on the photon embodied
vacuum, it easy to comprehend, for instance, that an s orbital extends to the nucleus even though the
electron itself, as the quantized action, would never be there. In this way the atomism, when describing
everything in terms of quanta, relates seemingly abstract theoretical notions to tangible constituents.

485 IV. DISCUSSION

486 The models of elementary particles, given in terms quantized actions, i.e., geometric entities, may at
487 first sight seem astonishingly simple when compared with expectations, for instance, generated by
488 quantum field theory. In particular if one is used to work with elementary particles only in terms of
489 calculations and diagrams, the tangible representations may seem even naive. Then again the
490 fundamental objects of string theory are open and closed strings that are superficially similar to the
491 quantized actions. It is worth recalling that the string sparked off when data was interpreted in terms of
492 “rubber bands”. We are motivated alike to present tangible models. Some one hundred years ago also
493 chemistry seemed opaque before the elements, each with its characteristic valences, materialized as
494 models that today pupils use to build models of compounds. The concrete models of subatomic
495 constituents serve the same purpose of comprehending by seeing, or better by having 3D models in
496 one’s hands.

497 The quantized actions corresponding to the particles comply with measurements, and hence the
498 models serve to represent outcomes of the quantum field theory calculations in the same manner as the
499 concrete atomic and molecular models assist to concretize quantum chemistry calculations. The
500 quantized actions can be recognized as the sources of the quantum fields. The surrounding quantum
501 field obtained from the quantum field calculation relates to the source’s distribution of quanta in the
502 same way as Gauss theorem relates the charge distribution of a source to the surrounding electric field.
503 In other words, the distribution of field quanta is the response taken by the surrounding vacuum to the
504 quantized source. Expressly the vacuum’s paired-photon density distribution is at the energetic balance
505 with the particle mass and the photon-phase distribution is at the energetic balance with the particle’s
506 electromagnetic characteristics. Interdependency of the photon density and phase distribution shows
507 up, e.g., so that the proton charged radius is smaller when probed with muons than when with
508 electrons.⁷² In a sense, permittivity is higher around the heavy muon heavier than the light electron.

509 In the historical perspective the pilot wave of a particle, proposed by de Broglie,⁵¹ corresponds to
510 the surrounding actions, i.e., paired photons that embody the vacuum. When considering calculations, it
511 is worth emphasizing that Maupertuis' principle of least action is a non-determinate equation because
512 driving forces and motions are interdependent.^{7,16,20} This means in some cases that seemingly subtle
513 changes in geometry may cause substantial changes in the particle's properties and vice versa. Therefore
514 ensuing iterative calculations can at times be problematic without convergence, e.g., in cases of
515 oscillations and resonant states.³¹

516 The mass of a particle can be regarded according to Eq. 3 as a geometric response taken by the
517 photon-embodied vacuum energy density. This notion parallels the meaning of Higgs field, but
518 attributes no special meaning to the Higgs particle. The tangible models of particles make it easy to
519 understand how the photons in the surrounding will adapt to the energy densities contained in various
520 particles. In particular, the models illustrate how seemingly slight changes in the quantum structure
521 might cause dramatic changes in the mass. For example, the transformation from electron to W^- boson is
522 accompanied with a huge change in mass. When the electron torus opens up by losing one loop for
523 neutrino, a tiny slot will emerge. The vacuum will adapt to it by placing a short wavelength photon into
524 the vacant slot, and hence the particle appears "heavy" in its surroundings. When such fine details
525 matter, it easy to guess that calculations of elementary particle masses, in particular those of 2nd and 3rd
526 generation, are tricky and the values are susceptible to subtle alterations in the particle's quantized
527 geometry.

528 The quantized structures of elementary particles provide insight to the standard organization of
529 elementary particles to three generations of matter as well as to the gauge bosons and the Higgs boson.
530 Each generation shares the same geometry of its basic constituent. Expressly the first generation
531 neutrino is the one-quantum planar loop. The 2nd generation neutrino is expected to be a more curved
532 loop and the 3rd generation loop is still a more curved away from the plane. We suppose the curvatures

533 of elementary bending modes of neutrino to match the ratio of muon and tau neutrino masses. We
534 understand the 2nd generation particles to share the same basic constituent of the muon neutrino and the
535 3rd generation particles that of the tau neutrino. Therefore the corresponding masses will primarily
536 reflect the curvature of the basic constituent that accrues along the geodesic. From this perspective there
537 is no apparent reason why the particles should limit of three generations. However, higher curvatures
538 correspond to higher energies, and hence to shorter lifetimes. It would be increasingly more difficulty to
539 make unambiguous discoveries of the putative higher generation particles.

540 When everything is described in terms of the quantized actions, also all interactions can be
541 described similarly, i.e., unified. Specifically, the strong interaction means the force, i.e., the energy
542 difference that will be needed to break apart the quantized actions that are bound in nucleons and other
543 hadrons. The weak interaction, in turn, concerns the force that is necessary to transmute one quantized
544 action to another via weak bosons, for example, in beta-decay. The electromagnetic interaction means
545 the force that is required to shift apart the phases of quanta that embody the surrounding energy
546 density, most notably the vacuum. Finally gravitational interaction entails the difference between a local
547 and the universal energy density, i.e., the free space embodied in the quanta. In this way gravity is
548 quantized and compatible with other interactions. The atomism by its commensurable account on
549 particles and interactions by actions relates them to each other via differences in energy densities. In
550 particular, the huge ratio of electrostatic to gravitational coupling constants, i.e., $\alpha/\alpha_G = e^2/4\pi\epsilon_0 G m e^2 =$
551 $4.17 \cdot 10^{42}$, where G is the constant of gravity, relates to the ratio of the radius of the Universe and the
552 radius of electron, and thereby provides insight to the hierarchy problem.³¹

553

554 V. CONCLUSIONS

555 The ancient idea that everything is composed of indivisible elements can be expressed in modern
556 terms of quantized actions. In this way elementary particles physics becomes as comprehensible as

557 chemistry is today with its models of molecules comprising atoms. Expressly models of mesons, baryons
558 and even exotic hadrons can be assembled from models of quarks and gluons as easily as models of
559 molecules can be built from models of atoms with valences. When having the models of particles in
560 hand one may demonstrate various nuclear reactions as simply as chemical reactions with models of
561 substrates and products. Thus, the quantized models of elementary particles are not introduced to
562 supersede calculations by quantum electro- and chromodynamics, just as the models of atoms were not
563 manufactured to displace calculations and simulations of quantum chemistry.

564 The scale-free description in terms of quantized actions benefits from the principle of least action
565 which maintains that the quanta will adopt the paths of least time in the prevailing surrounding energy
566 density. While this is true, it is not so obvious in practice how one would calculate a specific geodesic.
567 However, often the particle's properties and decay schemes suggest at least one structure, often only few
568 alternatives to be considered as a model. Eventual ambiguity among the alternative models is, of course,
569 characteristic of any inverse problem, but the models themselves may suggest experiments to remove
570 the ambiguity.

571 In the end it may not even be the most relevant task to construct detailed models of increasing
572 complexity. More and more versatile systems tend to have more and more alternative paths open for
573 evolution in quest of attaining balance with the surrounding energy density.^{6,7} The quest for
574 thermodynamic balance, i.e., evolution to diversity is observed already at the level of elementary
575 particles, for instance, as admixtures of states, resonances and oscillations as well as emergence of novel
576 particles in extraordinary conditions with increasing energy.^{31,Error! Bookmark not defined.,73} Thus for many a
577 substance, i.e., a system it would be only an elusive and futile aim to nail down anyone of its specific
578 stationary states, i.e., certain symmetry. After all there is nothing permanent except change.

579

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582

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