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The role of semiotic tools in mediating shared meaning making in the learning of mathematics pedagogy

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Abstract

This article investigates the role of semiotic tools as sign vehicles in mediating the socially shared meaning making process in the learning of mathematics pedagogy. Our particular interest is to investigate the evolving role of the social sign in learning within culturally organized activities. The participants in our study were a group of teachers participating in an in-service mathematics course whose pedagogy draws on the sociocultural perspective. A specific analysis method to unravel the nature of the semiotic tool as a sign vehicle was developed for the study. The results of the study suggest that socially shared meaning making around the semiotic sign vehicle consisted of seven phases, with a shared social sign as an end result indicating collaborative learning. On the whole, the semiotic sign vehicle investigated in this study supported the understanding of socially shared meaning making and hence teacher learning in the problem solving situation.

Keywords

Social sign, collaborative inquiry, mathematics education

Introduction

This study emphasizes the mediational role of semiotic tools in enhancing teacher learning in the collaborative problem solving situation in mathematics pedagogy. Specific attention is paid to the evolving role of social sign. A mathematical activity which emerged in an in-service teacher education course exemplifies the developmental phases of social sign in the meaning making process. Furthermore, we introduce the semiotic perspective and semiotic analysis to better understand learning in cultural and collaborative settings. An enhanced understanding

of the role of sign vehicles in learning could promote teaching and teacher education in modern society.

This paper raises mathematics education as the focus of research, with the aim of investigating teacher participation in culturally organised activities of mathematizing. The roots of the concept mathematizing in this study lie in the work of Bauersfeld, who approaches the meaning of participation in the following way: "Learning is characterized by the subjective reconstruction of societal means and models through negotiation of meaning in social interaction" (Bauersfeld, 1988, p. 39). In Bauersfeld's terms, participating in the processes of a mathematics classroom is participating in a culture of mathematizing. In the procedure of mathematizing (Elbers, 2003) the mathematical activity constructed in the teacher education classroom community plays a central role: it serves as a resource for participants to develop mathematical understanding. This process intertwines everyday knowing with mathematical tools and procedures.

In recent years mathematics education research too has paid attention to the importance of language and other semiotic tools in the learning of mathematics (Morgan, 2006). Discursive activity between teachers and students within classrooms has been researched (Cobb, Yackel & McClain, 2000). Less researched is the question of how teachers interact in the area of mathematics. Therefore this study investigates the social interaction of in-service teachers and applies semiotics to investigate the nature of mathematical activity negotiated in the course of the collaborative activity of these in-service teachers.

Lately, cognitively-oriented approaches to education have been challenged by sociocultural theories. The former have approached learning as an acquisition process which takes place as a result of the individual's active reconstruction of domain specific knowledge. Since the acquisition approach conceptualizes knowledge as a kind of property that can be transmitted, the goal of learning is seen as the individual enrichment of domain specific concepts and procedures (cf. Sfard, 1998). The sociocultural learning theories, in contrast to this, approach learning by examining learning in its culturally situated context (Cole, 1996a, 1996b; Vygotsky, 1962; 1978; Wertsch, 1991; Wertsch, del Rio, & Alvarez, 1995) and hence, define the learner as a cultural and historical subject embedded within, and constituted by, a network of social relationships and interactions. Learning and development, then, is explained by the changing nature of these relationships and types of participation in cultural activities (Goos, Galbraith, & Renshaw, 1999). From this perspective, teacher learning can be seen as an open-ended process with the possibility of diverse ways of acting, feeling and thinking (Renshaw & Brown, 1998).

In viewing learning as the process of becoming an active participant in various communities of practice, the sociocultural approach stresses the role of semiotic tools as mediators of modes of thinking (Vygotsky, 1962). However, we lack pedagogical understanding of the mediational processes of semiotic tools, and hence how the meaning and structure of social sign is co-constructed in collaborative learning (cf. Midtgarden, 2010). Therefore this study applies semiotic analysis as a method to investigate the evolving role of sign in the collaborative activity of in-service teachers in mathematics pedagogy. To conclude, we understand the semiotic tool as a vehicle that mediates meanings. In social learning this mediational process is constructed around signs. For the sign to become a social sign requires a shared understanding of the mediational role of signs. Social sign is negotiated in the collaborative meaning process of the collaborating participants.

Collaborative activity mediated by sign vehicles

The roots of sociocultural theories for learning and development lie in the cultural historical tradition of Soviet psychologist Lev Vygotsky. Vygotsky outlined the theoretical basis for the field of cultural-historical psychology in the 1930s (Vygotsky, 1962; 1978). Cultural-historical psychology is a theory that explains how personal mental functioning and development is related to historical, cultural, and institutional contexts. Sociocultural theories, including activity theory (Leontjev, 1981; Chaiklin & Lave, 1993) and cultural-historical activity theory (Cole & Engestrom, 1994; Cole, 1996b) are descendents of the cultural-historical theory. Parallel to the development of sociocultural thinking in the 1990s and 2000s (cf. also Wertsch et al., 1995; Kozulin, 2003; Säljö, 2004), our contemporary Timo Järvillehto (2009; 2012) has, in his general system theory, reflected on the role of the relationship between organism and environment in collaborative action. For Järvillehto, the elements of the social, material and intellectual environment, combined with the properties of the acting human being, create the conditions for mental activity, including learning and development. In Järvillehto's words, it is the relationship between human being and environment that explains learning and development. He also stresses the role of language in social interaction and in the creation of socially shared intentions, aims, knowledge and consciousness. In our study, we investigate the relationship of collaboratively acting human beings with sign vehicles, as conceptualized in the following section.

Sign, meaning and understanding

The basic concepts of general semiotics can be used to clarify the concept of semiotic tool, mentioned in the early cultural historical tradition and yet not sufficiently analyzed and applied in later educational research based on sociocultural theories. In semiotics there are two main traditions: the American and European traditions of semiotics, or if we refer to their respective founders, Charles Sanders Peirce and Ferdinand de Saussure, the peircean and saussurean traditions of semiotics. In this study we call upon the European tradition of semiotics: two-member sign theory and two forefathers of semiotics, Ferdinand de Saussure and Ernst Cassirer. In the European semiotic tradition a dyadic model of sign has been presented, and this model of the two-member sign has been commonly adopted in semiotics as a basic concept. The founder of European semiotics (or semiology), Saussure, presented it at the beginning of the 20th century. Soon after this, Cassirer refined the two-member sign in a way that is useful in the empirical analysis of mental meaning giving and shared meaning giving.

In Saussure's semiotics, *signe* is composed of *signifiant* and *signifié*. Common and generally used English translations of *signe*, *signifiant* and *signifié* are sign, signifier and signified, but they have also been translated in other, more concrete ways. In one English translation of Saussure's work (2005) the used terms are sign, signal and signification. In the same translation, sign and its two aspects have also been articulated with the terms sign, sound pattern and concept (Saussure 2005/1916.) In the English edition of Wilfried Nöth's handbook of semiotics (1990) Saussure's terms are translated as sign, sound image and concept, further concretizing to some extent the abstract terms sign, signifier and signified. The concept of sign in the semiotic theory of Ernst Cassirer consists of a concrete sign to be sensed, an expression (*sinnlicher Ausdruck*) and its contents, significance (*seeliche Inhalt, geistiger Gehalt*) (Cassirer 1964/1925; 1969).

The two-member sign and the terms used by Saussure and Cassirer are enlightening and useful in understanding sign, signifier, signified, meaning and understanding. They can be used when analyzing the process of shared meaning giving, as well as in the analysis of when shared understanding is created in social interaction, for example in collaborative learning. The two-member sign consists of a signifier that is perceptible or has been perceived, and of meaning that had to be or has been interpreted (or sign vehicle and meaning content, expression and content). Signifier and signified are two aspects of sign. Together they form the sign (Figure 1). Put in another way, the sign always has "two sides of the coin": the signifier always has meaning content that must be interpreted, and the meaning contents are always expressed with some perceptible sign vehicle, a signifier.

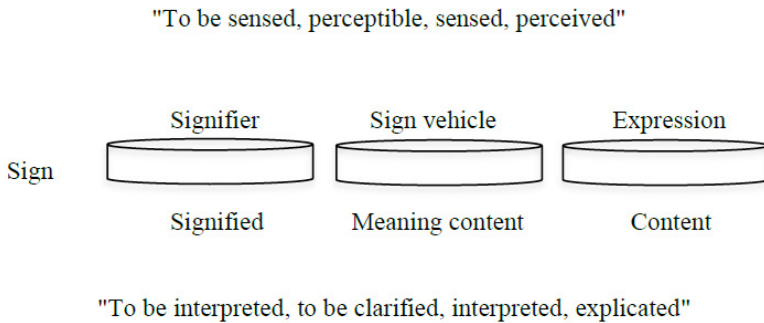


Figure 1. Two-member sign

We can say that we understand something when we are able to connect signified to signifier, or meaning content to sign vehicle, content to expression. In other words, we understand something when we have mental possession of both halves or aspects of the sign, both signifier and signified, and so we have a mental sign which means something; it represents something. When participants in cultural activity negotiate a shared understanding of the meaning content of a sign vehicle, a social sign is formed. We are able to talk about collaborative learning when a social sign as a shared meaning is created. The social negotiation process and the evolving role of a social sign in learning can be examined empirically by following empirical dialogue and linguistic expressions (i.e. use of sign vehicles).

Semiotics and pedagogical challenges to mathematics learning and instruction

Alongside discussion of the theoretical grounding for learning and development (Lave & Wenger, 1991; Wertsch, 1991; Wenger, 1998), there is also a growing interest in a sociocultural approach to mathematics education (Ball, 1993; Cobb, Wood, & Yackel, 1993; Sfard, 2001; Hoyles, 2002). The key constructs in defining the application of the sociocultural framework to mathematics pedagogy in this paper are the communicative approach to cognition (Sfard, 2001) and the mediational role of semiotic tools (Säljö, 1995) in the collaborative meaning making of the participants in the domain in question. The theoretical constructs represented by socioculturalists challenge the traditional views of mathematics learning and instruction, where pre-organized pieces of mathematical knowledge are transmitted to consumers. The task of instructional design in mathematics education from the sociocultural perspective is to give to participants the possibili-

ties to use mathematics in structuring and re-structuring their experiences, in social practices where mediational tools are put to use for specific purposes (Säljö, 1995). The stance adopted in this paper holds that promoting participatory student learning in mathematics also requires the teacher to go through participatory processes in similar types of activities. The communicative approach to cognition (Sfard, 2002) stresses the role of language in the collaborative meaning making of participants. In this paper Halliday's (1978, p. 2) formulation of "language as social semiotic" is applied to interpret language within a sociocultural context in which the culture itself is interpreted in semiotic terms. In the analysis of this study, the language took a specific meaning in communal problem solving and was further interpreted to construct cultural meanings across contexts. The role of language for collaborative inquiry is also reflected upon in the writing of Järvillehto (2000), who stresses the importance of the development of joint language as a tool for collaboration, as well as of the importance of the development of consciousness in the evaluation of collaborative action.

Method

Research Goals

The goal of this study was to use and to develop semiotic concepts for the analysis of emerging mathematics pedagogy in collaborative activity. As a case of social learning we studied mathematical problem solving processes in a collaborative learning situation with in-service teachers. An analytic tool for highlighting the semiotic mechanisms of collaborative problem solving was developed. The specific research goals for this study were:

- To apply semiotics for the analysis of collaborative meaning making.
- To construct a semiotic analytic method to highlight collaborative problem solving processes.
- To investigate the mediational role of sign vehicles in the shared meaning making in the collaborative learning of mathematics teachers.

Participants

The data for the study were collected from two in-service teacher education courses carried out at the Department of Educational Sciences and Teacher Ed-

ucation, Oulu University, Finland. Altogether twenty in-service teachers, who represented either early childhood education (10) or primary education (10), participated in this study.

Description of the professional activities

The activities presented in this study are part of a teacher education course aiming to give the participants tools to anchor their instruction around the collaborative application of semiotic sign vehicles. The sign vehicle selected for the activity was algebra tiles. The selection of this tool was due to its capacity to mediate the abstract domain of algebra in the mathematical curriculum. The specific activity was to model the algebraic expressions of polynomials with the help of algebra tiles. The activity itself involved collaborative inquiry and experimentation. During the course, the in-service teachers worked in self-selected small groups. The average size of the mixed-gender groups was four or five participants. The whole group of twenty teachers worked simultaneously in the same classroom, carrying out their research designs for executing the activities. In this paper, the empirical example of the usage of algebra tiles will be discussed.

Data Collection

The primary data for the study consist of videotaped and transcribed episodes of social interaction in the collaborative problem-solving situation. The language of the activity was Finnish. In the transcription the whole spoken problem solving period of the activity is recorded, excluding the silent moments of participation. The transcription was translated into English. In the activity the teachers solved how to factor $x^2 + 4x + 3$ with the help of algebra tiles. The algebra tiles are wooden pieces (semiotic tools = signs) where a big square means x^2 , one rod means $1x$ and a small square means 1 . In the activity in question, the role of algebra tiles as sign vehicles in the collaborative learning activity is investigated. The specific research question posed for the empirical study is the following: “*What is the mediational role of sign vehicles in collaborative meaning making?*”

Data analysis

This study applies semiotic analysis as a method to investigate the evolving role of the sign in the collaborative activity of in-service teachers. As a method, semiotic analysis investigates the usage of signs. In this study the interaction data

was analysed with the help of the theoretical structure of the sign. This was done by the reconstruction of the logical semiotic structure of the speech turns. In the analysis of the evolving role of the sign in the collaborative meaning making process, the logical phases of a social sign consisted of the following categories: *no elements of sign, elements of signifier, signifier element of sign, signifier connected to signified, elements of signified, sign and social sign*. These seven analytic categories are derived from the logical structure of a sign and make a distinction between a personal and a shared social sign. These categories form the logical phases of an evolving shared social sign, indicating how the shared understanding of the meaning content of a sign vehicle is gradually established. Table 1 summarizes the analytic frames and categories of the semiotic analysis method with examples from the data.

Table 1. The analytic method for analysing the evolving role of sign in collaborative problem solving in mathematics

<i>Cultural focus</i>	<i>Description</i>	<i>Example from the data</i>
Semiotic mode	Highlights the visibility of meaning making via mediational tools	so this colour connected with ordering these pieces carries the meaning
The evolving role of the sign	Description of the phase of evolving social sign	
Phase 1. No elements of sign	The sign vehicle and meaning content aspect of the sign cannot be traced in the speech turn.	I am totally unfamiliar with these
Phase 2. Elements of signifier	Elements of the sign vehicle aspect of the sign can be traced in the speech turn.	so we should organize this with the help of one tile, four rods and three ones
Phase 3. Signifier element of sign	Sign vehicle aspect of the sign can be traced in the speech turn	x plus one
Phase 4. Signifier connected to signified	Meaning content of the sign connected to sign vehicle can be traced in the speech turn	so this colour connected with ordering these pieces carries the meaning
Phase 5. Elements of signified	Elements of meaning content aspect of sign can be traced in the speech turn	
Phase 6. Sign	Meaning content of the sign can be traced in the speech turn	this is x squared, umm and these equal four x
Phase 7. Social sign	Meaning content of the sign can be traced in the speech turn collaboratively in social action	so there it is, so there it is

Results

The results of this study are discussed via a case-based description derived from one teacher group, to highlight joint reasoning through the application of cultural tools (i.e. use of sign vehicles and signs) and shared meaning making in the collaborative learning of mathematics pedagogy.

A case-based description

This case-based description highlights the collaborative processes of one teacher group when factoring polynomials with the help of algebra tiles. The extract characterizes the teachers' discourse as they negotiate and apply the usage of algebra tiles in collaborative problem solving. Table 2 shows the discourse data of the teacher group. The extract consists of 24 conversational turns in total, from a 3-minute continuous working period. Special attention is paid to the identification of semiotic mode in the teachers' discourse. This is followed by a micro-level investigation of three interaction episodes in the teachers' discourse. The analysis of the teachers' discourse reveals altogether three thematic episodes in the construction of an application for the usage of algebraic tiles. The themes for the episodes are problem solving with the help of algebra tiles (Episode 1), clarification through mathematizing (Episode 2), and clarification through hands-on activities (Episode 3). The analysis of the cultural focus of the teacher participation reveals the connection between the evolving role of sign and spoken language.

Table 2. Teacher participation

No	Name	Transcribed discourse	Cultural focus	
			Semiotic mode	The evolving role of the sign
Episode 1: Problem solving				
1	Maritta	so we should organize this with the help of one tile, four rods and three ones	elements of signifier	
2	Annikki	I am totally unfamiliar with these	no elements of signs	
3	Maritta	should we move this, so they don't hinder us		
4	Liisa	so this colour connected with ordering these pieces carries the meaning	signifier connected to signified	
5	Maritta	yes	signifier connected to signified	
6	Karra	this		
7	Maritta	that is how I would imagine	signifier connected to signified	
8	Maritta	this is x squared	sign	
9	Liisa	umm and these equal four x	sign	
10	Karra	so there it is	social sign	
11	Maritta	so there it is	social sign	
Episode 2: Clarification through mathematizing				
12	Liisa	but I don't understand this either	elements of signifier	
13	Maritta	but the length of this equals x and this one and this three	sign	
14	Liisa	yes, should we draw	signifier connected to signified	
15	Karra	x plus one	signifier element of sign	
16	Maritta	multiplies three x plus three	signifier element of sign	

17	Annikki	I fell off the cart	no elements of signs
Episode 3: Clarification through hands on activities			
18	Maritta	may I still advise, we had this x squared four x and three as given	signifier element of sign
19	Annikki	yes okay but this handicraft- task carried the important meaning	signifier connected to signified
20	Maritta	The handicraft-task was to collect these pieces into a connected region so that there are no holes	sign
21	Annikki	this was one, two, three, four x	sign
23	Maritta	and now edge multiplied by edge	sign
24	Annikki	okay, clear	social sign

Episode 1

In Episode 1 the teacher group started the activity by posing the problem. The episode suggests that the usage of algebra tiles as sign vehicles was new to all of the participants. Maritta was eager (6 turns of 11) in participating and tutoring the others. Annikki (turn 2) expressed her unfamiliarity with the usage of cultural tools and Maritta and Liisa made their thinking visible in their turns so that Annikki had the possibility to follow the joint problem solving. Karra was mainly silent but when participating (turns 6 and 10) he supported the group's problem solving by questioning and evaluating. In this episode the group reached the solution for the problem. Episode 1 shows the role of evolving social sign in the shared meaning making process.

Episode 2

Episode 2 highlights how the group of teachers deepened their understanding of the situation. Liisa starts the episode by saying "*but I don't understand this either*" referring to the mathematical meaning of algebra tiles. This turn (turn 12) raises the semiotic nature of interaction. In her turn (turn 13) Maritta ex-

plains the meaning of algebra tiles by tutoring “*but the length of this equals x and this one and this three*” Liisa (turn 14) extends the joint reasoning by suggesting the modelling of the solution by drawing. This leads Karra (turn 15) to join the discourse by writing the expression “ *x plus one*” and Maritta (turn 16) continues “*multiplies three x plus three*”. The mathematical modelling of the situation doesn’t help Annikki who in her turn (turn 17) says “*I fell off the cart*”. The end of Episode 2 highlights that although understanding deepened, shared meaning was not yet constructed.

Episode 3

Episode 3 starts with Maritta’s (turn 18) reaction to Annikki’s comment by tutoring “*may I still advise, we had this x squared for x and three as given*”. In doing so Maritta refers back to the starting information of the problem situation. In her turn (turn 19) Annikki expresses her inability to connect the bridge between the mathematical meaning of the situation and the hands-on activities with algebra tiles. Maritta’s tutoring turns (turns 20 and 23) connected with Annikki’s agreeing and thinking aloud turns highlights the interaction pattern where concrete activity was connected with the abstract nature of mathematical reasoning. Social sign and shared understanding were constructed.

Discussion

The pedagogical focus of this study was to engage the participants in cultural activity that encouraged problem posing and problem solving, social negotiation and joint knowledge construction. This is the key question in modifying classrooms as learning sites where inquiry is applied instead of knowledge being received. In these classrooms the origin for mathematics comes from the emerging nature of the social activity. In the social interaction networks of these classrooms mathematical thinking becomes visible through semiotic vehicles as demonstrated in this study.

From the theoretical and methodological point of view it is important to understand the mechanisms of inquiry processes and collaboration, as well as collaborative activities. In the activity investigated in this study, the teacher participants navigated between several representations for polynomials, such as symbolic notations, pictorial representations and concrete collapsible manipulatives. In this activity space, the teachers made visible their thinking with the help of the language used. The language used, in conjunction with the clarification through hands-on activities, led to the construction of social sign.

The pedagogical application of algebra tiles consisted of three continuously following episodes which were problem solving, clarification through mathematizing and clarification through hands on activities in this study. This result makes visible diverse learning paths for the learning of abstract mathematical ideas negotiated in the course of joint activity. Besides the mediational role of the algebra tiles, they served the participants as a tool to externalize their thinking. The social sign negotiated in the activity was the meaning for “how to apply algebra tiles in the factorization of polynomials”. The pedagogical consideration of the study suggests some requirements for semiotic tools applied in mathematics instruction. They need to be open enough to support and make possible participants’ diverse interpretations, trial and error and hence support the joint meaning making process. From the social point of view, participation instead of receiving emphasizes the multidimensionality of the social environment in classrooms. The pedagogical consideration of this study is to model for teachers how to take these factors into account in pedagogical design.

In this study we introduced the sign theory of general semiotics to further clarify the semiotic tools mentioned in sociocultural theories. This study showed that it is possible to analyse and understand the gradually evolving social learning process using semiotic analysis. In the future we will continue our study of the nature and structure of the concept of the semiotic tool, especially highly abstract signs used in mathematics and mathematics education, and the social learning process. Making a distinction between mental signs and shared social signs, psychosemiotics and social semiotics, will further clarify the concept of sign and the personal and shared meaning making process in learning. The results of this study suggest that teacher’s awareness of these distinctions have a possibility to make mathematical ideas visible and hence negotiable in pedagogical situations .

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