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Hidden attractors in electromechanical systems with and without equilibria

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Abstract: This paper studies hidden oscillations appearing in electromechanical systems with and without equilibria. Three different systems with such effects are considered: translational oscillator-rotational actuator, drilling system actuated by a DC-motor and drilling system actuated by induction motor. We demonstrate that three systems experience hidden oscillations in sense of mathematical definition. While some of these hidden oscillations can be easily seen in natural physical experiments, the localization of others requires special efforts.

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Keywords: Hidden attractors, drilling system, Sommerfeld effect, discontinuous systems

1. INTRODUCTION

The study of stability and oscillations in electromechanical systems requires the construction of mathematical model and its analysis. In addition to normal operation mode the system may experience unwanted oscillations which lead to its failure. Finding the basin of attraction of such oscillations can be a challenging task. Depending on simplicity of finding the basin of attraction in the phase space it is natural to suggest the following classification of attractors (Kuznetsov et al., 2010; Leonov et al., 2011, 2012; Leonov and Kuznetsov, 2013; Kuznetsov, 2016): *An attractor is called a hidden attractor if its basin of attraction does not intersect with small neighborhoods of equilibria, otherwise it is called a self-excited attractor.* Self-excited attractor's basin of attraction is connected with an unstable equilibrium. Therefore, self-excited attractors can be localized numerically by the *standard computational procedure* in which a trajectory, which starts from a point of an unstable manifold in a neighbourhood of an unstable equilibrium, after a transient process is attracted to the state of oscillation (i.e. to an attractor) and traces it. In contrast, hidden attractor's basin of attraction is not connected with unstable equilibria. For example, hidden attractors are attractors in the systems with no equilibria or with only one stable equilibrium (a special case of multistable systems and coexistence of attractors). Recent examples of hidden attractors can be found in *The European Physical Journal Special Topics "Multistability: Uncovering Hidden Attractors"*, 2015 (Leonov et al., 2015b; Shahzad et al., 2015; Brezetskyi et al., 2015; Jafari et al., 2015; Zhusubaliyev et al., 2015; Saha et al., 2015; Semenov et al., 2015; Feng and Wei, 2015; Li et al., 2015; Feng et al., 2015; Sprott, 2015; Pham et al., 2015; Vaidyanathan et al., 2015; Sharma et al., 2015)).

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Hidden oscillations appear naturally in systems without equilibria, describing various mechanical and electromechanical models with rotation. One of the first examples of such models was described by Arnold Sommerfeld in 1902 (Sommerfeld, 1902). He studied vibrations caused by a motor driving an unbalanced mass and discovered the resonance capture (Sommerfeld effect). The Sommerfeld effect represents the failure of a rotating mechanical system to be spun up by a torque-limited rotor to a desired rotational velocity due to its resonant interaction with another part of the system (Evan-Iwanowski, 1976; Eckert, 2013). Relating this phenomenon to the real world Sommerfeld wrote, "This experiment corresponds roughly to the case in which a factory owner has a machine set on a poor foundation running at 30 horsepower. He achieves an effective level of just 1/3, however, because only 10 horsepower are doing useful work, while 20 horsepower are transferred to the foundational masonry" (Eckert, 2013).

We consider three different systems, which have multi stability and experience hidden oscillations in sense of mathematical definition. At the same time we will show that some of these oscillations can be localized if physical nature of the process in such systems is taken into account.

2. TRANSLATIONAL OSCILLATOR-ROTATIONAL ACTUATOR

Following the works (Evan-Iwanowski, 1976; Fradkov et al., 2011) we consider the electromechanical "translational oscillator-rotational actuator" (TORA) system (see Fig. 1). It consists of DC motor which actuates the eccentric mass m with eccentricity l connected with the cart M . The cart is elastically connected to the wall with help of a string and moves only horizontally. The equations of the system are the following

$$\begin{aligned} (M + m)\ddot{x} + k_1\dot{x} + ml(\ddot{\theta} \cos \theta - \dot{\theta}^2 \sin \theta) + kx &= 0, \\ J\ddot{\theta} + k_\theta\dot{\theta} + ml\ddot{x} \cos \theta &= u, \end{aligned} \quad (1)$$

Here θ is rotational angle of the rotor, x is the displacement of the cart from its equilibrium position, u is motor torque, k is a stiffness of the string, k_1 and k_θ are damping coefficients, I is a moment of inertia.

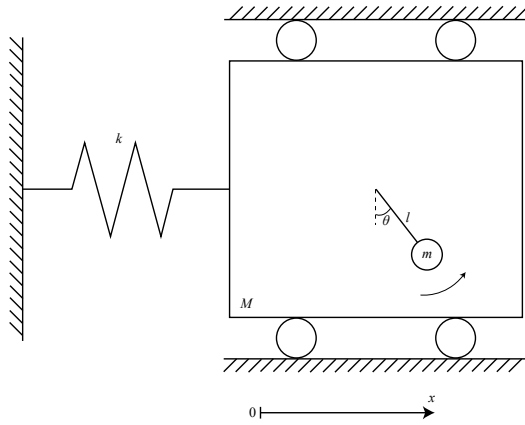


Fig. 1. Translational oscillator-rotational actuator scheme

Note that for $u \neq 0$ this system has no equilibria. Consider the following parameters of the system (Fradkov et al., 2011): $J = 0.014$, $M = 10.5$, $m_0 = 1.5$, $l = 0.04$, $k_\theta = 0.005$, $k = 5300$, $k_1 = 5$. For $u = 0.48$ the system experiences co-existence of attractors (i.e. multistability). The first attractor corresponds to Sommerfeld effect and it may be observed for initial data $\dot{x} = x = \theta = \dot{\theta} = 0$ (zero initial data represent typical start of the system, this effect can be easily found). For other initial data $\dot{x} = x = \theta = 0, \dot{\theta} = 40$ we observe another attractor which is normal operation – the achievement of desired rotational velocity of our mechanical system.¹ In Fig. 2 the transient process for both initial data is shown, in Fig. 3 we observe the attractors, which are obtained after the transient process. All numerical results in this article are obtained with the help of Matlab. Note that if we compare this result with the experiment of Sommerfeld, we see that an effective level of about 1/4 (comparing to normal operation) is achieved here when Sommerfeld effect occurs.

3. DRILLING SYSTEMS

Consider now another electromechanical system – drilling system. Drilling systems are widely used in oil and gas industry for drilling wells. The failures of drilling systems cause considerable time and expenditure loss for drilling companies, so the understanding of these failures is a very important task. Here we consider two mathematical models of drilling systems and study their behaviour after operation start. For drilling systems two different ways of operation start are possible: no-load start and start with load. No-load start means that at initial moment of time there is no friction torque acting on the lower disc. The start with load is start of the drilling with friction torque acting on the lower disc at initial moment of time (this case also corresponds to a sudden change of rock type).

¹ Both effects were modelled in (Fradkov et al., 2011), but in our work we give the information on parameters more accurate

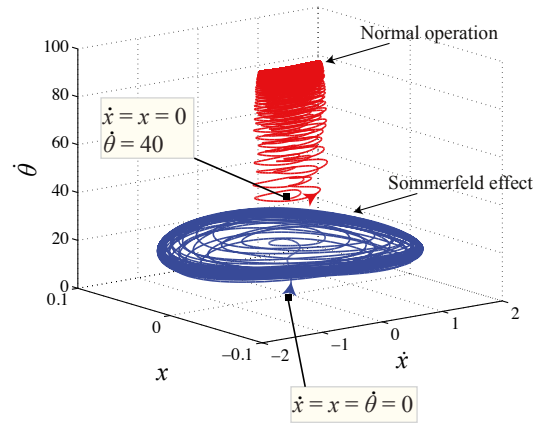


Fig. 2. Sommerfeld effect and normal operation in TORA

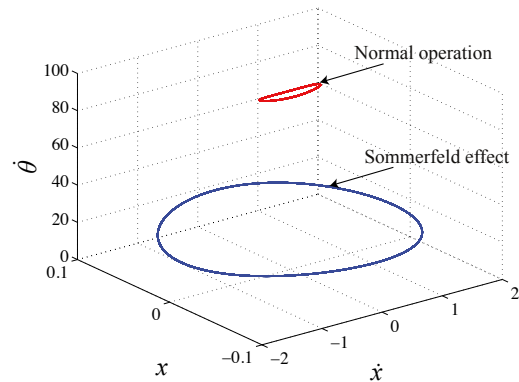


Fig. 3. Sommerfeld effect and normal operation in TORA – localization after transient process

3.1 Drilling system actuated by DC motor

In the (de Bruin et al., 2009; Mihajlovic et al., 2004) works the scientific group from Eindhoven University of Technology constructed and studied an experimental setup which consists of two discs connected with a steel string. The upper disc represents the rotary table of the drilling system and is actuated by a DC-motor (see schematic view of the system in Fig. 6). The lower disc represents bottom hole assembly.

For the construction of mathematical model of the system it is assumed that the drill string is massless and experiences only torsional deformation. The system is described by the following equations:

$$\begin{aligned} J_u \ddot{\theta}_u + k_\theta (\theta_u - \theta_l) + b (\dot{\theta}_u - \dot{\theta}_l) + T_{fu} (\dot{\theta}_u) \\ - k_m v &= 0, \\ J_l \ddot{\theta}_l - k_\theta (\theta_u - \theta_l) - b (\dot{\theta}_u - \dot{\theta}_l) + T_{fl} (\dot{\theta}_l) &= 0, \end{aligned} \quad (2)$$

where $\theta_u(t)$ and $\theta_l(t)$ are angular displacements of the upper and lower discs with respect to the earth, J_u and J_l are constant inertia torques, b is rotational friction, k_θ is the torsional spring stiffness, k_m is motor constant, v is constant input voltage. T_{fu} and T_{fl} are friction torques acting on the upper and on the lower disc, respectively.

Both friction torques T_{fu} and T_{fl} are obtained experimentally:

$$T_{fu}(\dot{\theta}_u) \in \begin{cases} T_{cu}(\dot{\theta}_u)\text{sign}(\dot{\theta}_u), & \dot{\theta}_u \neq 0 \\ [-T_{su} + \Delta T_{su}, T_{su} + \Delta T_{su}], & \dot{\theta}_u = 0, \end{cases} \quad (3)$$

where

$$T_{cu}(\dot{\theta}_u) = T_{su} + \Delta T_{su}\text{sign}(\dot{\theta}_u) + b_u|\dot{\theta}_u| + \Delta b_u\dot{\theta}_u \quad (4)$$

and

$$T_{fl}(\dot{\theta}_l) \in \begin{cases} T_{cl}(\dot{\theta}_l)\text{sign}(\dot{\theta}_l), & \dot{\theta}_l \neq 0 \\ [-T_0, T_0], & \dot{\theta}_l = 0, \end{cases} \quad (5)$$

where

$$T_{cl}(\dot{\theta}_l) = \frac{T_0}{T_{sl}}(T_{pl} + (T_{sl} - T_{pl})e^{-|\frac{\dot{\theta}_l}{\omega_{sl}}|^{\delta_{sl}}}) + b_l|\dot{\theta}_l|. \quad (6)$$

Here T_{su} , ΔT_{su} , b_u , Δb_u , T_0 , T_{sl} , T_{pl} , ω_{sl} , δ_{sl} , b_l are constant parameters. Note that T_{fu} and T_{fl} are multi-valued functions, thus we need to apply the theory of differential inclusions² and corresponding methods for numerical modelling of (2) (see (Piiroinen and Kuznetsov, 2008; Kiseleva, 2013)).

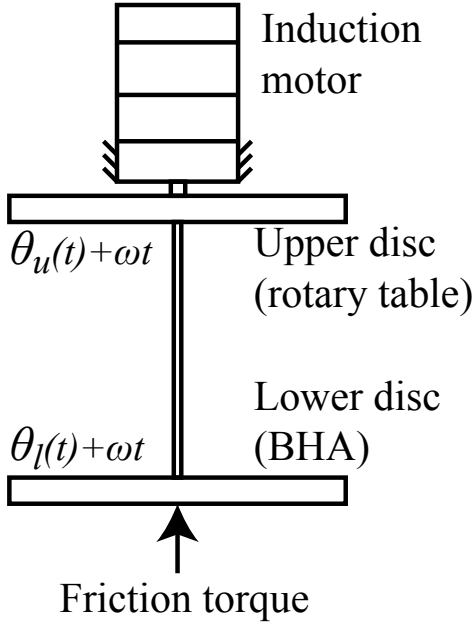


Fig. 4. Mathematical model of drilling system actuated by induction motor

Normal operation of the drilling system corresponds to rotation of both upper and lower discs with the same angular velocity with constant angular speed (i.e. the system reaches stable equilibrium state). Instead of normal

² A. Filippov introduced definition of solution for systems of differential equations with discontinuous right-hand side (Filippov, 1960). In (Gel'fand et al., 1978) generalisation of Filippov definition for theory differential inclusions and adaptation of stability theory for this new definition were done. Later this generalisation was included in (Filippov, 1985). Note that in many important cases (including system (2)) Filippov definition coincides with generalised Gel'fand-Leonov-Yakubovich definition, furthermore these definitions coincide with Aizerman-Pyatnitsky definition (see corresponding discussion in (Leonov et al., 2015a; Kiseleva and Kuznetsov, 2015))

operation system may experience unwanted oscillations, which lead to its failures.

For modelling (2) we use the following parameters (de Bruin et al., 2009): $k_m = 4.3228$, $J_u = 0.4765$, $T_{su} = 0.37975$, $\Delta T_{su} = -0.00575$, $b_u = 2.4245$, $\Delta b_u = -0.0084$, $k_\theta = 0.075$, $b = 0$, $J_l = 0.035$, $T_{sl} = 0.26$, $T_{pl} = 0.05$, $\omega_{sl} = 2.2$, $\delta_{sl} = 1.5$, $b_l = 0.09$. For initial data $\theta_u - \theta_l = 0$, $\dot{\theta}_u = \dot{\theta}_l = 6.1$ (no-load start: both upper and lower discs rotate with the same angular speed without angular displacement; such initial data correspond to stable equilibrium state with $T_{fl} \equiv 0$) after transient process the system enters normal operation mode (see Fig. 5). But for the same parameters and for initial data $\theta_u - \theta_l = \dot{\theta}_u = \dot{\theta}_l = 0$ (start with load: discs don't rotate and there is no angular displacement between them) after transient process the system starts to experience stable hidden oscillations.

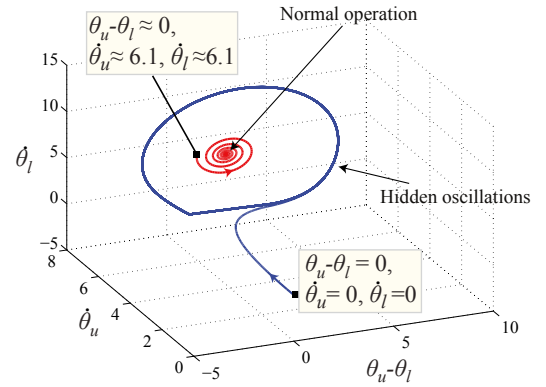


Fig. 5. Hidden oscillation and normal operation (corresponds to stable equilibrium state) in drilling system with DC motor

3.2 Drilling system actuated by induction motor

Consider now the modification of the drilling system studied above. Suppose it is driven by an induction motor (see schematic view of the system in Fig. 6; patents for such systems: (Staege, 1936; Hall and Shumway, 2009; Hild, 1934)). In order to take into account the dynamics of the motor we modify equations (2) by excluding terms $T_{fu}(\dot{\theta}_u) - k_m v$ and by introducing the equations of induction motor (see e.g. (Kiseleva et al., 2014; Leonov et al., 2014)):

$$\begin{aligned} J_u \ddot{\theta}_u + k_\theta (\theta_u - \theta_l) + b (\dot{\theta}_u - \dot{\theta}_l) \\ - nBS \sum_{k=1}^3 i_k \sin \left(\theta_u + \frac{2(k-1)\pi}{3} \right) &= 0, \\ J_l \ddot{\theta}_l - k_\theta (\theta_u - \theta_l) - b (\dot{\theta}_u - \dot{\theta}_l) + T_{fl} (\omega + \dot{\theta}_l) &= 0, \\ L \dot{i}_1 + (R + r) i_1 &= -nBS \dot{\theta}_u \sin \theta_u, \\ L \dot{i}_2 + (R + r) i_2 &= -nBS \dot{\theta}_u \sin \left(\theta_u + \frac{2\pi}{3} \right), \\ L \dot{i}_3 + (R + r) i_3 &= -nBS \dot{\theta}_u \sin \left(\theta_u + \frac{4\pi}{3} \right), \end{aligned} \quad (7)$$

where $\theta_u(t)$ and $\theta_l(t)$ are angular displacements of the upper and lower discs about the magnetic field rotating

with constant speed $\omega = 2\pi f/p$, where f is the motor supply frequency, p is the number of pairs of poles (usually not less than 8 pairs) (Leonhard, 2001); n is the number of turns in each coil; B is an induction of magnetic field; S is an area of one turn of coil; i_k are currents in coils; R is resistance of each coil; r – variable external resistance; L – inductance of each coil; J – the moment of inertia of the rotor. Note that in contrast to the previous model with DC motor here angular displacements of the upper and lower discs with respect to the earth are $\theta_u(t) + \omega t$ and $\theta_l(t) + \omega t$. Friction torque T_{fl} acting on the lower disc is defined by (5), where $\dot{\theta}_l \rightarrow \theta_l + \omega$.

Let us model system (7) with the following parameters: $T_0 = 0.25$, $c = 10$, $\omega = 8$, $J_u = 0.4765$, $J_l = 0.035$, $k = 0.075$, $a = 2.1$, $b = 0$, $T_{sl} = 0.26$, $T_{pl} = 0.05$, $\omega_{sl} = 2.2$, $\delta_{sl} = 1.5$, $b_l = 0.009$. For initial data $\theta = \theta_u = \theta_l = 0$, $\omega_u = -\dot{\theta}_u = 0$ and $\omega_l = -\dot{\theta}_l = 0$ (no-load start: rotation of both discs with the same speed with respect to the earth without angular displacement) after the transient process the drilling system enters normal operation mode (see Fig. 6). But for initial data $\theta = 0$, $\omega_u = 8 = \omega_l = 8$ (start with load: initially discs don't rotate with respect to the earth and there is no angular displacement) after the transient process the system starts to experience hidden oscillations, which may lead to break-down.

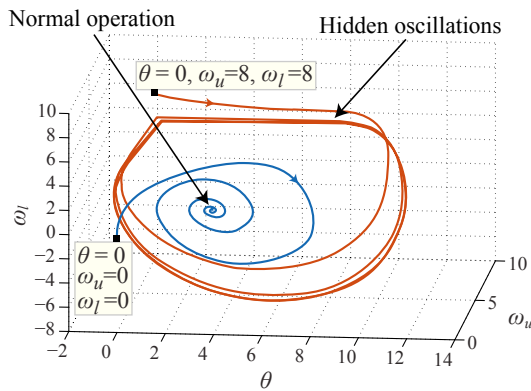


Fig. 6. Hidden oscillations and normal operation (corresponds to stable equilibrium state) in drilling system actuated by induction motor

4. CONCLUSIONS

We modelled three different electromechanical systems. All of them have hidden oscillations in sense of mathematical definition. While some of these hidden oscillations can be easily seen in natural physical experiments, the localization of others requires special efforts. For example, for TORA system zero initial data correspond to typical start of the system, so Sommerfeld effect can be easily localized. In our examples for drilling systems no-load start leads to normal operation and start with load (or the change of rock type) leads to unwanted hidden oscillations. Hence better understanding of physical nature of the mathematical models may make it easier to find hidden attractors.

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