Real Options Analysis as a tool for Start-Up Company Investment Valuation

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ABSTRACT

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Abstract

The aim of this Master's Thesis was to research whether the Real Options Analysis method works as a tool for start-up company investment valuation. In order to answer the main research question the business plan of the case company was outlined.

This is a quantitative single-case research and the research method is both constructive and descriptive. A tool for the investment valuation was constructed and the valuation process, the business plan itself and the Real Options Analysis was described. For background, start-up companies, investment valuation and the Real Options Analysis were researched. In the investment valuation section of the research the basic Net Present Value –method is illustrated. Other investment valuation methods are described shortly, too. After theory the tool for the investment valuation was constructed. Finally, the option to expand and the investment as a whole was given a valuation. The valuation was turned over to the case company.

The first-phase investment was a webstore that tested the traction of the company's brand and the demand for its merchandize. An expansion option – a real option – was attached to the investment: if the revenues are satisfactory, a further investment will be made turning the webstore into a market place. The case company accepted the investment valuation and thus validated the construction of the case study. The valuation and the in-depth analysis made for the investment highlighted that the investment wasn't sufficiently profitable even though the potential good outcome was rather optimistic and the company decided not to initiate the investment plan.

Keywords
Real Options Analysis, Start-up company, investment valuation, net present value

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Tiivistelmä

Pro Gradu -tutkielman tavoitteena oli tutkia Real Option Analysis -metodin sopivuutta start-up yhtiön investoinnin arvonmääritykseen. Jotta tutkimuskysymykseen saatiin vastaus, tuli myös case-yhtiön liiketoimintasuunnitelma selvittää ja edelleen mitkä ovat liiketoiminnan potentiaaliset rahavirrat, joihin arvonmääritys perustuu.

Tutkimusmenetelmä oli kvantitatiivinen case-tutkimus ja tutkimusmetodi oli konstruktiivinen ja deskriptiivinen. Tutkimuksessa konstruoitiin työkalu ja arvonmääritys case-yrityksen suunnitellulle investoinnille. Toisaalta tutkimuksessa kuvailtiin arvonmääritystä yleisesti sekä Real Options Analysis -metodia. Tutkimuksen pohjaksi tutustuttiin start-up yhtiöihin, arvonmääritykseen ja Real Options – metodiin yleisesti. Arvonmäärityksen teoria-osuudessa tutustuttiin lyhyesti muihin arvonmääritysmetodeihin, eritoten Net Present Value -metodiin. Teoria osuuden jälkeen rakennettiin haastattelujen ja yhteisen työnteon avulla työkalu investoinnin arvonmääritykseen. Lopulta kasvuoptiolle ja edelleen koko investoinnille tehtiin arvonmääritys, joka luovutettiin case-yhtiön käyttöön.

Ensimmäisen vaiheen investointi oli verkkokauppa, jolla testattiin case-yhtiön brändin toimivuutta ja tuotteiden menekkiä. Ensimmäisen vaiheen investointiin liitettiin kasvu-optio: mikäli kauppa käy, tullaan palvelua edelleen kehittämään markkinapaikaksi, joka sisältää myös aluksi tehdyn verkkokaupan. Yritys hyväksyi arvonmäärityksen ja täten validoi tutkimuksen konstruktion. Arvonmääritys ja sen perustaksi tehty syvällinen analyysi kuitenkin osoittivat investoinnin olevan kohtuullisen kannattamaton vaikka kasvulukemat hyvässä tapauksessa asetettiin korkeaksi. Tästä syystä yritys päätti olla toteuttamatta investointia.

Asiasanat
Real Options Analysis, Start-up company, investment valuation, net present value

Säilytyspaikka Jyväskylän yliopiston kirjasto

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1 BACKGROUND/INTRODUCTION

Since starting my studies in business at the Jyväskylä Business School (Jyväskylän Kauppaoppilaitos) I've been interested in creating something new and fresh to the Finnish and global markets. Starting from marketing I got excited about launching a new product or service to the market. In Jyväskylä School of Business and Economics of University of Jyväskylä I switched my focus to accounting and made my Bachelor's Thesis on Balanced Scorecard (BSC). At the same time I ran my own business providing both supplies as well as services to the Finnish art market. The company succeeded rather well in my leadership and we eventually opened a store in Tampere and shifted to a larger selection of products and services before my departure in the spring of 2012.

The entrepreneurial experience influenced my preferences and after a year of auditing at a Big Four company I felt sure that I should do my thesis in the sphere of management accounting with some features from financial accounting as well. I wanted to combine both entrepreneurial activities and accounting and I found the connecting link with research potential in Real Options Analysis. My first encounter with ROA was in Russia when I took a risk management course that orbited around ROA. I understood that this relatively new valuation method might justify and monetize parts of investments that fall outside the scope of Discounted Cash Flow method, for example. Also uncertainty of Start-Ups and their future cash flows has a relation to ROA and its emphasis on uncertainty. Combining both ROA and Start-Up industry I found a research field that both interests me and has room for more research.

Start-Up companies have been in the Finnish and international news more and more often. The most visible example of hype around the Finnish start-up community is the Slush Conference held annually in Helsinki. According to the Slush company website the conference has grown substantially and in the end of November 2014 approximately 14 000 people attended the event including entrepreneurs, investors and media representatives. Many global news outlets were present at the 2014 event. The participation amounts to 500 reporters out of which about half are representatives of foreign press (Kauppalehti, 17.11.2014.) The Forbes Magazine used the companies exhibiting at the event as catalysts for new trends in the global start-up scene (Forbes Magazine, 19.11.2014.)

The economic impact of start-up companies on both the gross domestic product and the employment has been scrutinized extensively especially due the recent economic crisis of 2008 and its aftermath including the economic sanctions constituted against the Russian Federation.

The economic impact of small and middle-sized enterprises (SMEs) in Finland is great and growing. According to annual statistics on companies in Finland compiled by Statistics Finland SMEs employ 64 % of all private sector employees

in 2012. All revenues combined they contribute 53 % of private sector revenues nationally (Statistics Finland, 2013.) Furthermore the Federation of Finnish Enterprises calculates that out of total added jobs in the private sector from 2001 to 2012 about 93 % originated in SMEs. Start-up companies by definition are categorized as SMEs. The definition of a start-up company is in the chapter 2.

Start-up companies, their features and success factors have been studied thoroughly during the last decade (e.g., Stücki, 2014 and Littunen, 2000). Apart from short definition and case company description the research will be outlined so that start-up companies and their features will remain out of the scope of the research.

A more interesting topic is the start-up company investment valuation. If the company has only one idea that constitutes an investment plan you could argue that the investment valuation is de facto valuation for the whole company. The company has one business plan and it is this business plan that is being valued. Brealey, Myers and Allen (2011) emphasize this fact when explaining the Net Present Value method for valuing investments (p. 130-131).

Many quantitative researches both in international (e.g., Festel et al., 2013) and national (Miettinen & Niskanen, 2015) setting have been made based on this assumption and the assumption is logical: in the beginning the company has no measurable assets. The accounting legislation reinforces this point of view. According to the Finnish Company Act chapter 2 section 6 the subscription price for shares can be paid with other assets besides cash but undertaking to perform work or services can't be used for this purpose. Thus, there's no value before the actual work is done and the possible investment gives the value for this work and the business plan.

Cumming and Dai (2011) write that start-up company valuation is the central matter and negotiation point for both investor and founders of the company. For both investors and company founders the valuation is important in terms of return on investment for the investors and value of work and future structure of ownership for founders.

The financing that a valuation provides is crucial to transform a company with an idea to a functional and operating entity with actual cash flows (Gunter et al., 2013.) The financing of start-up companies is shortly described in chapter 2.2 and will otherwise be outlined out of the scope of the research.

In times when innovation and job creation is stressed there needs to be more research to provide start-up companies valuations for their needed investment and ultimately for company valuation (Festel et al, 2013), There has been extensive research both on start-up company definitions and their particular features and also in certain types of valuation methods for start-ups, in particular

the Discounted Cash Flow (DCF) method. As Festel et al.(2013) write there are multiple points of criticism against the DCF method. All these points derive from the high uncertainty surrounding the business plan.

My thesis is that in order to give a start-up investment a proper valuation this uncertainty has to be addressed more profoundly. To attempt to achieve this goal Real Options Analysis will be used to value case company's investment. The Real Options Analysis has been used to valuate investment and opportunities for quite a while. Prominent users of the approach are e.g. Kone and Boeing. Kone Corporation is a leading elevator manufacturer and service provider in the world with huge R&D investments yearly (Annual Report 2014). Kone Corporation's usage of ROA is studied by Collan and Kinnunen (2009) and the company states also in their Annual Report (2014) the following:

"KONE's Risk Management and Strategy Development functions jointly coordinate and develop a systematic assessment of risks and opportunities within core business planning and decision-making processes."

Boeing Corporation's American division's necessity and implementation of Real Option Analysis is thoroughly researched by Copeland and Antikarov (2001). The need comes from uncertainty, vast amounts of money involved in multi-period investments and the opaque pricing of airplanes. By using Real Options Analysis the company tries to evaluate the price for customer airlines.

Recently there have been one master's thesis (Oinonen, 2010) done in the field of Real Options Analysis in Aalto School of Business. This master's thesis researches the valuation of emerging market investments using ROA. As in my thesis there's a strong emphasis on uncertainty thus the chosen valuation method.

1.1 Topic and research question

My thesis will be made on Real Options Analysis and its application to a Finnish start-up company's investment valuation. Thesis' topic will be Real Options Analysis as a tool for Start-Up Company Investment Valuation. The thesis will include the theory needed to clarify what is the Real Options Analysis and how does it differ or incorporate other valuation methods.

The main research question is: *How does a Real Options Analysis work as a tool for investment valuation for the case company?*

In order to be able to answer the main research question the thesis will answer the following sub-question:

- What is the business plan of the case company and its cash flows?
- How valuable is the option to expand?

1.2 Real Option Analysis

There are multiple valuation methods and most have been applied to start-up investment valuations. The discounted cash flow (DCF) methods such as Net Present Value method are generally the most popular valuation methods in finance (Brealey et al. 2011.) The use of DCF in start-up company valuation is common. The method has its limitations especially if the case company is just founded and there are no past cash flows to analyze. The difficulty to forecast reliable cash flows, future growth rate and discount rate are the main points of criticism in the DCF method (Festel et al., 2013) In other words uncertainty is hard to include into the model. Festel et al. have responded to this criticism with focusing on the capital asset pricing model in setting the discount rate to input the uncertainty.

The real options analysis approach takes another view on uncertainty. It is a binomial method to value options. The basic idea is a simple question: what will happen next? Instead of valuing financial assets the real option analysis applies the option approach to investment valuation calculating in the uncertainty in good and bad states of nature in the following periods. The approach can be used to both investments and company valuations.

1.3 Research method

This is a quantitative single-case research. Vilkka (2007) writes that in quantitative research information is observed numerically. The aim of quantitative research is to describe, illustrate, map, compare or forecast a natural or human phenomenon. The main feature of a quantitative research is objectivity. The research material is commonly gathered using inquiries. These inquiries might include both numerical and verbal questions.

In this research the research material will be gathered by using both an inquiry and interviews. The inquiry will cover the numerical data needed for the Real Options Analysis and the interviews will provide the background information and necessary explanations for the numbers as well as general information on the case company.

The research method is both descriptive and constructive. The descriptive method derives from the fact that the case company, its business plan and uncertainties have to be described in order to construct a solution for the company's investment valuation. Kasanen et al. (1993) state that the constructive method aims to construct a solution for a problem that is known beforehand but

the means to reach the solution for this problem is unknown. The construction will be validated if the valuation will be used to get financing for the investment. In the conclusions the validity will be discussed more thoroughly.

The research material for the theory is compiled from research literature concerning ROA. This material is also used to describe the case company so that it is applicable to ROA usage. Most of the data collection is done by extensive interviews with the case company's management. In addition to the interviews a financial information questionnaire was submitted to the management. This questionnaire was filled semi-autonomously so that all the relevant information was gathered.

1.4 Research Plan

The research can be divided into three general parts. The first part introduces the reader to the research. The second part focuses on the theory on Net Present Value method and finally on the Real Options Analysis. These two methods are also compared to highlight their differences and similarities. The third part is the application of ROA to the case company's investment valuation.

In introductory part the research question, method and plan is introduced along with short description of ROA. Also, a presentation of the company will be given to supplement the context of the research.

The first part of theory will begin with basic theory on start-ups: what are their characteristics and how are they financed. The second part is compiled from investment making theory and most commonly used valuation method: Net Present Value. Strong emphasis on Net Present Value (NPV) method is made as a basis for ROA. Also the IRR approach is shortly explained as another application of NPV. Third part will present the case company, its business plan and the ROA itself and its application.

Since ROA is a complicated and mathematical model for investment valuation an example calculation is introduced and supplemented throughout NPV and ROA section.

In the applied section the ROA based investment valuation will be constructed. In order to achieve this the two complementary research questions have to be answered. First the business plan is laid out according to the interviews and material gathered from the company management. Secondly the uncertainties of the investment are discussed. Thirdly the cash flows are calculated using financial accounting methodology. The ROA application is constructed from the business plan, cash flows and uncertainties surrounding it.

The final result of the research is the ROA tool tailored for the investment. The calculations are exhibited when necessary to give the reader a clear picture how the tool was constructed and how it functions mathematically. The tool is a

Microsoft Excel - document including the necessary information and mathematical formulas suitable for person with basic knowledge of profit and loss calculations.

2 START-UPS

2.1 Start-up company definition

There are several different interpretations of what a start-up company is. Defining a startup company, the interpretations can roughly be divided into descriptive and quantitative interpretations.

According to Timmons and Spinelli (2008) start-up companies are raw companies that have an innovative idea that develops into a high-growth company. The success relies on strong leadership from the main entrepreneur and on building a team with complementary talents. Giardino et al. (2014) write that startups are newly created companies with little or no history facing high volatility in technologies and markets. The environment of startups is dynamic and unpredictable forcing the management to act quick, try to avoid failures and find a niche in the market that enables a sustainable income. The line of business is commonly emphasized. They often operate in one or more high-technology sectors (Bürgel et al., 1999). The failure rate of startups is overwhelming: according to Giardino et al. sixty percent of startups fail in the first five years of their existence.

The more quantitative approach to start-up definition is provided by European Commission (n.d.). Start-ups can be either micro, small or medium-sized companies. The commission seems to posit that start-up phase is commonly experienced in micro companies. In this study a start-up company is defined as a micro company which fits the case company as well.

Company Category	Employees	Turnover	or	Total Balance Sheet
Medium-sized	<250	≤ € 50 m		≤ € 43 m
Small	< 50	≤ € 10 m		≤ € 10 m
Micro	< 10	≤ € 2 m		≤ € 2 m

2.2 Start-up financing

Just like any other business, a start-up company needs financing to realize its investment needs. The financing of start-up companies differ greatly from older and more established companies. The financing comes from smaller amount of sources especially from business angels or venture capitalists(risk investors) and from friends and family (Brealey et al., 2011) The small amount of outside investments derives from the lack of historical data and the business idea that is

commonly not tested previously. And, as emphasized earlier, there's no assets on the balance sheet of the start-up company (Berger & Udell, 1998).

The phases of financing can be defined by age, size and financial history. Berger and Udell (1998) present four phases for start-up financing that go hand in hand with the age of the company: ”infants”(0-2 years), ”adolescents”(3-4 years), ”middle-aged”(5-24 years) and ”old”(25 years or more). Brealey et al. (2011) group the phases into four groups: seed financing (family and friends), early investment rounds (business angels), later investment rounds (venture capital firms) and finally the public listing of stocks i.e. initial public offering.

3 INVESTMENT AND ITS VALUATIONS

3.1 Investment decision

A company – no matter in which phase of its business – needs real assets to make products or services that it sells to its customers. An easy example of a real asset today is a computer with which programmer can design a website that has been ordered by its client for a product launch. The decision to purchase a real asset is called investment decision. All the investment decisions taken by a company or the planning process of these decisions during a period are called *capital budgeting* or *capital expenditure (CAPEX) decisions.*

In order to get the real asset in question the company has to finance its investment through financial assets or securities. These include bank loans, corporate bonds or stocks to stockholders. The company gets financing and makes the investment. Not only does the company need tangible assets – something you can touch and see – the company might need to do research and development(R&D) investments. A good example of this kind of investment decision is a biochemical company's decision to research possibilities of peat for heating purposes. These investment decisions fall under capital budgeting, too.

The difference between investment and financing decisions must be made also. The financing decision is the decision taken when the need for financing has been set in the form of investment decision. The company can decide to borrow money from banks (debt financing) or raise money from present or future stockholders (equity financing). These decisions are closely related to company's strategy regarding its capital structure (Brealey et al., 2011)

Pacta sunt servanda, the company has to repay the financing to banks, bondholders or as dividend or stock repurchase to stockholders at some point in time. The financing for the investment decision will be spent up front but the future revenues inflicted by the investment might start coming to company treasury in one year or even later. In other words there's a gap between the payment for the investment and its subsequent revenues that make up for the investment. The company's managers have to plan the financing so that the company remains viable in the period between, too. Otherwise the company may become insolvent and even worse bankrupt and the revenues from the investment will not materialize. Insolvency might come from bad decision making too: are the future revenues too small, what about the margins and maintenance costs? The question of investment decision isn't this straight forward, though. The manager also has to think whether the asset is needed and how will it fit to the existing set of assets.

Moreover it is a question of strategy, does this investment decision lead towards our strategy and is this the right time and place to make the decision; how expensive is financing and will the subsequent cash flows make up for the

investment financing as well as for profit and growth? To make a good investment decision one has to base it to solid estimates of future revenues and costs.

According to the Limited Liability Companies Act of Finland chapter 1 section 5 "the purpose of a company is to generate profits for the shareholders, unless otherwise provided in the Articles of Association." Thus the managers attempt to steer the company so that the return on equity (ROE) is as high as possible in both the short and long term. To reach this goal the managers need to invest in real assets that are worth more than what they cost (Brealey et al., 2011.) To find these assets the company's management has to look for assets and evaluate them.

The question of valuation is essential. Some assets, like real estate or even gold bullion, have an easily acquirable price that can be taken straight from well-functioning markets. The valuation of a research and development project or purchase of a factory equipment is far less convenient. Various questions arise: how many products can we produce in a year, what cost of capital should we use, what will the electricity prices be in five years' time and how will the fixed costs such as salaries change in a ten year period? An investment valuation has to be conducted to answer these questions and evaluate what an investment is worth and what will be the return on investment.

3.1.1 Different methods to value an investment

There are several investment valuation methods. The most discussed and widely used one is the Net Present Value (NPV) method and the Internal Rate of Return (IRR). Internal rate of return or discounted cash flow rate of return is the discount rate that gives a zero NPV (Brealey et al., 2011). Both are used by approximately 75 % of firms and approach the valuation the same way and if used properly should give the same answer.

The idea of the internal rate of return is to accept an investment that has a higher internal rate of return than the applicable opportunity cost of capital. Mathematically the internal rate of return is the discounting rate with which you get a zero NPV. In other words, it is another application of the NPV.

$$Net\ Presentvalue = \frac{Cashflow}{1 + IRR} - Investment = 0$$

The payback period method is another method used to value an investment. Its main task is to calculate how quick the investment can be paid back thus making it a tool used mostly to consider the effects and pay back times of financing. The method doesn't take into consideration the opportunity cost of capital and the time value of money; the payback period method doesn't include discounting. These defects make it unattractive to most investment valuations, especially those that have long time spans (Kinnunen et al., 2007). The payback period method is usually supplemented by a cutoff date which constitutes the payback rule: "a project should be accepted if its payback period is less that some specified

cutoff period" (Brealey et al.., 2011). Also, discounted cash flows could be added along with the cutoff period. Still, the cutoff period ignores all cash flows after the cut off period.

The basis of valuations is the cash flows generated by the investment project. Of course you could make the calculations based on accounting income but this would include both the capital expenditures and depreciation. Although not directly usable, book values are important addition to the investment valuation and the valuation itself is commonly derived from forecasted financial accounting of the investment (Brealey, 2011)

3.1.2 Net Present Value(NPV)

Investment decisions have to be based on facts - and if not possible - solid estimates of future cash flows and costs. The most common and traditional way to calculate the value of an investment is to use the Net Present Value method. The method is widely used and already in the end of 1970's the method was used by over 85 % of companies (Copeland & Antikarov, 2001.) The basic idea of net present value is exhibited in below

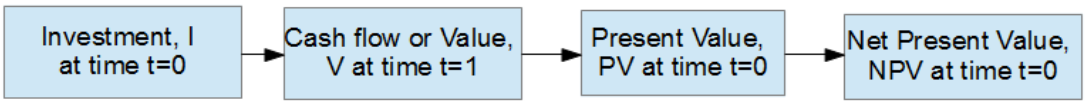


FIGURE 1

The process starts with the investment outlay, I. What amount of money must be paid in the beginning (t=0) to make the investment. This could be empty business premises and the affiliated costs for renovating the premises into a modern cafeteria that you're planning to sell at a future time, t=1. After the real estate transaction and renovations the enhanced property has another value, preferably one that is higher than the initial money paid for the real estate and its renovation. A professional investor would acquire an independent appraisal of the planned cafeteria premises to evaluate the estimated future value, V of the renovated property. Using basic mathematics the profit would be the difference between the resale value and the investment. Unfortunately the investment calculation has to include the cost of capital to take into account the lost opportunity of capital used (Kinnunen et al., 2007)

Brealey et al. (2011) define the three principles of NPV as follows:

1. An euro today is worth more than an euro tomorrow; the time value of money
2. Value depends on the forecasted cash flows not on accounting methods or managers preferences; opportunity cost of capital
3. The present values are all measured in today's euros so they can be

summed up; the present value.

One of the most quoted basic principles of finance is that an euro today is worth more than an euro tomorrow. This principle is derived from the lost opportunity to make a profit elsewhere that you turn down when investing in this particular project; an investor could start earning interest on the euro today. The rate with which the present value is *discounted* to present day can be called discount rate, hurdle rate, opportunity cost of capital, or if both debt and equity are taken into consideration weighted-average cost of capital or WACC.(Brealey et al., 2011. p 51 and 244)). All the cash flows are taken into account and discounted from the year of occurrence to the starting point. After you have the cash flows discounted the investment is subtracted to get the net present value (Kinnunen et al., 2007).

Furthermore the cash flows are less vulnerable for managerial misconduct or other bias. There are several ways to adjust earnings and this can be a tool for tax planning along with giving a better picture of company's profitability. Which capital outflows influence only one year and which are considered capital investment and thus influence the profit of several accounting periods?

Thirdly the investment and their NPVs can be summed up or their values separated. This is an important feature because you can compare projects in different combinations. For example, a combination of two projects might have a combined positive NPV but separately only one of these projects have a positive NPV (Brealey et al., p. 131-143).

Mathematically net present value equates to value at time t=1 over 1 plus the discount rate. The numerator could also be written as FCF or free cash flows and the denominator could be 1 plus weighted-average cost of capital (WACC).

$$Present\ value_{t=0} = \frac{value_{t=1}}{1 + discount\ rate} = \frac{FCF_{t=1}}{1 + WACC}$$

When the present value of the cash flows has been discounted to the present day we can subtract the initial negative cash flow(the investment outlay, *I*) and we get the Net Present Value(NPV) of the investment.

$$Net\ \ Present\ Value = \frac{FCF_{t=1}}{1 + WACC} - I$$

In the real estate example described above the resale value one year after the purchase and renovations could be higher(t=1) than the investment outlay, I, at time t=0 but when the opportunity cost of capital is taken into consideration the net present value of the project might become zero or negative depending on the discounting rate used.

If there are perpetual cash flows from the investment a sum of infinite geometric series is used instead (Copeland & Antikarov, 2001). This means that the initial investment outlay produces cash flows infinitely to the company. But because of the discounting the sum of infinite geometric series winds up to a definite number according to its mathematical nature. Below is first the formula for a sum of infinite geometric series and then its application to NPV (Weissstein, 2015).

$$\text{Sum} = \frac{a - ar^{n+1}}{1 - r} = \frac{a}{1 - r}, \textit{as long as} -1 < r < 1$$

In general form the geometric series is the following

$$\text{Sum} = a + ar + ar^2 + ar^3 + \ldots + r^n$$

Where a is the first term of a geometric series and r is the multiplier. Infinite geometric serie formula is applied in NPV:

$$NPV = -I + \sum_{t=1}^{N} \frac{(FCF_t)}{(1 + WACC)^t} \textit{where} a = FCF_t, r = \frac{1}{(1 + WACC)^t}$$

An easy example of this formula is provided. A company makes an investment with an outlay of 800 euros which provides perpetual free cash flows of 100 euros per year. Weighted average cost of capital is 10 per cent per annum. Depreciation of the investment yearly is compensated by replacement investment of equal size. Although the cash flows are infinite, they will eventually arrive at a precise sum, in this example 1 100 euros because of discounting. Thus, the net present value is:

$$NPV = 800 - \frac{100}{1 - \frac{1}{1{,}1}} = -800 + 1100 = 300$$

3.1.3 Discount rate

Discount rate itself is a mathematical term that signifies a percentage with which the discounting is done. In the world of finance discount rate can be thought of as a cost of capital or - as previously mentioned – opportunity cost of capital. The company cost of capital is usually estimated as a Weighted-Average Cost of Capital (WACC). WACC is the average return investors demand for investment in the company's debt or equity. The cost of equity and debt can be hard to calculate especially if the company isn't public – or even worse – not operational. Below is the formula for WACC:

$$WACC = \frac{D}{D+E} r_f + \frac{E}{D+E} r_f$$

To tackle the problem of cost of debt needed to calculate WACC, a comparable debt with the same risk must be found or outlined which is assumed to be yielding the same amount of cash flow to its maturity as the case company's debt would if issued. Essentially, the cost of this comparable debt is used as the cost of debt for the case company's WACC (Copeland & Antikarov, 2001).

Capital Asset Pricing Model (CAPM) can be used to calculate the cost of equity. The CAPM makes the assumption that when making investments the investors demand higher returns when the risks involved are higher. The formula of CAPM is

$$C = R_f + ß(R_M - R_f)$$

where
C = cost of equity
R*f* = Risk-free return
R_M = Expected market return on all risky assets
ß = Beta of the target company calculated against the market index

The key factor in CAPM is the company beta which mathematically means covariance (ß) between the investment and the market portfolio. According to Festel et al.(2013) the beta for start-up companies cannot be derived from past values and accounting or by comparison of a peer group. This is rational especially for technology companies considering that start-up companies usually do not have similar companies to compare to – the product, business model or both are new to the market. Festel el al. (2013) use 39,5 percent as the rate of return required by the investors, or as cost of equity for average start-up investment.

Risk-free rate is traditionally the yield of a long-term government bond. In the United States the 10-year bond is the applied bond and in Europe it is the German Republic's 10-year Bund(Brealey et al., 2011) According to Ernst & Young's whitepaper(E&Y, 2015) in the year 2015 the risk-free rate is as low as 0,2 percent.

3.2 Option

Simplified, an option is a right to do something. This right has a price which is the value of the option. Generally options are divided into two: call options and put options. A call option gives its owner a right not an obligation to buy an asset at a certain, predetermined price that is called either the strike price or the exercise price. A put option is the opposite of a call option. It gives its owner the right to sell an asset at a certain price.

If these options can be exercised only at maturity, i.e. end of the contract period, they're traditionally known as European options. The other type of option is the American one that can be exercised any time before maturity (Brealey et al., 2011).

The economics of an option is straightforward. A call option is in the money if the exercise price is lower than the value of the underlying asset. If an American call option on a stock has an exercise price of ten euros when the stock is trading at twenty euros, the call option is in the money and is worth the difference between its value and the exercise price, 20 € - 10 € = 10 €. A put option is in the money when the exercise price is greater that the value of the underlying asset.

Options in ROA context differ significantly in terms of the underlying asset and the flexibility of the asset. Difference between financial and real option is that the owner of a financial option cannot affect the value of the underlying asset. But,

the management that operates a real asset can raise its value and thereby raise the value of all real options that depend on it (Copeland & Antikarov, 2001).

3.2.1 Option valuation with NPV approach

A simple option, such as one period deferral option, can be valued by using the NPV approach to option valuation. The basic idea is to first calculate the net present value of the project without flexibility and then evaluate the net present value with flexibility. The value of the deferral option is the difference between the two values (Copeland & Antikarov, 2001). The net present value of the investment with flexibility will be evaluated using the Net Present Value rule that selects the maximum of expectations, thus the negative values will not be selected to the discounted value. The rule is mathematically:

$$NPV\ rule: MAX(t=0)[0, E_0 V_T - X]$$

Using the previous example in chapter and its figures and assuming that there's a 50-50 chance of free cash flow being either 50 or 150 yearly. If it is possible one should wait and see what the cash flows will be perpetually and then make the decision. The possibility to defer if the cash flows are not satisfactory is the value of the deferral option. The NPV for infinite geometric series is applied. Note that both the investment outlay as well as the value of the cash flow is discounted because the decision is made after one year.

$$NPV(with\ \ flexibilit\ y) = .5MAX\left[\frac{-800}{1{,}1} + \sum_{t=1}^{\infty}\left(\frac{150}{1{,}1^t}\right), 0\right] + .5MAX\left[\frac{-800}{1{,}1} + \sum_{t=0}^{\infty}\left(\frac{50}{1{,}1^t}\right), 0\right]$$

The above equation indicates that we select either the positive net value of both the up-state and the down-state or zero meaning that only positive or zero outcomes are chosen. By deducting the formula it becomes:

$$NPV = .5MAX\left[\frac{-800}{1{,}1} + \frac{1650}{1{,}1}, 0\right] + .5MAX\left[\frac{-800}{1{,}1} + \frac{550}{1{,}1}, 0\right]$$

The outcome indicates that in the down-state the value is negative and that the formula selects zero. The maximums are selected:

$$NPV = .5\,MAX\,[773{,}0] + .5\,MAX\,[-955{,}0] = 0{,}5 * 773 + 0{,}5 * 0 = 386$$

As mentioned above the value of the deferral option is the difference between the value of NPV without flexibility and the value of NPV with flexibility, 386-300=86. Had you had more volatility, for example the same 50-50 chance but with up-state of 125 and down-state of 25 the value of the deferral option would have been greater. This is an interesting fact in option valuation demonstrating the effect of volatility in option valuation.

3.3 Real options

The problem in applying Net Present Value method for evaluating investments is its connection to the real world with several changing variables and underlying uncertainty that these possible changes and scenarios impose on investment and its success. Trigeorgis writes that traditional discounted cash flow approaches, such as the standard net present value rule, cannot properly capture management's flexibility to adapt and revise later decisions in response to unexpected market developments (1996). The NPV method uses expected cash flows and discounts these to the present day using the discount rate. Thus, uncertainty of cash flows is not explicitly modeled in the NPV method. In real world there are various cash flows that might or might not materialize during the life of the investment project. Furthermore there are multiple choices to be made along the way instead of following a certain static operating strategy. The NPV method precommits to an irreversible investment today without flexibility in the future. The method uses only information available today. The Real Options Analysis method takes into account different paths of future, managerial flexibility and the underlying uncertainty. The management may have the possibility to expand, defer, contract, abandon or otherwise change the initial investment project.

Both approaches – NPV and ROA – take into consideration all cash flows of the investments from beginning to end, both discount cash flows back to the present and both use the opportunity cost of capital. Thus, both are discounted cash flow methods of valuation. The fundamental difference between the two methods is that NPV doesn't include flexibility in decision making which is the basis of ROA. NPV could be described as a special case of ROA: it is an approach that assumes no flexibility of management in the investment. In reality there are no certain cash flows and there are several different paths of possible events in an uncertain world.

3.3.1 Real Option Definition

In order to define the real option analysis one must start with the definition of real option. A real option is the right, but not the obligation, to take an action such as deferring, expanding, contracting or abandoning at a predetermined cost – exercise price – for a predetermined period of time, the life of the option. As in traditional valuation methods the investment and its valuation is multi phased event and can last many years with different cash flows in each period. In real options analysis the practitioner assumes that the investment isn't inflexible and there will be many possibilities to alter the previous business decisions or to take on new ones (Copeland & Antikarov, 2001). For example, if a company invests in a new plot of land and factory complex on it there are several options attached to the original investment: option to sell the land, option to switch or expand the production or to construct a new factory on the plot if there's free space still

available. The traditional investment valuation assumes that the cash flows are inflexible and predetermined and discounts them back to the present day giving the net present value of the investment.

The value of a real option depends on various variables. Below are the six variables in the Real Options Analysis:

1. Expected present value of cash flow of investment; a rise in the present value of the investment increases the NPV(without flexibility) and subsequently also the value of ROA will increase
2. Exercise price/Investment cost; if the exercise price, i.e. the investment outlay increases the NPV of the investment decreases which reduces the value of ROA
3. Time to expire; the longer the time there's to acquire more information about the uncertainty the more it increases the valuation of ROA
4. Uncertainty(Volatility) about the present value; with managerial flexibility an increase in uncertainty gives a rise in the value of ROA
5. Risk-free interest rate; as the risk-free rate increases, the value of the option also increases
6. Cash Flows (dividends) lost due to competitors who have fully committed; the cash flows lost to competitors will obviously decrease the value of ROA.

Copeland et al., 2000.

In NPV method the best outcome (MAX) is selected at the beginning of investment (time t=0) if it has a positive net value, estimated value at time t=0 subtracted by exercise price X, the investment outlay.

$$NPVrule: MAX\ (t=0)[0, E_0 V_T - X\ ]$$

A deferral call option can be valued using NPV approach and the above mentioned equation as demonstrated in chapter 3.3.1.

Real Option Analysis takes a different approach. In a call option the best possible outcome will be selected at time t=T when the state of nature is known at that time. Mathematically it means expectation of maximums instead of maximum of expectations.

$$ROArule: E_0\, MAX\ (t=T)[0, V_t - X\ ]$$

3.3.2 The risk-adjusted discounted cash flow method

There are several methods for valuing real options. There is the replicating portfolio approach emphasized by Copeland and Antikarov (2001). Then there's the landmark Black & Scholes -method for option valuation, which can also be applied to real options. In this research the replicating portfolio approach is used. Nonetheless, the Black & Scholes -method is briefly demonstrated for convenience and to show its close resemblance to replicating portfolio approach. Finally the decision tree model is described in order to give the valuation its temporal dimension.

An example of a deferral option gives a good picture of the different approaches and practicality to mathematical formulas. The example case has following features:

Investment: a company has planned a machinery investment worth 70 euros that is irreversible and the equipment is bespoken for company needs. Thus, the salvage value – the value should it be sold, for example - of the investment is zero. The company has the possibility to initiate the investment now or to defer until the end of the year. The cash flows are perpetual and replacement investments net out the depreciation of machinery. The risk-free rate of capital is 8 %. The cash flows are uncertain and have 50-50 chance of being either 100 or 40. The risk-adjusted discount rate is unknown.

Risk-free rate: 8 %

Investment outlay: 70

Up-state probability at end of period, t=T: 0,5

Up-state cash flow:100

Down-state probability at end of period, t=T: 0,5

Down-state cash flow:40

First the net present value is calculated. In this phase we know the cash flows and their probabilities along with investment outlay. In order to calculate the present value and the net present value the risk-adjusted discount rate is needed. One method to acquire the rate would be the *Capital Asset Pricing model.* In the method practitioners search for company-level betas that have similar risk to the investment valuated (Brealey et al., 2011). Another way to calculate the rate is the risk-adjusted discounted cash flow method that uses similar or twin securities that have similar payouts, thus the law of one price can be used. It states that in functioning market to prevent arbitrage profits, two assets with similar payouts in every state of nature are each other's substitutes and must have exactly the same price or value.

A twin security can be found. The security has the following value and correlated cash flows:

Up-state cash flow: 20

Down-state cash flow: 8

Market price for twin security: 12

The correlation can be verified by calculating the ratio between up and down states of both the twin security and the investment. The ratio in both cases is 2,5. Now the risk-adjusted discount rate, k, can be calculated with basic present value equation:

$$V_0 = \frac{q(V_u) + (1-q)(V_d)}{1+k}$$

$$12 = \frac{.5(20) + .5(8)}{1+k} \rightarrow k = 0{,}167$$

The present value of the investment can be discounted with the risk-adjusted discount rate.

$$PV = \frac{.5(100) + .5(40)}{1+0{,}167} = 60$$

The present value of the outlay on the other hand is 70/(1+0,08)=64,81 and thus the net present value of the project is 60-64,81=-4,81 indicating that we should abandon the investment.

3.3.3 Replicating portfolio approach

Yet another way to calculate the net present value is the replicating portfolio approach which will be used for option pricing, too. The approach calculates the present value using portfolio of *m* shares of the twin security and *B* bonds to replicate the payoffs of our project: how many shares of the twin security and bonds must one hold in order to replicate the same payoffs as the investment itself. It is essentially a synthetic portfolio that uses the law of one price as its theoretical basis. The twin security proposition will be analyzed more thoroughly in chapters 4.1.4 and 4.1.7. With the same payoffs and values as above the payoffs of the replicating portfolio must be following:

$$\textit{Replicating portfolio payoff at the up state}: m(20) + B(1+r_f) = 100$$

$$\textit{Replicating portfolio payoff at the down state}: m(8) + B(1+r_f) = 40$$

These two unknowns can be solved and they become m=5 and B=0 and the present value is equal to the one calculated with risk-adjusted discounted cash flow method:

$$\textit{Present value of the replicating portfolio} = m(12) + B(1+r_f) = 5\text{x}12 + 0 = 60$$

Thus, the net present value is the same, too.

In addition there's a third approach called neutral probability approach. It discounts certainty-equivalent cash flows at the risk-free rate and is

mathematically equal to replicating portfolio approach. The only difference is in which phase the risk-adjustment is done (Copeland & Antikarov, 2001.)

3.3.4 Option valuation using the replicating portfolio approach

If the project can be deferred to the end of the year, t=T, the following table can be constructed with payouts from precommitment and deferral.

	Precommit	Investment	Net Precommit	Defer
Up state	100	70	30	MAX[30,0] = 30
Down state:	40	70	-30	MAX[-30,0] = 0

One way to value the real option would be to use the decision tree analysis (DTA) with risk-adjusted discount rate as calculated above. However this approach has its limitations and the use of replication portfolio approach is preferred as shown next. The assumption is made that the investment has the possibility to defer until the end of the year to see what is the state of nature in order to make the best possible decision by selecting the maximum value in the end of the period. The net present value is estimated by discounting the deferral cash flows with the risk-adjusted discount rate, 16,7 % in this case.

$$NPV = \frac{,5(30)+,5(0)}{1+0,167} = \frac{15}{1,167} = 12,86$$

Since the net present value of the investment without flexibility is -4,81 the value of the deferral option according to DTA is 12,86-(-4,81) =17,67. Unfortunately the previously calculated risk-adjusted discount rate isn't appropriate for the DTA approach. Thus, the approach violates the law of one price because the cash flows aren't correlated to the twin security's cash flows anymore; given the deferral the payouts for the option are 30 and 0 whilst for the net cash flows they're 30 and -30 as demonstrated in by comparing the last two colums in table x.

Replicating portfolio approach to valuing deferral option must be used to circumvent the DTA's limitation in terms of inaccurate risk-adjusted discount rate.

To replicate the payouts we use a portfolio composed of m shares of the same twin security, with the price of 12 and B euros of the risk-free bond whose present value is 1 per bond.

$$\textit{Replicating portfolio payout at the up state}: m(20) + B(1 + r_f) = 30$$

$$\textit{Replicating portfolio payout at the down state}: m(8) + B(1 + r_f) = 0$$

In up-state the replicating portfolio pays 20(in down state 8) on every share and 8 per cent on every risk-free bond (same as in down state). The two unknowns in the equation are: m = 2,5 and B = -18,52 which means that to replicate the payouts with flexibility you need 2,5 shares of the twin security and to borrow -18,52 with

the risk-free rate of 8 per cent. Because both the replicating portfolio and the investment have the same payouts, by the law of one price, their present value should be the same. We can derive the present value by multiplying the number of shares and bonds with their present value:

$$\textit{Present value of the replicating portfolio}: 2{,}5*12 - 18{,}52*1 = 11{,}48$$

The value of flexibility is the difference between the precommitted investment and the one with flexibility: 11,48-(-4,81) =16,30. Notice the rounding error. Now that the value of the deferral is known the correct risk-adjusted discount rate can be calculated:

$$PV = 11{,}48 = \frac{0{,}5*30 + 0{,}5*0}{1+k} \rightarrow k = 0{,}301$$

The result shows the accurate discount rate as opposed to the one used in DTA. DTA approach uses the wrong discount rate because it assumes it stays constant throughout the decision tree and do not consider the fact that the discount rate changes based on where in the decision tree the calculations are made. With the correct discount rate, the value could have been calculated using the DTA approach.

The value of flexibility can be calculated also from option payouts. Following table of option payouts can be constructed:

	Net Precommit	Defer	Option payout
Up state	30	MAX[30,0] = 30	0
Down state:	-30	MAX[-30,0] = 0	30

Out of which payout equations can be constructed:

$$\textit{Replicating portfolio payout at the up state}: m(20) + B(1+r_f) = 0$$

$$\textit{Replicating portfolio payout at the down state}: m(8) + B(1+r_f) = 30$$

This equates to m = -2,5 and B = 46,30. Again the present value can be calculated

$$\textit{Present value of the option}: -2{,}5*12 + 46{,}39 = 16{,}30$$

The resulting present value of the option is the same as the difference between NPV and NPV with flexibility which means that the value of the option can be calculated also from the differential cash flows that it generates.

Let C_u be the option payoff in up-state and C_d in down-state. Equations of payouts can put in the following form showing that m is actually a hedge ratio of the option payouts:

$$m = \frac{C_u - C_d}{V_u - V_d} = \frac{\textit{Incremental option payoff}}{\textit{Change in the value of the twin security}}$$

3.3.5 Black-Scholes option pricing model

The Black-Scholes option pricing model was invented by F. Black and M. Scholes with the help of R.C. Merton in 1973(Brealey, 562-570). The model consists of two parts:

$$Value\ of\ call\ option = [delta * share\ price] - bank\ loan$$

According to Copeland and Antikarov (2001) it has the same idea as the replicating portfolio. This opinion is shared also by Brealey et al. (2011). In its mathematical form Black & Scholes is written:

$$C_0 = N(d_1) * S_0 + N(d_2) * Xe^{rT}$$

$$d_1 = \frac{\ln(S/X) + r_f T}{\sigma\sqrt{T}} + \frac{1}{2\sigma\sqrt{T}}$$

$$d_2 = d_1 - \sigma\sqrt{T}$$

Where:

S_0 = The price of the underlying (e.g., a share of common stock)

$N(d_1)$ = The cumulative normal probability of unit normal variable d_1

$N(d_2)$ = The cumulative normal probability of unit normal variable d_2

X = The exercise price

T = Time to maturity

r_f = Risk-free rate

e = The base of natural logarithms, constant = 2,17...

Although the Black-Scholes formula looks quite different to the replicating portfolio approach it is both mathematically and conceptually very similar. The replicating approach can be simplified:

$$mV_0 - B_0 = C_{0,}$$

In Black-Scholes formula the first term is actually the number of units of the underlying asset necessary to form a mimicking portfolio and the second term is the number of bonds each paying 1 unit of currency at expiration. The idea behind both the Black-Scholes formula and the replicating portfolio is the same. The starting point of the approaches differs: Black-Scholes starts from Itô calculus whilst the replicating portfolio approach is an algebraic approximation (Copeland & Antikarov, 2001).

3.3.6 Differences between financial and real options

Copeland and Antikarov (2001) name three important differences between financial and real options. The first difference between financial and real option

is the issuer of the option. The issuer of a financial option is typically a financial institution enabling side betting on the asset value. The issuer has no control over the underlying asset. Real options differ because they are issued by the company management that control the underlying asset.

Both the financial and real options are right to take an action. The second difference between financial and real options is the underlying asset used in option valuation. In Financial options the underlying asset is typically a common stock, index or a bond. These assets are traded securities which makes it easier to estimate their parameters and get the necessary information. Historical data and the security price is available to calculate the option value.

In real options the underlying asset is a tangible asset, for example, a R&D project or a business division. The price of the underlying asset is not typically traded and its price is not obvious. To counter this a Marketed Asset Disclaimer assumption is made that is explained in the next chapter.

The final difference is the risk and the option holder's possibility to change it. The rate of return on a stock is typically out of the stockholder's control. In real options the management has the ability to change the risk and change the uncertainty of the underlying at least to a certain degree. This ability derives from the possibility to affect competitors'' actions.

3.3.7 Marketed asset disclaimer

Above the valuation of an option was done by using the replicating portfolio approach which assumes that a twin security with correlating payouts will be found. Using the twin security the option payouts are replicated with the help of risk-free borrowing. It is well to doubt the possibility of finding a perfect or even closely correlated twin security that fits the often complex nature of investment projects valued with ROA. Copeland and Antikarov (2001) point out that it is not realistic although in the early years of ROA application world commodities were used as the underlying risky asset. The application implied that the volatility of the underlying project without flexibility was the same as the observed volatility of the world commodity. For example, the price of copper was assumed to be the same as the volatility of the gold mine that had the right to defer its start of operations – unfortunately the volatilities do not match. The unrealistic nature of this assumption becomes even more obvious with R&D-investments: how can you find a twin security for new technology product that hasn't been launched yet?

Modern application of ROA uses the present value of the investment without flexibility itself as the twin security or the underlying risky asset. Indeed, what would imitate the possible payouts and its volatility better than the investment itself? Copeland and Antikarov call this assumption the Marketed Asset Disclaimer. Instead of searching for a twin security, the ROA approach uses the

payoffs of the investment without flexibility to determine the value of the option.

When substituting the twin security's payoffs to the ones provided by the investment we get the following replicating portfolio payoffs:

$$\textit{Replicating portfolio payoff at the up state}: m(100) + B(1 + r_f) = 30$$

$$\textit{Replicating portfolio payoff at the down state}: m(40) + B(1 + r_f) = 0$$

Out of which the unknowns can be solved as m = 0,5 and B = -18,52. Once the present value of the investment (60) is included the present value of the project with flexibility becomes:

$$\textit{Present value of the investment with flexibility}: m(60) + B = 0{,}5 * 60 + (-18{,}52) = 11{,}48$$

This is the same result as calculated with the twin security but it is more practical: it uses the investment's present value without flexibility (100) to calculate the present value with flexibility. The set of assumptions is the same as with the NPV calculation comparability being the most important amongst them. If a security is comparable in terms of possible rates of return to value a regular option why wouldn't the NPV calculation and its rates of return be? And since the NPV analysis already assumes that the present value of the investment would be the value it fetches were it a marketed asset, ROA can make the same assumption.

3.3.8 The risk-neutral probability approach

Second approach for evaluating real option is the risk-neutral probability approach that starts out with a hedge portfolio consisting of one share of the underlying asset and a short position in m shares of the option that is being evaluated. The hedge ratio m is chosen so that a gain in the value of underlying asset is offset by the loss in the value of the short position and vice versa. In fact, if the m is chosen correctly the ensuing portfolio is riskless. To illustrate the payouts in both the up state and the down state, following table is constructed:

End-of-period state	Hedge portfolio payouts	Payoff of underlying
Up state	100-mMAX[100-70,0]	100
Down state	40-mMAX[40-70,0]	40

Next formula equating the hedge portfolio payouts will be formed indicating that if the right hedge ratio m is selected, the portfolios will provide the same cash flows and be, in fact, riskless.

$$uV_0 - mC_u = dV_0 - mC_d$$

$$100 - m(55) = 65 - m(0)$$

$$m = \frac{(u-d)V_0}{C_u - C_d} = \frac{(1{,}67 - 0{,}67)60}{30 - 0} = 2$$

Where:

u=up movement (u60=100)

d=down movement (d60=40)

V_0=starting value

C_u=call value in up state (100-70=30)

C_d=call value in down state (MAX[40-70,0]=0)

To solve the present value of the call option, an equation must be made from the present value of the hedge portfolio:

$$V_0 - mC_0 = 60 - 2C_0$$

The riskless hedge portfolio will earn the risk-free rate and the resulting payoff will be equal in both up and down state. Mathematically the above present value of the hedge portfolio will be multiplied by one plus risk-free rate (r_f, 8%) which will equal to the payout in the up (or down) state:

$$(V_0 - mC_0)(1 + r_f) = uV_0 - mC_u$$

$$C_0 = \frac{\left[\frac{(100-60)}{1{,}08} - 60\right]}{-2} = 11{,}48$$

The equation provides the correct present value for the option. If the hedge ratio m is substituted by its definition in equation above the value of call option becomes:

$$C_0 = \left[C_u \left(\frac{(1+r_f) - d}{u-d} \right) + C_d \left(\frac{u - (1+r_f)}{u-d} \right) \right] / (1+r_f)$$

The expressions in parentheses can be described as ”risk-neutral” probabilities, p and p-1, which sum to one. The risk-neutral probabilities are not the same as objective probabilities that reflect with which probability an event will occur. They're merely a mathematical concept that adjust the cash flows to a form that they can be discounted at the risk-free rate.

The equation arrives at its final form indicating that the value of the option is equal to the expected payouts multiplied by probabilities that adjust them for their risk. In effect, the numerator is a certainty-equivalent cash flow that can be discounted at the risk-free rate.

$$C_0 = \frac{[pC_u + (1-p)C_d]}{(1+r_f)}$$

Both – the risk-neutral probability approach and replicating portfolio approach can be used to evaluate real options along with Black & Scholes model.

3.3.9 Event tree and Decision Tree – ROA is a multi-period model

In ROA the emphasis is on multiple possibilities and decisions that come about

when the investment is done or is being deferred or when new information about the nature of reality can be obtained and a decision to expand will eventually be made. Since the goal is to achieve a value of the whole investment the different scenarios that might occur must be recognized and their effects calculated with proper and reasoned possibilities based on thorough thinking (Copeland & Antikarov 2001).

Event tree is an useful and commonly used method to visualize and eventually calculate the present value of an investment (Copeland & Antikarov, 2001). It is one application of binomial method which starts off by reducing the outcomes in the next period to two distinct outcomes: an up move and a down move. In other words the binomial method simplifies the outcome in the next period to a good and a bad outcome.

When the investment is done only certainty is the irreversibility of the investment and its initial outlay: how much capital is needed and on what costs will it be allocated. The sales figures for the first year can often be estimated rather well into good and bad scenarios.

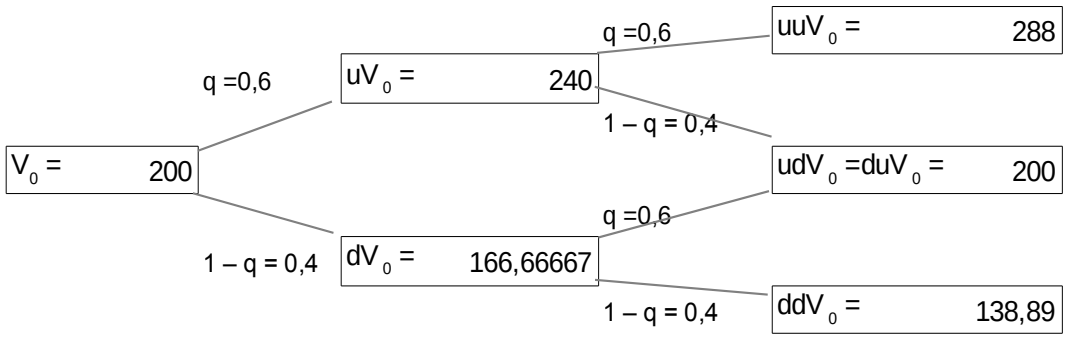
An event tree establishes the parameters for both good and bad outcomes. An event tree doesn't have any decisions built into the three. The purpose of the event tree is to model the uncertainty of the project that drives the value of the underlying asset. The consolidated approach for valuing uncertainty as described by Copeland and Antikarov (2001) combines all the uncertainties into the single uncertainty of the value of the project. At this point there is still no flexibility included in the valuation.

When the event tree is formed the decisions that management can made in each situation are embedded into the event tree. The event tree then becomes a decision tree.

Before forming a decision tree with flexibility an example of a two-period event tree without flexibility is provided. The decision tree describes an investment that runs for two years and has the following parameters:

Parameters				
Current value		200		
Upside prob		0,6	upside change	
Downside prob		0,4	dside change	

The below table indicates that the present value of the investment either goes up by 1,2 or down by 1/1,2 with probabilities of this happening standing at 60-40, respectively. Also the expected present values in each year are calculated using the probabilities provided.



Expected present values

V_0	V_1	V_2
200	210,66667	221,9022222

FIGURE 2

For example, the value at node dV_0=200*1,2=240. The expected present value V_2=0,6*240+0,4*166,67=210,67.

The probabilities are objective and not risk-neutral thus the cost of capital can be calculated to be 5,33 %.

$$r = \sqrt{\frac{V_2}{V_0}} - 1 = 0{,}0533$$

If results from first year are encouraging maybe there's reason to scale up the operation and invest more to achieve even greater growth or to just merely see if the revenues are maintainable or will they diminish after initial success. On the other hand if the results do not reach the wanted threshold maybe there's a chance the next year will better or that there's no feasibility even to continue with the plan for a one more year. In latter case there might be salvage value in selling technology or equipment needed in the investment to recoup some of the outlay to minimize losses. When these separate possibilities are logically organized and visualized a decision tree is formed to suit the ROA.

When the decisions that management is capable of making are put into the event tree nodes a decision tree is formed. The decision tree actually demonstrates the payoffs from optimal decisions. Therefore its payoffs constitute the payoffs from the option being valued.

A decision tree with similar parameters as in the figure 1 above with an American call option with an exercise price of 195 is provided. The nodes are named and the payoff of the option, value minus the exercise price is added.

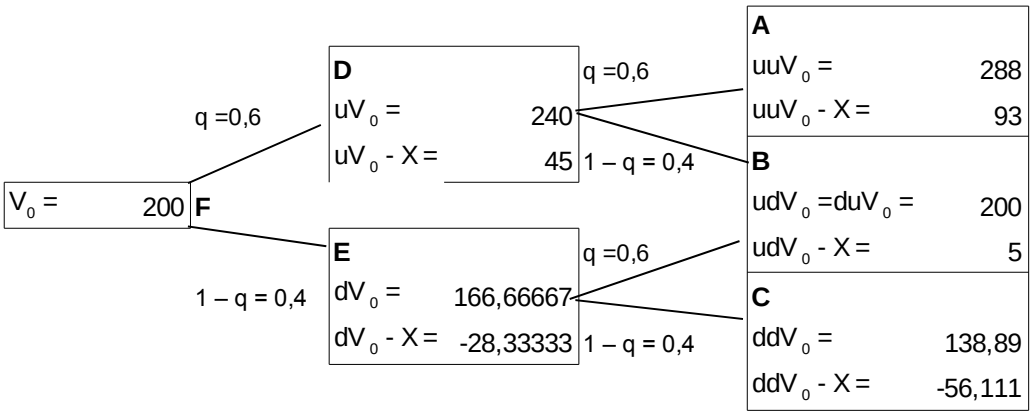


FIGURE 3

Now a replicating portfolio is formed for nodes D, E and F. The nodes A, B and C are the nodes where the decision tree ends and there's no option to value. However these nodes are integral part of valuing the nodes D and E. The payoffs of these nodes must be used to value the possibilities of the previous node until the starting point is reached at node F. The node F can be valued when the maximum payoffs of the nodes D and E are valued. The replicating portfolio for the node D is:

$$mu^2 V_0 + (1 + r_f)B = 93 = C_{uu}$$

in the up state and

$$mudV_0 + (1 + r_f)B = 5 = C_{ud}$$

in the down state. C_{uu} is 93 because of the ROA rule that selects the maximum of either the value at node D subtracted by exercise price or the value of the option at node D. Since there's no more options at the end of the event tree, the value of the option is zero. Thus, the value at node D subtracted by exercise price is chosen. A simultaneous equation can be formed by combining both the up and down state equations. The number of shares m can be solved

$$m = \frac{C_{uu} - C_{ud}}{V_0 u(u - d)} = \frac{93 - 5}{200 * 1{,}2\left(1 - \frac{1}{1{,}2}\right)} = 1$$

Which states that M is equal to the difference between option payoffs in up state and down state divided by the value of the underlying at the beginning of the period(at node D, V_0u) multiplied by the difference between up movement and down movement.

Then, the number of risk-free bonds can be calculated

$$B = \frac{C_{uu} - mu^2 V_0}{1 - r_f}$$

If you input the value of m to the equation you eventually get

$$B = \left[\frac{uC_{ud} - dC_{uu}}{u - d}\right] \div \left(1 - r_f\right) = \left[\frac{1{,}2 * 93 - \frac{1}{1{,}2} * 5}{1{,}2 - \frac{1}{1{,}2}}\right] \div \left(1 + 0{,}03\right) = -189{,}32$$

Which says that the needed number of bonds, B, is equal to the down movement times the payout of the option in the down state minus down movement times the option payoff in the up state; divided by the difference between up and down movement and 1 plus the risk-free rate.

In this example the negative amount of bonds means that in order to replicate the payoff the portfolio should borrow 189,32 shares at the risk-free rate instead of lending the money by holding bonds.

The value of the option at node D is C_D= muV_0+B = 1*1,2*240 – 189,32 = 50,68. The payoff if the call is exercised at node D on the other hand is 240-195 = 45. Since the value of the option is greater the decision should be to hold the option (i.e., the option is kept alive to exercise it later)

Similar calculations are performed in node E. Now, the formula is still the same but the up and down state payoffs are different as is the value of the underlying in the beginning. For the number of shares m:

$$m = \frac{C_{ud} - C_{dd}}{V_0 d\left(u - d\right)} = \frac{5 - 0}{200 * 1/1{,}2\left(1 - \frac{1}{1{,}2}\right)} = 0{,}81$$

And for the number of bonds:

$$B = \left[\frac{uC_{dd} - dC_{ud}}{u - d}\right] \div \left(1 + r_f\right) = -11{,}03$$

The value of the option at node E is C_E= mdV_0 + B = 2,60.

The value of the American call option at node F, at the root of the decision tree, can be calculated from the following replicating portfolio:

$$muV_0 + \left(1 + r_f\right)\text{B} = 50{,}68$$

$$mdV_0 + (1 + r_f)B = 2{,}60$$

Again, the maximum is chosen for the payoff at both nodes. For example, at node D the payoff is MAX[uV_0 -X; 50,68]=MAX[45; 50,68] = 50,68.

Now, the number of shares m needed to replicate the payoff is

$$m = \frac{C_D - C_E}{V_0(u-d)} = \frac{50{,}68 - 2{,}60}{200\left(1 - \frac{1}{1{,}2}\right)} = 0{,}656$$

The number of risk-free bonds B is

$$B = \left[\frac{uC_E - dC_D}{u-d}\right] \div (1 + r_f) = -103{,}55$$

Therefore, the value of the American call option is $C_F = mV_0 + B = 27{,}56$.

3.3.10 The four-step process for valuing real options

The process of valuing real options can be divided roughly to four steps. The four steps according to Copeland and Antikarov(2001) are

1. Compute base case traditional present value without flexibility at t= 0.
2. Model the uncertainty using event trees and understand how the present value develops over time, still no flexibility.
3. Identify and include the managerial flexibility into the event tree creating a decision tree.
4. Conduct the Real Options Analysis

The first step is traditional net present value analysis of the project at the beginning of the project. The free cash flows of the project are estimated and the possible initial investment is identified and measured.
The second step is to form the event tree and incorporating the uncertainty to it. This step aims to model the uncertainty that drives the value of the underlying risky asset. At this point there's still no flexibility but the nodes are already in place.
The third step is to develop the decision tree by inputting the managerial flexibility into the event tree. In practice the possible decisions that management can take are put into the nodes of the event tree. As written above, by putting the decisions into the nodes the event tree becomes a decision tree. Now the flexibility is incorporated.
The final step is the Real Options Analysis itself. The aim is to calculate the payoffs and value them using either replicating portfolios or risk-neutral probabilities.

The case investment will be valuated using this four-step process and replicating portfolios. Both the replicating portfolio and the risk-neutral probabilities

approach give the same result, however.

3.3.11 The option to expand and other real options

Real options can be found where there is uncertainty and managerial flexibility. The important factor in recognizing real options is to realize that the net present value technique systematically undervalues projects because the technique fails to include the value of flexibility (Copeland and Antikarov, 2001)

There are multiple types of real options and their combinations. A deferral option – as discussed in previous chapters – is a right to delay a projects with the investment being its exercise price. In other words it is an American call option.

An option to abandon is the right to abandon the project for a fixed price if the market conditions decline severely. This option is common in capital-intensive industries such as airlines and in a certain degree financial services. It is an American put option.

When the management has the option to contract the scale of operations if the market conditions decline it has an option to contract. An option to contract can be highly valuable in new product launches or when opening a new plant. It is a partial put option.

Switching options are of portfolios of American call and put options that enable the management to switch between two modes of operation according to the market conditions.

An option to expand is an American call that allows its owner to scale up the operations for by incurring a follow-up cost or investment. As a whole the investment opportunity with an option to expand built in it can be viewed as a combination of a base-scale project with a call option on future investment. If the value of future cash flows from the follow-up investment exceed the investment outlay the option to expand should be exercised.

There are also compound options that are options on options. Furthermore, options that have several uncertainties are called rainbow options. An investment can have many options linked to it. In a product development and launch to the market here might be a possibility to expand the project if it the project succeeds by investing in other versions of the succesful product. If the demand doesn't turn out to be sufficient there might be a need to contract the operations or even abandon the project for a fixed – undoubtedly smaller sum of money that the initial investment.

3.3.12 Uncertainty

There are several methods for discovering uncertainty in the future events that can be used in the valuation of investments. In the context of real option valuation, Copeland and Antikarov(2001) point out two different ways for finding the uncertainty in the investment: using historical data and using subjective estimates by experts or management.

This is a rather broad view and in general, several methodologies have been developed for estimating the future development of the investment. These methodologies include, for example, econometric models, rule-based forecasting, role playing and conjoint analysis.(Armstrong, 2001).

4 THE CASE

4.1 Case company

The case company is a start-up company that aims to develop an art marketplace on the internet. The company has officially been established in 2010 but the business plan was put on hold due to founders being students and lacking time to embark on full-time entrepreneurship. Some technological and marketing efforts were accomplished, though. The initial plan is to sell art on commission on behalf of artists, private art owners and gallerists. In addition the company will sell art related merchandise in order to supplement revenues creating more stable and predictable revenue streams. As in every business plan also in this business plan lies uncertainties which ROA tries to tackle.

4.1.1 History

The company was founded in Tampere, Finland in the end of 2010 by six persons. The founders were three business students, one information technology student, an IT architecture professional and a new media professional. The first steps were taken in an accelerator called Protomo that is financed by the Finnish State. A working prototype without payment system was done within few months. The company and the marketplace was marketed in an arts fair in 2011 and gathered significant consideration amongst exhibitors.

”The greatest achievement in terms of marketing was the first-hand reactions and feedback – which were mostly positive and encouraging - and a mailing list of over hundred artists, private art owners and gallerists. This list was later complemented via internet form with which one could add oneself to the mailing list”

CEO of The Company

After the positive response the company reached out to both private and public investors to gather financing for the development of final technology to start the business. According to CEO Tauriainen there were a few serious negotiations about financing and he had the impression that financing deal could be achieved in the short term.

Unfortunately because of personal reasons owing to most of the founders still being students the company decided to postpone the launching of the marketplace in order to give adequate time for founders to finish their studies. It was decided that when the students – CEO Tauriainen included – approach the end of their academic careers the business plan will be revisited in order to see

whether it is still viable at that point of time. This research is made to order for making conclucions whether the investment has a positive net value and whether the venture is worth a try.

4.1.2 Business model

The business model and its revenues can be divided into four revenue streams:

1. physical art sales directly and in auction
2. digital art sales,
3. merchandize sales and
4. sales of related services.

Physical art sales is the corner stone of the business model. The idea is to sell art on commission on behalf of clients. *The sales method is comprised of both art sales with predetermined price and in auction.* The art sold are typical items found in galleries, e.g. paintings and original prints. The management has also considered whether to sell both physical and digital art as a proprietor but has concluded that the lack of knowledge of the Finnish art market is too risky to justify the potential revenues in the beginning.

Digital art refers to intellectual right to a certain image or pattern that can be considered as art. The sales method is similar to the one in physical art sales. The functioning of this revenues stream is however more uncertain which will be reflected in the calculations.

Merchandize sales – although supplementary in long-term – will nevertheless provide steady flow of revenues. The merchandize sales will be used as an indicator to determine whether the business has gained traction in terms of marketing and the phase two investment should be approved.

Sales of related services include optimization for art sellers on the market place and other marketing related services. First all the services will be tailored for each customer separately. According to CEO Tauriainen more emphasis on the productization of services will be done when the company gathers customer feedback and gains more information on demand for these services.

4.1.3 Present day and future

In the end of 2014 all the students have either finished or quit their studies or are in the last months of their academic studies. The plan has changed into a less-riskier one with a wait-and-see feature included in it. The ROA will reflect this change and the real option is the option to expand embedded in the revised business plan. Below is a picture depicting the process for constructing the investment valuation and the business process around it:

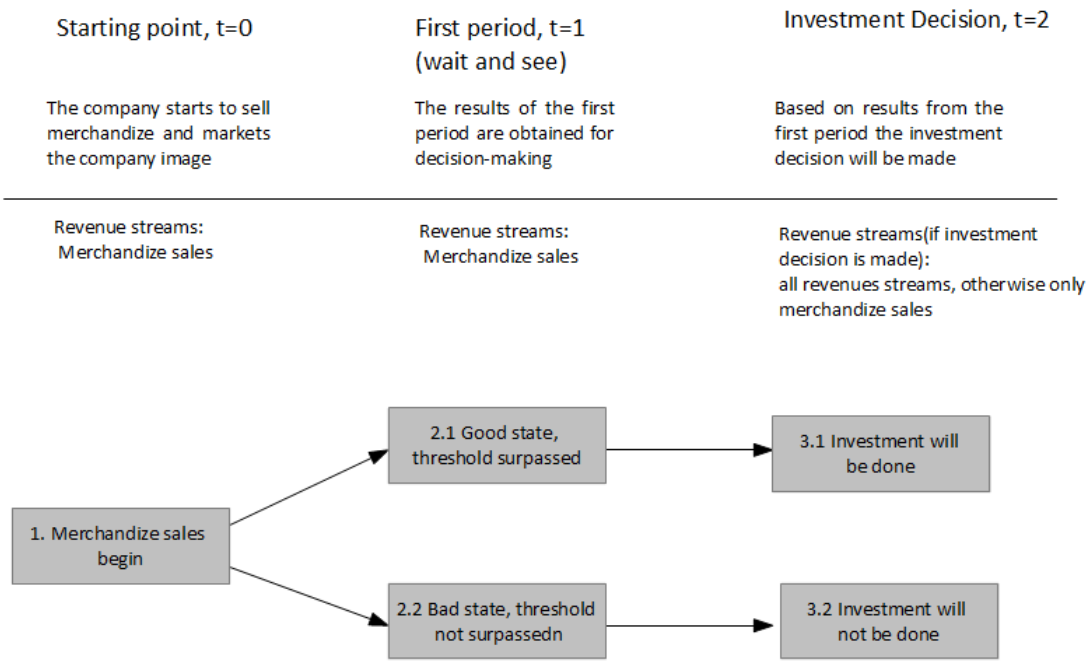


FIGURE 4

In the beginning, the company takes the decision to open up a webstore to sell merchandize and some art work (everything is included in the merchandize revenue stream). The merchandize is mostly prints with some other art related products added to the selection. The art sales are small in quantity. In this research the first-phase is a fact and a starting point and the research will focus on the following phases. According to the business plan the next phase is to wait and see whether the webstore will be successful or not. To define the successfulness of the webstore the company has set a threshold, a certain level of cash flow in the first period that the sales should generate in order to be successful. The consequence in the up state – the threshold has been surpassed - is the investment. In the down state the investment will not be executed.

”To appeal to potential investors we have devised a road map that includes both a lower risk level for us as entrepreneurs and a way to show some cash flows with which future projections can be made more reliably. And, maybe more importantly, to show investors that the brand has traction.”

CEO of the company.

4.2 The company characterization

The company's financial needs and its age put it into seed finance or early investment round phase of start-up finance. The view of the management confirms the financial context:

”The pitches for financing will most definitely be aimed at angel investors, or other wealthy individuals. We have thought about getting some financing from relatives also which would give the outside investors more comfort into investing in this venture: 'they and their family have a big stake in this too, I guess they mean business.' If the funding round goes well with angel investors maybe we will look into that too. Primarily it is the business angel sector we're pitching, though”

CEO of the Company

4.3 The investment plan

The investment plan and – actually – the business plan is to first open up a web store selling merchandize and some art work. If the demand for products and the traction of the brand is sufficient a further investment to develop and launch an art marketplace. The investment valuation is done before committing any capital to the project.

The idea is to calculate whether it is rational to make the initial investment – setting up the webstore – taking into consideration both the cash flows that the webstore generates and the option to expand and its potential cash flows. The option to expand is the real option researched in this case study. The following exhibit demonstrates the

Naturally to test the feasibility of the brand and materialize the revenues a certain amount of marketing and sales promotion must be done. This will be included in the investment calculations of the first-phase.

To determine whether the demand for products and the traction of the brand is sufficient a threshold must be set. The threshold in this context means the attribute in a decision. The threshold determines the action to be taken at this point. In the case if the threshold is surpassed the further investment will be done and if the threshold is not surpassed the investment will not be done and the project will be abandoned.

The threshold will be the combined revenues in the first-phase of the investment plan: merchandize sales. Although the costs related – costs of goods sold – must be taken into consideration in the general profitability calculations the management has decided to use the revenues as the threshold to quicken the decision making process. When the revenues are the single factor there's no need to calculate the actual costs and the decision can be made as soon as the state of nature becomes known.

The threshold will have a certain period in which it is measured, this period is one year at least in the planning phase. This means that the threshold must be surpassed in one year to make the investment decision happen.

4.4 Real option description

The real option included in the investment can be classified as a growth option (as termed by Trigeorgis, 1996) or option to expand (Copeland and Antikarov, 2001). The term option to expand will be used in this research. One could suggest that there is after all an abandonment option in the investment plan, too. There might be an interested art merchant looking for a platform to sell art and related merchandise. Thus, the case company could be able to sell the first-phase webstore at a fixed price. However, this is highly uncertain and incorporating the abandonment option into the investment valuation could complicate the thinking especially when pitching the investment opportunity to potential investors. In principal the investment could include a deferral option, too. The reasoning behind it would be even more dubious, though.

"Since Real Option Analysis is rather unorthodox in Finland it is really important to keep the calculations as simple as possible in order to maintain the explicitly of the valuation and moreover its reliability."

CEO of the company

4.5 Four-step process to value the real option

The net present value of the project is calculated from the estimates provided by the CEO of the company. The CEO was given assistance because he doesn't have business degree and only knows rudimentary finance.

4.5.1 Net present value of the investment

First the cash flows from the webstore were analyzed. The typical item was given a median price and the cost of goods sold ratio was set based on example calculation of an item sold in the webstore.

The cash flows, COGS and the free cash flows in each case are assumed to be perpetual.

WEBSTORE FREE CASH FLOW					
All prices not including VAT					
Price			25,00 €		
Amount sold	Up state		900	Down state	400
	Probability		50,00%		50,00%
	REVENUE CASH FLOW		22 500,00 €		10 000,00 €
COGS, Cost of goods sold					
	Transportation paid by customer				
	Purchase price	50,00%	11 250,00 €		5 000,00 €
	Overhead ratio				
	Including CAPEX	20,00%	4 500,00 €		2 000,00 €
	No CAPEX				
	FREE CASH FLOW		6 750,00 €		3 000,00 €

The up state revenue cash flow, 22 500 € sets the threshold for the investment too. If it is surpassed the option to expand is exercised. This is a management decision and its reasoning is the management's view that below this amount of threshold the traction of the brand isn't well enough reasoned.

Next the webstore investment outlay is outlined. The webstore investment consists the webstore design and the marketing campaign to market the webstore.

”The investment to set up the webstore consists two parts: the technical and graphical design of the webstore so that it can process orders securely and its user interface is as user friendly as it can be. The costs relating to design comes from wages for both the in-house coder and CTO and to exterior graphic designer. The amount is rather conservative since wages are minimal in the beginning and the graphic designer is a close friend.

The second part of the investment is the marketing campaign of the webstore.

Although we're more or less acquinted to many key players and art media a push for a broader market needs to be done. This is essential because this way we can test our brand's traction and the general demand for art pieces and merchandise in the internet. The cornerstone of the marketing campaign is a social media campaign that highlights the art exhibition that we will set up at the Helsinki Expo and Convention Center. This will also be the launch of the webstore. In addition to this year round marketing will be done in specialized magazines."

CEO of the company

WEBSTORE INVESTMENT OUTLAY

Webstore design		10 000,00 €
Marketing campaign		
	Social media	2 000,00 €
	Exhibition	2 000,00 €
	Traditional media	2 000,00 €
Total investment outlay		16 000,00 €

The discount rate for the case company will be the weighted-average cost of capital (WACC).

As Festel et al.(2013) write it is essential to note that usually most start-ups use only equity financing thus making the cost of capital or WACC the cost of equity. According to Miettinen and Niskanen (2015) the role of trade finance is important for startup companies in Finland. From a sample of 288 startup companies the average share of trade finance stood at 34 percent of total debt. The overall debt ratio was 56 percent. The CEO of the company thinks differently and the aim of the company is to be almost fully equity funded with some trade credit.

"We try to finance the company without significant debt. The debt that we employ is mostly trade finance and we maintain that it has a zero percent rate on it if we pay in the window giving zero interest rate. We will not issue bonds at least in the beginning."

CEO of the company

Although the CEO thinks that there are no financing costs for trade credit it is nearly impossible to think that all the trade payables will be paid before due date and that there would be no interest payments needed for this debt. Thus, to give a more reasonable cost of capital also the trade finance is assumed to have a cost. Chuldek (2011) calculates that the real interest rate of trade credit is somewhere between 4 and 6 percent despite its negative image in literature. This also

explains why trade credit constitutes a large part of companies' mix of capital structure.

"We chose to use cost of equity as the WACC. Many investors have given us the green light for this and it is indeed our plan to use mostly equity financing in the first phases of the business plan."

CEO of the company

The equity part of capital is much harder to estimate. As described above Festel el al. (2013) use 39,5 % as the average rate of return required by the investors in startup companies. This cost of equity will be used as the WACC. One could argue for a lower discount rate in times of quantitative easing by both the ECB and FED, though.

The traditional NPV formula for perpetual discounted cash flows is used:

$$NPV = -I + \sum_{t=1}^{N} \frac{(FCF_t)}{(1+WACC)^t} \; where a = FCF_t, r = \frac{1}{(1+WACC)^t}$$

The free cash flow is FCF = 0,5*6750 + 0,5*3000 = 4875

Thus, the net present value can be calculated

$$NPV = \text{-}16\,000 - \frac{4875}{1 - \frac{1}{1{,}395}} = 1216{,}47$$

4.5.2 Event tree and the cash flow estimation

The up state and down state are symmetrical and follow the random walk of probabilities. The management decided that both scenarios are as likely. Either the value of the investmen will go up 1,5 times or down symmetrically 1/1,5. At this point the event tree doesn't include any flexibility and management isn't able to make decisions. It is merely the net present value visualized.

				B		
				uV_0 =	1 825,16 €	
			0,5			
	V_0 =	1 216,77 €	**A**			
Investment value t=0		1 216,77 €	0,5	**C**		
Up movement		1,5		dV_0 =	811,18 €	
Down movement		0,66666667				

FIGURE 5

The event tree states that in the next year the value has either gone up by 50 % or down by 50 %. This means that the free cash flow from the merchandise sales goes up or down respectively if calculated using the NPV method.

To double check the event tree and the basis for the decision tree a replicating portfolio can be done using the up-state and down-state values as the payoffs. This test gives the same net present value.

4.5.3 Decision tree

To form the decision tree the decision has to be outlined and its consequences, too. If the aforementioned threshold is passed the management will make the decision to expand the operation by executing the second-phase investment to generate the revenues from the three other business lines. The management has forecast the revenues for each line of business to be following:

1.Physical art

Revenues

A.The price of an average physical art piece

		Up state	Probability		Down state	Probability	
Average commission	Price	350,00 €	0,6		80,00 €	0,4	
Approx 30 % of price							
	Risk-adjusted price			242,00 €			

B. Sale volume of an average year

		Up state	Probability		Down state	Probability	
Average volume	Volume	600	0,4		200	0,6	
	Risk-adjusted volume			360			

RISK-ADJUSTED REVENUE	87 120,00 €

2.Digital art

Revenues

A.The price of an average ´digital art piece

		Up state	Probability		Down state	Probability	
Average commission	Price	300,00 €	0,5		200,00 €	0,5	
Approx 50 % of price							
	Risk-adjusted price			250,00 €			

B. Sale volume of an average year

		Up state	Probability		Down state	Probability	
Average volume	Volume	100	0,4		50	0,6	
	Risk-adjusted volume			70			
RISK-ADJUSTED REVENUE				17 500,00 €			

3.Merchandize

Revenues

A.The price of an average merchandize

		Up state	Probability		Down state	Probability	
Average piece of merchandise	Price	25,00 €	0,5		25,00 €	0,5	
	Risk-adjusted price			25,00 €			

B. Sale volume of an average year

		Up state	Probability		Down state	Probability	
Average volume	Volume	900	0,5		400	0,5	
	Risk-adjusted volume			650			
RISK-ADJUSTED REVENUE				16 250,00 €			

4. Services

Revenues

A.The price of an average service

		Up state	Probability		Down state	Probability	
Average service	Price	2 000,00 €	0,5		1 000,00 €	0,5	
	Risk-adjusted price			1 500,00 €			

B. Sale volume of an average year

		Up state	Probability		Down state	Probability	
Average volume	Volume	15	0,5		10	0,5	
	Risk-adjusted volume			12,5			
RISK-ADJUSTED REVENUE				18 750,00 €			

The management was given the task to outline good and bad scenarios for both the average price of one unit and the volume of units in each business in one calendar year. From these two possibilities both a risk-adjusted price and volume was calculated forecasting the risk-adjusted revenue.

Next the management estimated the cost of goods sold for all the three expanded business lines. The costs were first calculated for each business line and afterwards combined into one. Cost of goods sold, COGS for physical art, digital art and services:

COGS, OVERHEAD(NOT INCLUDING MERCHANDIZE)

1. IT infrastructure and licence costs	5 % of revenues		6 981,00 €
2. Salaries			
	Tech		28000
	Sales/Marketing		16800
	Back office and administrative cost		28000
	Total salaries		72800
3. Sales/Marketing costs			10000
4. Office and/or warehouse costs	€/month		
	Office/warehouse premises	800,00 €	9 600,00 €
	Insurance	200,00 €	2 400,00 €
	Legal costs	300,00 €	3 600,00 €
	Accounting costs	500,00 €	6 000,00 €
	Total office/warehouse costs		21 600,00 €
			111 381,00 €

In overhead, the salaries are activity-based and the number of employees is estimated to be 1. However the salary without side costs is only 52 000 € which indicates that the salaries might need to go up at some point. The salaries include the mandatory side costs with a ratio of 40 %.

The cost of goods sold are estimated for the risk-adjusted revenues. Since the labor intensity is much higher in physical art, digital art and services, the overhead ratio and the cost of goods sold is much higher than for the merchandise revenue stream. Furthermore, in the phase one of the business plan the salaries are paid hourly which is the reason behind small sum allocated for the salaries.

If the marketplace is easily scalable – as is the plan according to the company – the overhead-ratio should reduce for the other revenue streams. Also, the management thinks that there should be ways to reduce costs generally when the ideal business model is found.

On the other hand the purchase price is calculated to be 50 % of one euro of merchandise revenue while in other revenue streams there's no purchase price because both the physical and digital art revenues and the revenues arising from services do not involve proprietary commerce. The art revenue is generated from

commissions and services are purely service revenue.

Note, although mathematically the down state includes the option to expand the threshold is set so that if it is not surpassed, the option to expand is not economically viable in the down state.

The expansion rate is calculated so that the expanded value of the investment at node B is equal to the free cash flow generated by the second phase of the business plan including all the revenue streams:

FREE CASH FLOW CALCULATION OF THE SECOND PHASE INVESTMENT

REVENUES		
	Physical art	87120
	Digital art	17500
	Merchandize	16250
	Services	18750
	TOTAL REVENUES	139620
COGS, physical art, digital art and services		111381
COGS merhcandise		11375
NO CAPEX		0
FREE CASH FLOW		16864
PRESENT VALUE		59557,67089

Cost of goods sold and revenues for merchandise revenue is the same as in the NPV valuation above. The revenue stream of merchandise sales must be included otherwise the phase two wouldn't incorporate all of the necessary revenue streams and the expansion rate wouldn't be correct.

Once the present value is known the equation to get the necessary expansion rate to suit the decision tree can be formed:

$$uV_0 * expansion\ \ rate = Present\ value\ of\ \ the\ second\ -\ phase$$

The expansion rate is approximately 32,63.

When the expansion rate is known and the decision tree correlates to the estimated cash flows the second phase investment outlay is put into the decision tree. The second phase investment outlay is:

2ND PHASE INVESTMENT OUTLAY

Marketplace design	25 000,00 €
Trademark	2 000,00 €
Marketing campaign	
Social media	5 000,00 €
Exhibition	3 000,00 €
Traditional media	10 000,00 €
Total investment outlay	45 000,00 €

”The investment to the marketplace is mostly just hours and hours of coding, debugging and testing. Also some work has to be outsourced to graphic designer and maybe we need help with user interface too.”

CTO of the company.

Now that both the cash flows and the investment outlay, the exercise price, is in place the replicating portfolio to value the option to expand can be formed and the real options analysis can be performed.

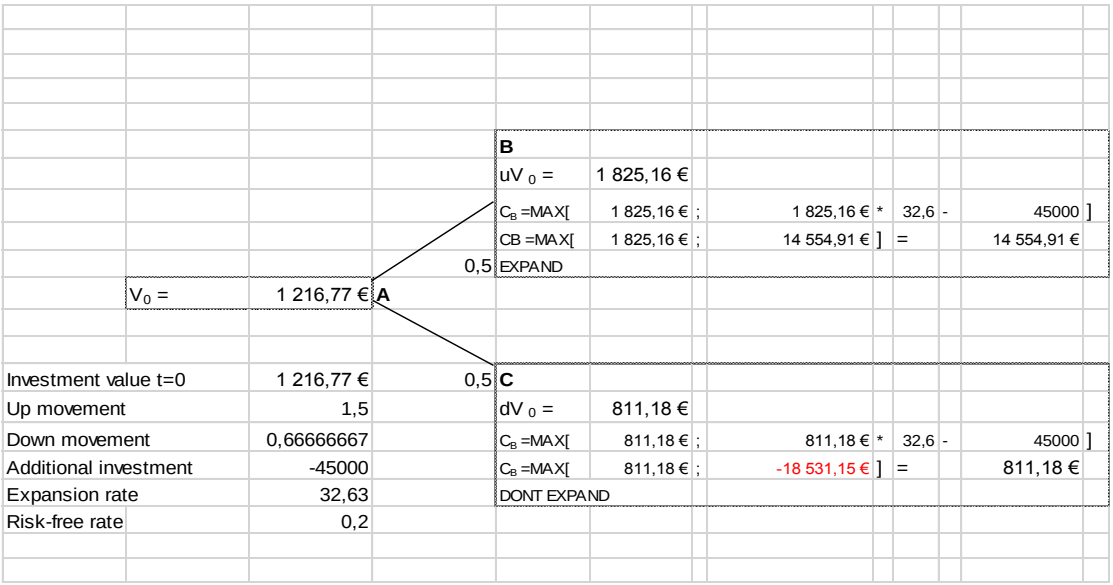


FIGURE 6

4.5.4 Real options analysis

The option valuation can be done using both the replicating portfolio approach and the risk-neutral probabilities. Both methods give the same result. In this research the emphasis is on the replicating portfolio analysis. In the end the result can be double-checked with risk-neutral probabilities. The decision tree gives the following replicating portfolio:

$$muV_0 + (1 + r_f)B = C_b = 14554{,}91\,€$$

$$mdV_0 + (1 + r_f)B = C_C = 811{,}18\,€$$

The risk-free rate is 0,2 %. The amount of shares m needed to form the replicating portfolio is m = 13,55 and the amount of risk-free bonds is B = - 8486,50. Again indicating that we would borrow at the risk-free rate.

Thus, the option valuation can be done:

$$C_A = mV_0 + B = 13{,}55 * 1216{,}77€ - 8486{,}50€ = 8005{,}97€$$

The value of the investment is the value of the option to expand and the net present value of the investment without flexibility. The value of the option to expand is 6789,20 €.

The valuation could have been done also by using either the option payoffs in the replicating portfolio or by using the risk-neutral probabilities approach.

”Considering the cash flows and the base-case valuation the result makes sense. The value of the option is almost six times bigger than the net present value of the website investment. This highlights the fact that the option to expand is rather lucrative – at least in comparison to the net present value of the first-phase investment.”

CEO of the company

5 CONCLUSIONS

5.1 The case finale

The construction of this research is validated if the investment valuation will be taken into use and if it will be used a tool to reach a financing deal for the investment. The construction was partly validated. On the other hand the management of the company was satisfied with the research and the valuation as its outcome. And on the on the other hand the project was abandoned because the valuation for the investment was lower than previously anticipated. The project was abandoned because the outcome wasn't as encouraging as was previously anticipated.

The process of valuation was easy enough to understand and the management felt that all the hard questions were asked. Actually, when the management was tasked to outline both good and bad cases in both phases of the project the reality and the hardness of the business plan started to come to light. The amounts of art that should be sold to reach decent profitability were seen hard to attest and showcase to potential investors.

And on the other hand the company took the valuation into use and based its decision on the research. This is the most important validator of the research. The real option model gave value for the market potential and when the numbers were crunched it also showed the reality of business plan. The process itself was also emphasized a great deal during the final conversation regarding the research: the process showed the weak spots and how much the volatility of each variable affects the outcome. The Excel tool for the management was many times used with different numbers just to see the value in each step of the business plan.

5.2 End notes and further research

The suitability of the Real Options Analysis for the investment valuation has been criticized recently due to the possibilities for the management to inflate the valuation. This is understandable: the option can described so that almost every investment looks profitable when there is a convenient option or several options attached to it. Particularly in terms of accounting the ROA approach has been criticized in a paper commissioned by the Finnish Employment and the Economy (Anttonen et al., 2011). This criticism further underscores the need for ROA usage to be studied in Finland for startup companies and investments in general.

A necessary further research would be multiple case study. The research should have different types of firms and their financial information would be disclosed as needed for the research. It would be beneficial to see how close to these valuations the ROA valuation comes to especially if the time period of the research is long enough for some of the firms to get actual investments. This research could compare the valuation done using ROA to the valuation assigned to the company by investors. Naturally a bigger quantitative research would be ideal to diminish the random errors in valuations and see the results from various companies from different fields of business.

Another line of research could be to use one case company with ROA and other valuation methods in order to compare the valuations and to find out where the differences originate. Afterwards, the valuation assigned by the investors could be compared to each of these valuations done by different methods.

The problems of the model are obvious but so are the answers. The possibility to inflate the investment valuation is always there. This is why it is essential for the reader to understand how the model works and analyze the numbers behind the valuation critically. Otherwise the ROA approach is no different to financial options and the mathematics behind it is similar. The trust issue is visible in Finland also in the work of auditors: at my work place we tried to appraise customer's R&D investment to the company's balance sheet but the auditors hesitated citing the law and uncertainty. Although this is understandable it doesn't mean that the state of legislature and opinions shouldn't change. There needs to be more valuations done using real options and more research. The startup companies – hailed as saviors to the economic troubles Finland is in - oblige us.

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