DEPARTMENT OF PHYSICS
UNIVERSITY OF JYVÄSKYLÄ
RESEARCH REPORT No. 2/2014

# GAUGE THEORY PHASE DIAGRAMS FROM HOLOGRAPHY

## BY
## TIMO ALHO

Academic Dissertation
for the Degree of
Doctor of Philosophy

*To be presented, by permission of the
Faculty of Mathematics and Natural Sciences
of the University of Jyväskylä,
for public examination in Auditorium FYS-1 of the
University of Jyväskylä on April 4, 2014
at 12 o'clock noon*

Jyväskylä, Finland
April 2014

# Preface

The research in this thesis has been carried out at the Department of Physics, University of Jyväskylä, during 2010-2013. The work has been supervised by Doc. Kimmo Tuominen, whom I would like to thank for the opportunity to participate in this research at the forefront of theoretical physics, and for his guidance.

I would also like thank Prof. Keijo Kajantie, who has for all practical purposes been a co-advisor of this work, for his guidance and inspiration. Much of the work in this thesis has been done in collaboration with Prof. Elias Kiritsis and Dr. Matti Järvinen, whose work has cleared the path for the studies done here, and to whom I'm grateful for their collaboration. I would also like to thank Prof. Nick Evans for the opportunity to work with him on the subject of dynamic AdS/QCD. I am grateful to Doc. Esko Keski-Vakkuri and Dr. Ari Hietanen for their valuable comments on this manuscript.

I wish to thank Prof. Jan Rak for first introducing me to physics research. I want to thank the whole Department of Physics for the exceptional atmosphere both in terms of physics and practical everyday matters. In addition to the faculty, who have provided an inspirational learning environment, I would like to thank the office staff who make things "just work". My fellow physics students still at the department, and those who have already moved on to other jobs, known as the Holvi Collaboration, I would like to thank for a lively social life, exciting conference cruises, and for countless discussions, some of which have even concerned physics.

Finally, I would like to thank my family and my friends for their love and support. Last but not least, but I would like to express my love and gratitude to Minna, who lovingly but sternly played no small part in making sure that I finish the final stretch of writing this thesis efficiently and on time.

This thesis has been supported by the Vilho, Yrjö and Kalle Väisälä foundation, the University of Jyväskylä, the Department of Physics, the Helsinki Institute of Physics, the Graduate School in Particle and Nuclear Physics GRASNPANP/PANU and the ESF Holograv Programme.

# Abstract

This thesis consists of an overview and four research papers. The subject is the application of gauge/gravity dualities to strongly coupled quantum field theories, and especially thermodynamical computations in a class of bottom-up holographic models.

In the overview section, we first briefly review the elements of gauge/gravity dualities. We then present in some detail a simple model, dynamic AdS/QCD, where the effect of fermions on a holographic system is input via the running of the quark anomalous dimension. We find that this is enough to reproduce all of the common lore concerning walking technicolor theories.

We then review the Improved Holographic QCD (IHQCD) model for Yang-Mills theories, and present a model of a quasi-conformal quantum field theory based on it. We compute the phase diagram of the model, its particle spectrum, and finally study the melting of the bound states via the spectrum of quasinormal modes at finite temperature.

Finally, we review Veneziano QCD (VQCD), which extends IHQCD with an explicit model for fermions, and present methods for computation of the phase diagram and various thermodynamic quantities in the model. The phase diagram is computed at both finite temperature and finite chemical potential, where we find a phase diagram broadly in line with expectations from field theory approaches. We also briefly touch upon the zero temperature finite density structure of the theory, although a more detailed investigation is left for later.

Overall, we find that the holographic method gives results that are quantitively in line with those from other approaches to quantum field theory, while allowing more tractable calculations in the strong coupling regime. This gives confidence that while the holographic approach is not a controlled approximation, it will be able to give significant insights to strongly coupled field theories. A more detailed matching of the potentials in VQCD, together with a detailed investigation of its zero temperature structure, is left for future work.

# List of publications

This thesis is based on the work contained within the following publications:

I J. Alanen, T. Alho, K. Kajantie, and K. Tuominen, "Mass spectrum and thermodynamics of quasi-conformal gauge theories from gauge/gravity duality", Phys.Rev. **D84**, 086007 (2011), arXiv:`1107.3362 [hep-th]`.

II T. Alho, M. Järvinen, K. Kajantie, E. Kiritsis, and K. Tuominen, "On finite-temperature holographic QCD in the Veneziano limit", JHEP **1301**, 093 (2013), arXiv:`1210.4516 [hep-ph]`.

III T. Alho, N. Evans, and K. Tuominen, "Dynamic AdS/QCD and the Spectrum of Walking Gauge Theories", Phys.Rev. **D88**, 105016 (2013), arXiv:`1307.4896 [hep-ph]`.

IV T. Alho, M. Järvinen, K. Kajantie, E. Kiritsis, C. Rosen, and K. Tuominen, "A holographic model for QCD in the Veneziano limit at finite temperature and density", (2013), arXiv:`1312.5199 [hep-ph]`, to appear in JHEP.

In [I] the author did most of the numerical work and all of the numerics for computing the quasi-normal modes, including the development of the method described in Appendix B of the paper. He also participated in the interpretation of the results and in the writing of the paper.

In [II, IV], the author participated in deriving the equations of motion and boundary conditions in a form amenable to numerical methods, wrote the numerical code which allowed semi-automated computation of the phase diagrams, and did most of the numerical work. The author discovered, together with M. Järvinen, the existence of a non-analytic horizon structure in the full background solutions, at zero temperature and finite chemical potential. The author had a leading role in the interpretation of the results and participated in the writing of the papers.

In [III],the author participated in the numerical work, interpretation of the results and in the writing of the paper.

# Contents

# Chapter 1

# Introduction: Nature and strongly coupled field theories

*"What do we know . . . of the world and the universe about us? Our means of receiving impressions are absurdly few, and our notions of surrounding objects infinitely narrow. We see things only as we are constructed to see them, and can gain no idea of their absolute nature. With five feeble senses we pretend to comprehend the boundlessly complex cosmos, yet other beings with wider, stronger, or different range of senses might not only see very differently the things we see, but might see and study whole worlds of matter, energy, and life which lie close at hand yet can never be detected with the senses we have."*

–H.P. Lovecraft, "From Beyond", 1920–

## 1.1 An overview of the fundamental theories

Since Lovecraft's time, we have been able to extend our understanding of nature immensely beyond that immediately available from our natural senses. The fundamental structures that give rise to the world around us have been explained to astonishing precision by physics. Our everyday technology is a testament to our success in understanding the structure of matter in terms of quantum mechanics and atomic theory. The results from collider experiments show that in terms of individual particles the understanding is very detailed, and the matching of particle cosmology, such as big bang nucleosynthesis, to astronomical observations tells that even at very large scales our understanding is accurate. We have strong reasons to believe that fundamentally all of this is nothing but elementary particles and their interactions.

Perhaps the most obvious force in our everyday life is gravity. The physical theory which describes gravitation well enough to explain why the reader of this text isn't currently floating freely, how planets, stars, galaxies and larger structures of the universe are formed, and how all of this came about from a near-singularity at the beginning of time, is called the general theory of relativity (GR). Whereas all other fundamental theories of physics are descriptions in terms of quantum fields propagating and interacting in a background spacetime, general relativity is a theory of the structure of spacetime itself. Since all the areas of application for GR this far concern objects that are very large compared to the elementary particles, it is not necessary for the description to be quantum mechanical. Probing the quantum mechanical properties of gravity is expected to require energies on the order of the Planck scale, which is an energy scale at which the strength of gravity becomes comparable to that of the other fundamental forces. The Planck scale is about $10^{15}$ times greater than the highest energies achieved in particle accelerator experiments this far. In GR, at the semi-classical level, spacetime is assumed to be a passive background to the quantum field theories describing elementary particles at scales that are small but still much larger than the Planck scale, and then the classical equations of GR determine how all these small areas of space join together to form the larger whole. As the theory is not quantum mechanical, these equations are (partial) differential equations, for which approximation methods are known. Therefore, while the practical difficulties can be formidable, solutions for GR can in principle be found to any desired accuracy, if sufficient computing power is available.

The fundamental interaction from which the structure and behavior of matter arises, from the atomic scales upwards and in the background dictated by gravity, is the electromagnetic interaction. At such scales, the atomic nuclei can be considered simply as a class of charged particles coupled to the electromagnetic field. Electromagnetism is described in physics by a quantum field theory, known as quantum electrodynamics (QED), which is a quantum mechanical description of charged particles that obey a special type of symmetry known as an abelian gauge symmetry. At the fundamental level, chemistry is then simply the quantum electrodynamics of bound states of atomic nuclei and electrons, and all properties of macroscopic matter come from the the interactions of these bound states.

Among quantum field theories, QED belongs to a category of weakly coupled field theories. This means that, at least at low energies, the theory can be described by degrees of freedom which behave almost as free particles, and it is possible to construct its solutions by a systematic expansion around the free particle solution. This method of perturbative solutions has allowed a thorough investigation of QED, and has given it the status of the theory with the most accurate match between calculation and experiment in the history of science: the match between the theoretical and experimental determination of the anomalous magnetic moment of the electron is better than 1 part in $10^9$.

Nuclear reactions are not described by QED. The inner structure of the nucleus consists of two types of particles, the electrically neutral neutron, and the positively charged proton. The proton and the neutron are composite objects themselves, consisting of two types of particles called the up quark and the down quark. The quarks are believed to be elementary particles.

The strong interactions which bind them to nucleons and nucleons to nuclei, are described by a quantum field theory called quantum chromodynamics (QCD). The difference to QED is that now the gauge symmetry has been generalized to a non-Abelian symmetry, which technically says that the order of the various symmetry operations do matter, unlike in the Abelian case. This leads to the analog of the electromagnetic field in QCD interacting with itself.

In addition to QCD, an examination of the nucleus reveals another class of interactions, also described by a theory based on a non-Abelian gauge symmetry, although a different one than in QCD. This so called weak interaction concerns both the quarks and the electrons, and requires the existence of another particle, the neutrino. The interaction is called weak, since at long ranges its effects are exponentially suppressed, and even at nuclear scales it is less significant than the strong force.

In terms of its consequences, QCD is very different from QED: the fact that the quarks are confined into nucleons (or more generally, hadrons, all of which except for the nucleons are unstable and decay to stable particles), at the low energies which concern ordinary matter, means that the degrees of freedom that are directly observable are nothing like the solutions of the corresponding free field theory. Therefore it does not make sense to build up solutions to QCD by computing corrections to the free field theory, as none of the corrections need to be small, at least at energies where hadrons are bound.

However, in QCD the coupling constant decreases as a function of the interaction energy. Such a theory is called asymptotically free. In high energy collisions, this means that perturbation theory can actually be used to describe the process in terms of quarks and the force carrying gluons, and our current strong belief in QCD as the correct description of hadrons and their interactions relies mainly on this. The mapping between the hadrons and the elementary particles still has to be fixed by a separate measurement, but once that is done at a single energy scale, the evolution of the mapping as a function of energy can be derived from QCD.

Methods for describing directly the strongly coupled, low-energy part of QCD, are needed for a more complete understanding. One such method, which can describe static quantities such as hadron masses, lifetimes, thermodynamics and so on, at the expense of immense computational effort, is lattice field theory[1]. Another method for modeling the low-energy features of QCD are effective field theories, which model the low energy part of the interactions by constructing field theories with the appropriate symmetries and matching the necessary coefficients to experiment.

In light of all this, it is clear that our understanding of nuclear interactions, and therefore the fundamental structure of ordinary (and unordinary!) matter is mainly limited by our capability to do calculations in strongly coupled quantum field theories. The subject of this thesis is a method for solving certain strongly coupled theories exactly: gauge/gravity dualities. The approximation in using

[1]Lattice field theory can, in principle, be also extended to dynamics, but no concrete implementations exist at the moment

this method is that while a strong coupling theory is solved, it is not possible in general to control exactly which theory that is. When modeling QCD-like theories, this makes it a non-controlled approximation, since the quantitative results do not carry guaranteed error limits with them. Nonetheless there is reason to believe that qualitative features may be well represented, and it turns out after the fact that even the quantitative match is better than would be naively expected. In addition to their usefulness as computational tools for strongly coupled quantum field theories, gauge/gravity dualities have immensely interesting connections to string theory, which is a theory that combines gravity and particle physics into a unified theory of quantum gravity and particle interactions

## 1.2 The Standard Model of particle physics

The Standard Model of particle physics describes the interactions of the elementary particles by a quantum gauge field theory with the symmetry group $U(1)_{\mathrm{Y}} \times SU(2)_{\mathrm{W}} \times SU(3)$. Fermion fields describe the matter particles. A complex scalar field spontaneously breaks the electroweak $U(1)_{\mathrm{Y}} \times SU(2)_{\mathrm{W}}$ to its $U(1)_{\mathrm{EM}}$ subgroup, which is responsible for electromagnetism [1]. The remaining unbroken $SU(3)$ is the gauge group of QCD.

There are three matter generations, which are identical except for their masses. Before considering the Higgs vacuum expectation value, the fermions are all massless, so they have well defined chirality. This allows for the $SU(2)$ part of the electroweak gauge group to act only on the left-handed components. All the left handed fermions are $SU(2)$ doublets, and all the right handed fermions are singlets. All the quarks are in the fundamental representation (anti-fundamental for the anti-quarks) of the $SU(3)$ of QCD, which confines them to hadrons. The gauge bosons of QCD are the eight gluons.

The Higgs field is a complex scalar weak doublet. The Higgs potential has an unstable local maximum at zero value of the field, and a stable minimum at a non-zero expectation value. This breaks the electroweak symmetry, making the electroweak gauge bosons massive expect for a certain combination of one of the $SU(2)_{\mathrm{W}}$ generators, say $\tau^3$, and the gauge boson of $U(1)_{\mathrm{Y}}$. This combination is the photon of electromagnetism, which generates an unbroken symmetry in the stable vacuum. Two of the bosons corresponding to broken generators, $W^{\pm}$, are electrically charged, with charge $\pm 1$ respectively, and one, the $Z^0$, is electrically neutral.

Expressing the interaction terms between the fermion fields and the photon in terms of the eigenstates of $\tau^3$ gives the electric charges of the fermions in the stable vacuum. The eigenvalues of the doublet states are[2] $\pm\frac{1}{2}$, and that of the singlet states is 0. Combining this with the $U(1)_{\mathrm{Y}}$ hypercharge assignments given

[2]The choice of normalization is here such that the electric charge operator is $Q = \tau_3 + Y$, where $Y$ is the hypercharge. The normalization $Q = \tau_3 + Y/2$ is also commonly used, as then the charges of the leptons are integers.

in table 1.1 leads to the electric charges observed in nature. The left-handed

| Particle | $SU(3)$ representation | $SU(2)_{\mathrm{W}}$ representation | $U(1)_{\mathrm{Y}}$ hypercharge |
|---|---|---|---|
| $L_L$ | $\mathbf{1}$ | $\mathbf{2}$ | $-\frac{1}{2}$ |
| $Q_L$ | $\mathbf{3}$ | $\mathbf{2}$ | $\frac{1}{6}$ |
| $e_R^-$ | $\mathbf{1}$ | $\mathbf{1}$ | $-1$ |
| $u_R$ | $\mathbf{3}$ | $\mathbf{1}$ | $+\frac{2}{3}$ |
| $d_R$ | $\mathbf{3}$ | $\mathbf{1}$ | $-\frac{1}{3}$ |

Table 1.1: Gauge group representations and hypercharge assignments for the first generation SM fermions.

electron $e_L^-$ and neutrino $\nu_L$ are the $-\frac{1}{2}$ and $+\frac{1}{2}$ components, respectively, of the lepton doublet $L_L$, and the left-handed up-quark $u_L$ and down-quark $d_L$ are the $+\frac{1}{2}$ and $-\frac{1}{2}$ components, respectively, of the quark doublet $Q_L$. For all of these states except the neutrino, there is a corresponding right handed $SU(2)_{\mathrm{W}}$ singlet state. Their electric charges are the same as their hypercharges. Note how the Higgs mechanism allows the left handed electron and neutrino to have different electric charges even though they are different states of the same particle: the symmetry breaking identifies them by their $\tau^3$ eigenvalues. The same holds for the up and down quark.

At this stage, all the fermions are still massless. Adding explicit fermion mass terms is not possible due to the fact that the weak interactions are chiral. The masses are added by introducing Yukawa interactions between the fermions and the Higgs boson, which become mass terms proportional to the Higgs vacuum expectation value. Each of the fermions have different Yukawa couplings, which gives them their observed masses. The couplings are also different between each fermion generation, breaking their degeneracy.

The resulting theory is in excellent agreement with experimental tests, and remains so after the discovery of the Higgs boson in the LHC, with no signs of discrepancy between theory and collider experiments so far. There are, however, some theoretically unsatisfactory features about the Standard Model, and taking cosmological observations into account, it is clear that the Standard Model must be extended in order to explain all of the experimental data.

## 1.3 Beyond the Standard Model

The Standard Model of particle physics is compatible with all known precision measurements. However, we know with certainty that this is not the full story. On the observational front, cosmology tells us that most of the mass of the universe is non-baryonic and electromagnetically non-interacting, known as dark matter, and most of the energy density in the universe is dark energy [2]. The Standard Model has no particle state corresponding to dark matter, and it predicts either zero or order $10^{76}\mathrm{GeV}^4$ [3] vacuum energy density, depending on whether one considers the zero-point energies of the fields to contribute to gravity or not. It is then clear that Standard Model must be extended to explain these phenomena[3].

From the theory point of view, the Standard Model is not completely satisfactory. For example, its structure, with three generations of leptons and quarks, only left handed neutrinos and a non-simple gauge group, seems ad hoc. Is there a deeper explanation than simply compatibility with measurements, that would explain this pattern? The mass of the Higgs particle, since it is a scalar, is subject to renormalization with terms proportional to the square of the cutoff. In the Wilsonian view of renormalization the bare mass is to be set at the highest accessible scale, *i.e.* the Planck scale, and so the Higgs particle gains huge corrections to its mass. This requires a a very fine tuning of the mass at Planck scale in order to achieve the observed mass at low energies. This is known as the hierarchy problem.

During the last four decades, there has been an immense theoretical effort to find answers to these and many other questions posed by the Standard Model. These efforts have produced a large array of models "Beyond the Standard Model". Among them are supersymmetric extensions of the Standard Model, variations of theories with a composite Higgs, and attempts at grand unification which would explain the electroweak and strong forces via symmetry breaking in a larger theory. With the first two years of LHC running, a number of these models have been ruled out or at least severely constrained [4]. However, there are many models which still remain viable, many of them with either fine tuning of the parameters, or due to the fact that theoretical calculations are not well under control, mostly due to strong coupling.

A proposed class of extensions of the Standard Model which purports to solve the hierarchy problem, and also has dark matter candidates, is technicolor. Technicolor stems from the observation that the Higgs mechanism of spontaneous symmetry breaking has been seen before, in the theory of superconductivity. The Ginzburg-Landau effective field theory approach describes superconductivity as the spontaneous symmetry breaking of the gauge symmetry of electromagnetism by a complex scalar field, quite analogously to the Higgs mechanism. In superconductivity, a microscopic explanation for the symmetry breaking is the Bardeen-Cooper-Schrieffer theory of superconductivity, where superconductivity results from pairing of electrons into massive bound states. The scalar field of the Ginzburg-Landau theory can be identified with the electron pair condensate. It is therefore natural to assume that the Higgs mechanism is simply a low-energy description of a BCS -like formation of a condensate which breaks the electroweak symmetry.

A similar mechanism actually already exists in the Standard Model without the Higgs mechanism, as the chiral condensate of QCD already spontaneously breaks the electroweak symmetry. The breaking is, however, too weak, as the

[3]For the dark energy problem it is possible that some understanding of quantum gravity will also be needed

induced gauge boson masses are smaller by three orders of magnitude compared to the experimental values. The original technicolor then postulated another QCD-like theory, which would have a larger value of the chiral condensate and couple to the electroweak sector in the same way as QCD. The quarks of this theory are called techniquarks. In the Standard Model, the Higgs field also generates the fermion masses. In technicolor models, an extended technicolor (ETC) theory, which adds yet another gauge group under which both the ordinary quarks and the techniquarks are charged, is usually introduced for this purpose. At low energies, this can be described by a four-fermion interactions, and the term with two techniquark fields and two ordinary quark fields would play the role of a mass term, due to the existence of the techniquark condensate.

This simple model is in conflict with observations. In order to give a large enough mass to the heaviest quark, the top quark, the scale related to ETC cannot be too high. The limit turns out to be too low, as the effects of the ETC theory in terms of flavor changing neutral currents would already have been observed.

The way out of this is to tune to the technicolor theory such that its renormalization group flow takes it very near, but not quite to, an infrared fixed point. This enhances the value of the techniquark condensate, allowing larger masses while the ETC scale can remain large and in accord with the experimental constraints. Also the near-conformality of the field theory helps reduce the S-parameter, which describes deviations from the Standard Model, and which is experimentally known to be small. Since the renormalization group flow near the would-be fixed point is very slow, *i.e.* the condensate walks instead of running, these are called walking technicolor theories. [5–7]

The difficulty in analyzing such theories stems from the fact that the essential features, such as confinement of the techniquarks and the formation of the chiral condensate, are all strong coupling dynamics. In the case of QCD, access to experimental results allows gaining intuition about the inherently strong coupling features in order to study the theory and build effective models. That this is not available for technicolor makes it difficult to compute the exact consequences of the models. In lattice simulations of near-conformal theories, the extremely slow evolution of the coupling constant makes extrapolation to the continuum limit challenging [8–18]. This is why all methods applicable to strong coupling physics are very valuable for technicolor phenomenology, and as such we will often in this thesis consider the results obtained from the holographic models in light of technicolor models.

## 1.4 Gravity

General relativity generalizes the concept of Minkowskian spacetime in special relativity to allow curvature, resulting in a Lorentzian manifold. Generalizing the concept of a free particle to mean an object which moves along the geodesics of spacetime, gravitation is found to be a fictitious force resulting from the curvature of that spacetime.

This explains the effects of gravity on matter, but it does not explain how matter affects the gravitation, *i.e.* the curvature of spacetime. That connection is given by the Einstein field equations,

$$R_{\mu\nu} - \frac{1}{2} R \; g_{\mu\nu} + \Lambda \; g_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}, \tag{1.1}$$

where $R_{\mu\nu}$ is the Ricci tensor, $R$ is the Ricci scalar, $g_{\mu\nu}$ is the metric tensor, $\Lambda$ is the cosmological constant, and $T_{\mu\nu}$ is the energy-momentum tensor. These equations can be derived from the Einstein-Hilbert action

$$S = \int \mathrm{dx}^4 \sqrt{-\mathrm{g}} \left[ \frac{\mathrm{c}^4}{16\pi \mathrm{G}} (\mathrm{R} - 2\Lambda) + \mathcal{L}_{\mathrm{m}} \right], \tag{1.2}$$

where $\mathcal{L}_m$ is the Lagrangian from which the energy-momentum tensor $T_{\mu\nu}$ results.

With the Einstein field equations, given a classical energy-momentum tensor for the system, the gravitational problem is fully determined, even though solving the equations can in practice be very involved. The story of theoretical gravity does not end here, however. First of all, the Einstein field equations predict their own failure: they generically have solutions where the energy density diverges and the spacetime becomes singular, such as is the case in the Schwarzschild solution. The Penrose-Hawking singularity theorems effectively guarantee that our universe contains areas where the Einstein field equations predict singularities, so they cannot be banished by simply decreeing that they are not physically acceptable [19]. The second major problem with GR is that it is incompatible with quantum field theory, if the quantization is carried out by any of the well-known methods. This is manifested as non-renormalizability of gravity when it is quantized canonically as a field theory in the Arnowitt-Deser-Misner (ADM) formalism [20].

The two problems above are most likely connected: whenever singularities tend to form, the volume of spacetime where the curvatures are large is small, and in such short scales quantum effects should be significant for the gravitational field. These are precisely the same effect which are non-renormalizable in the naive quantization of gravity. Therefore, it is generally expected that in a correct quantum theory of gravity these singularities are softened or even eliminated.

The quest for a consistent quantum theory of gravity has turned out to be longer and more demanding than expected. Approaches beyond gravity as a quantum field theory have included loop quantum gravity, which is in essence a canonical quantization of gravity by taking background indepence as a starting point, non-commutative geometry, which considers the geometry to have a quantum uncertainty and causal dynamical triangulations, which attempts to explicitly construct a model of spacetime from more primitive building blocks. The specific theory of quantum gravity that we are concerned with here is also the most well studied: string theory.

## 1.5 String theory

In the Standard Model of particle physics, the elementary objects are point particles. While they are described by a theory quantized fields, the point particle interpretation is deeply embedded, for example via the Fock space representation of the field theory Hilbert space via single particle states. The string is considered to be the elementary object of the theory. In its simplest form, the action of a single string is taken to be its surface area that it sweeps out as it moves in spacetime. This is the Nambu-Goto action:

$$S_{\mathrm{NG}} = -T \int \mathrm{d}A, \tag{1.3}$$

where T is the string tension, which is in principle the only undetermined parameter of the theory, and $\mathrm{d}A$ is a differential surface area of the string world-sheet with respect to the metric of the spacetime in which the strings move.

The degrees of freedom in classical string theory in $D$ spacetime dimensions are the $D$ spacetime coordinates of the string world sheet, parametrized by the two coordinates on the world sheet. Quantizing this theory leads to a 2 -dimensional conformal quantum field theory for the $D$ -scalar fields. A detailed analysis of such a theory leads to the conclusion that in order for the Lorentz algebra to close, $D$ must be 26. Unfortunately, in such a theory there is a tachyon in the spectrum of the string vibrations, which cannot be consistently decoupled, leading to an instability of the theory. Also, there are no fermions in the spectrum of spacetime excitations, which is in conflict with observations.

Both of these problems can be cured at once by supplanting the $D$ bosonic fields with $2D$ fermionic fields, which combine to form supergravity multiplets on the world-sheet. This allows projecting out the tachyonic states, and introduces states which are spacetime fermions. The critical dimension, where Lorentz symmetry is preserved on the quantum level, is now 10, and in this case, it is actually enhanced to supersymmetry.

The various modes of oscillation give a lot of structure to string theory. It turns out there are five different theories in which the tachyons can be projected out. In all string theories, the low-energy spectrum of closed string excitations contains a spin-2 boson, which gives rise to General Relativity in the classical limit. Therefore string theory is a candidate for not only a unified theory of particle interactions, but also a theory of quantum gravity.

The various types of string theories are connected by dualities. The current consensus is that the dualities are actually strong enough to suggest that they are all limits of a single theory in 11 dimensions, called M-theory. As the string theories themselves lack a non-perturbative definition, there is no general definition for M-theory.

# Chapter 2

# Gauge/gravity dualities

> *"Men of broader intellect know that there is no sharp distinction betwixt the real and the unreal; that all things appear as they do only by virtue of the delicate individual physical and mental media through which we are made conscious of them; but the prosaic materialism of the majority condemns as madness the flashes of super-sight which penetrate the common veil of obvious empiricism."*

–H.P. Lovecraft, "The Tomb", 1917–

Gauge/gravity dualities are a class of conjectured equivalences between quantum field theories on flat space and theories with gravity on higher dimensional manifolds. In the best justified form of this duality the higher dimensional theory is type IIB super string theory on an $\mathrm{AdS}_5 \times S^5$ background space-time, and the conformal $\mathcal{N} = 4$ super Yang-Mills theory in 4D flat space [21]. This is called the Maldacena duality. Since the $S^5$ in the higher dimensional space-time, often called the bulk, is compact (and usually taken to be small), this duality is between a 4D theory without gravity, and a 5D theory with gravity.

In addition to the remarkable connection between two theories of different dimensionality, the correspondence of the various limits in this duality is also very interesting. The strong coupling limit of the field theory corresponds to a limit where all massive fields in the bulk theory become infinitely massive and therefore decouple. That is, the limit of the field theory which is difficult to compute, produces a considerable simplification in the bulk theory. Furthermore, taking the number of colors in the field theory to infinity corresponds to the classical limit of the bulk theory, allowing the computation of full quantum field theory observables by solving a classical theory.

This is of course not only very interesting from the theoretical point of view, but also as a computational tool for strong coupling calculations in field theory. Unfortunately, the $\mathcal{N} = 4$ super Yang-Mills in the Maldacena duality is rather removed from the Standard Model, and also from plausible extensions of it. However, while the details of the duality are intimately connected to string theory, supersymmetry and conformality, its overall structure is not. As a matter of fact, one can write down the so called master formula for any theory in a space with a boundary at infinity and use it to formally define another theory on that boundary, provided the quantum numbers of operators on the boundary and fields in the bulk match. Of course, the question of whether the boundary theory thus defined is consistent, and especially whether the duality works the other way around in defining the bulk theory given the boundary, is very involved. These issues may well spoil naive generalizations of the duality. There are, however, indications that holography might generalize to beyond string theory, and rather be a general property of theories of quantum gravity [22], as also suggested by the the rather general arguments behind the holographic principle. See also [23] for an approach to generating a 5D holographic formulation of QCD from the worldline formalism of the field theory description.

Independent of theoretical attempts to justify dualities in more general settings than the Maldacena duality, there have been two closely related very active research programs: for one, trying to modify the string theory side such that the arguments behind the original duality can be more or less applied while introducing phenomenologically more realistic ingredients, such as breaking supersymmetry and conformality, to the field theory. This is called the top-down approach. The other approach has been to take the master formula and the holographic dictionary from the Maldacena duality, and apply it to bulk theories chosen by fiat to reproduce some desired features in the boundary field theory, and leaving the justification of this procedure to the results. This is called the bottom-up approach.

In the rest of this chapter we will introduce some of these concepts in more detail (see for example [24, 25] for pedagogical introductions to string theory and AdS/CFT, respectively), and in the last section we will present a specific bottom-up model which has also been the subject of the work done in paper [III] of this thesis.

## 2.1 The holographic principle

A precursor to gauge/gravity dualities is the holographic principle of quantum gravity. In its more restricted form, the holographic principle says that a black hole can be entirely described in terms of the degrees freedom on its horizon. In a more general form, it claims that quantum gravity and all quantum field degrees of freedom in a general spacetime volume can be described by a theory on the boundary of that volume [26].

The roots of the holographic principle go back to the black hole information paradox. According to Hawking's famous semi-classical quantum gravity argument [27, 28], black holes emit Hawking radiation, which is thermal radiation with an almost black body spectrum. As the intensity of the Hawking radiation is inversely proportional to the mass of the black hole, this means that a black hole in vacuum will eventually evaporate to nothing but black body radiation. This poses a problem for unitarity in quantum gravity: suppose a particle in a pure state enters a black hole. The black hole gains a certain amount of mass, and continues its evaporation. If no more particles enter the hole, it will eventually evaporate entirely, leaving only thermal radiation behind. The original pure state has now been transformed into a mixed state, which has destroyed information and broken unitarity [29]. Which is wrong: Hawking's argument, or the very structure of quantum mechanics when applied to quantum gravity?

Attempts to resolve the paradox lead to the conjecture of black hole thermodynamics [30, 31], according to which a black hole is a thermodynamic state with an associated entropy and temperature. Furthermore, the entropy of a black hole must be proportional to its horizon area rather than its volume. Interpreting the entropy as the amount of information needed to describe the black hole, this means that the number of degrees of freedom in the black hole must also be proportional to the horizon area, and therefore the black hole must be describable by a theory referring only to its surface.

Now consider a lump of matter that is just about to fall into a black hole, and enclose both the lump and the black hole with an imaginary surface. If the rest of the enclosed volume is empty, one can just wait for the matter to cross the event horizon, and then shrink the surface to touch the new event horizon everywhere. Then by the above argument the black hole, which now includes the lump of matter, can be described by a theory on the imaginary surface. However, if the full theory describing the black hole and the lump of matter together is unitary, then we could time-evolve the theory in reverse to recover a description of the black hole and the lump of matter on the original enclosing surface. Since this thought experiment did not use any properties of the lump of matter itself, it seems that it should extend to any system. Therefore a theory which unifies quantum gravity and quantum field theory should be describable overall by a theory on a surface rather than a volume. This is the holographic principle.

The above argument is of course of the handwaving kind, and while the original arguments by t'Hooft, Susskind and others are less so, they are by no means rigorous [26, 32]. They do not give an explicit construction for the holographic theory nor do they prove its existence. The first concrete definition for a holographic theory came from a setting rather different to black hole physics in Einstein gravity: a string theory in an $\mathrm{AdS}_5 \times S^5$ background was suggested to have a holographic description in terms of the four-dimensional $\mathcal{N} = 4$ super Yang-Mills theory. This is the Maldacena duality [21].

## 2.2 The Maldacena duality

Among the solutions of the low-energy theory of string theory, supergravity, there are $p$-branes, which are solutions that are singular along a $p$-dimensional subspace of spacetime. A 0-brane is a point singularity, a 1-brane is a line singularity, a 2-brane is a membrane (from which the term $p$-brane is derived) and so on.

It turns out that these same objects appear as endpoints for open strings with Dirichlet boundary conditions.

In addition to appearing in the low-energy approximation, $p$-branes can also be identified in the full string theory as Dirichlet-branes, which are endpoints of open strings. Due to this identification, they are called D$p$-branes. D$p$-branes generate gauge theories very elegantly, essentially via a refinement of the Kaluza-Klein mechanism. Consider a stack of N coincident Dp-branes as endpoints of open strings. The ends of the strings appear as point-like particles from the viewpoint of the branes, while the strings themselves are free to move in the higher dimensional bulk spacetime. The endpoints of the strings on the brane carry $SU(N)$ charges, whose electrodynamics are described by the Dirac-Born-Infeld (DBI) -action. The DBI -action gives a non-linear generalization of Yang-Mills theories, and reduces to the ordinary Yang-Mills action at small field strengths. Therefore, at the low energy limit, the resulting theory becomes the dimensional reduction of $D = 10$, $\mathcal{N} = 1$ $U(N)$ super Yang-Mills theory to $p$-dimensions.

This leads to a justification of the Maldacena duality in the following way [24]: Consider $N$ parallel coincident D3-branes in Type IIB string theory, where 5 dimensions are compactified on a sphere. As mentioned above, the low-energy theory of the open strings on the brane is a supersymmetric gauge theory on the world volume of the brane. In this case, the dimensional reduction leads to $\mathcal{N} = 4$ super Yang-Mills in the four spacetime dimensions of the D3-brane. The low-energy limit of the closed string degrees of freedom is the free type IIB supergravity. Furthermore, on the low energy limit these two degrees of freedom become decoupled, and therefore the whole system is described by the $\mathcal{N} = 4$ super Yang-Mills and the supergravity theory, with no interactions between them.

On the other hand, one can take the low energy limit first and get directly type IIB supergravity with the p-branes embedded in the bulk. Considering the low-energy, *i.e.* long wavelength, spectrum of excitations around this background, it turns out that excitations which start out in the bulk do not see the brane, since it is much smaller than their wavelength. On the other hand, excitations which are near the brane are redshifted to long wavelength from the point of view of an observer at infinity. These excitations are separated from the bulk by a potential barrier, and become confined to the near-brane volume. In that volume the metric becomes $\mathrm{AdS}_5$ times the compact $S^5$ with the brane at the boundary at infinity of the $\mathrm{AdS}_5$. Note that from the point of view of a near-brane observer, the excitations can be of arbitrarily high energy. Therefore in this view we have again two decoupled theories, one of which is the type IIB supergravity, and the other is the full type IIB string theory but on an $\mathrm{AdS}_5 \times S^5$ background.

In order for these two limits to be consistent with each other, it is necessary that the resulting theories are the same. Since both exhibit a free type IIB supergravity in flat space, the remaining parts, type IIB string theory on $\mathrm{AdS}_5 \times S^5$ and $\mathcal{N} = 4$ super Yang-Mills must also be equivalent. Since at the limit the brane is at the boundary of $\mathrm{AdS}_5$, the spacetime background of the gauge field theory is identified with that boundary.

Note that while the inspiration for the duality comes heavily from string theory physics near D3-branes, the branes do not appear in the final conjecture. The statement is simply that the string theory on the $\mathrm{AdS}_5$ background is equivalent to the field theory on a lower dimensional flat spacetime, which is conformally isomorphic to the boundary of the $\mathrm{AdS}_5$.

There are many features of this duality that can be immediately checked for consistency. The most obvious are global symmetries. The SYM theory has an unbroken conformal symmetry in four dimensions, which is isomorphic to $SO(2,4)$. On the other hand, the $\mathrm{AdS}_5$ background of the string theory is symmetric under the same group. The $S^5$ part of the background has an $SO(6)$ symmetry, which matches up with the R-symmetry of the field theory. The remaining symmetries are the supersymmetries, and both sides have 32 supercharges, completing the matching of the symmetry groups.

To go further, an operational definition of the duality is needed in order to compute quantities on both sides of the duality and compare them. Such a definition is given by the master formula and the holographic dictionary [33, 34]:

$$\langle \exp \int \phi_0 \mathcal{O} \rangle_{SYM} = \mathcal{Z}\left[\lim_{z\to 0} z^{\Delta-4}\phi(\vec{x}, z) = \phi_0(\vec{x})\right]_{\text{String}}. \tag{2.1}$$

On the left side, the expectation value is with respect to the super Yang-Mills theory path integral, $\mathcal{O}$ is an operator in that theory, and $\phi_0$ is an external field sourcing that operator. On the right side there is the generating functional of the string theory on-shell amplitudes, where $\phi(\vec{x}, z)$ is a field in the string theory bulk, $z$ being a coordinate such that the boundary at infinity of the $\mathrm{AdS}_5$ is at $z = 0$. The exponent $\Delta$ is the conformal weight of the operator $\mathcal{O}$.

Given a holographic dictionary, which identifies the string theory fields in the bulk which are dual to corresponding operators in the super Yang-Mills theory, this prescription allows computing any field theory observables given knowledge of the string theory partition function as a function of the boundary asymptotes of the fields.

### 2.2.1 The supergravity and classical limits

While the the master formula gives a concrete formulation for the duality, it is of little computational use directly: evaluating either the string theory partition function or the $\mathcal{N} = 4$ SYM path integral for arbitrary sources is an unsurmountable problem at the moment[1]. The step necessary for making headway is an analysis of the relation between the couplings of the two theories.

An analysis of the DBI -action shows that the string coupling $g_s$ is proportional to the square of the Yang-Mills coupling. The constant of proportionality depends on the normalization of the Yang-Mills generators, but with a certain often used choice we have

$$g_{\mathrm{YM}}^2 = 4\pi g_s. \tag{2.2}$$

[1]Although it is conjectured that $\mathcal{N} = 4$ SYM may actually be an integrable theory [35, 36].

From the near-brane -limit we can read off the radius $L$ of the $\mathrm{AdS}_5$ as

$$L^4 = 4\pi g_s N \alpha'^2, \tag{2.3}$$

where $N$ is the number of branes on the stack, and $\alpha' = l_s^2/2$, where $l_s$ is the fundamental string scale.

From Eq. (2.2) we see that the 't Hooft coupling $\lambda = g_{\mathrm{YM}}^2 N$ is just

$$\lambda = 4\pi g_s N, \tag{2.4}$$

and combining with Eq. (2.3) we get

$$\lambda = \frac{L^4}{\alpha'^2}. \tag{2.5}$$

These relations show very interesting behaviour at certain limits: let us first of all take the limit $N \to \infty$, while keeping the 't Hooft coupling fixed. Eq. (2.4) then shows that this leads to $g_s \to 0$. That is, the string loops are suppressed, which means that the string theory becomes classical. Here we have one of the most striking, and useful, features of holography: the dual of large $N$, but *fully quantum mechanical*, field theory is a *classical* string theory. In addition to it's implications on the foundations of physics, it makes it at least much more tractable to evaluate the right side of Eq. (2.1), since now the problem is reduced to finding the saddle points of the action, *i.e.* solving differential equations.

On the other hand, taking the limit $\lambda \to \infty$ while keeping $L$ fixed, that is, holding the geometry unchanged, corresponds to $\alpha' \to 0$. Since the masses of the string excitations are proportional to $1/\sqrt{\alpha'}$, this leaves only the massless modes in the spectrum. This is the supergravity limit.

Combining these limits gives the most useful form of the duality from the point of view of strong coupling applications. In that case the dual to the super Yang-Mills field theory, at infinite 't Hooft coupling and infinite $N$, is classical supergravity on an $\mathrm{AdS}_5$ background. Solving classical field theories, while still technically non-trivial, is in principle well understood. In addition, for a classical solution the boundary conditions in Eq. (2.1) can be straighforwardly evaluated, yielding the expectation values of operators in the field theory dual.

### 2.2.2 Tests and applications of the duality

At the classical supergravity limit, some straightforward tests can be carried out by comparing calculations on both sides of the duality. While in general, the computations are at opposite limits, the large amount of symmetries protect some observables, allowing a reliable comparison. For example, conformal symmetry constrains the form of free field propagators in the super Yang-Mills theory. Computing these propagators from the dual supergravity description one gets precisely the required form for scalars, vectors and higher rank fields. All such tests which have been carried out have indeed found a match, thus supporting the duality at least on the classical supergravity limit.

Once faith in the duality has been established, one can begin to use it to compute field theory observables for large $N$, large $\lambda$, $\mathcal{N} = 4$ super Yang-Mills. The already classic results include the ratio of shear viscosity to entropy density, $\frac{\eta}{s} = \frac{1}{4\pi}$, which was also conjectured to be a lower limit for any field theory with a gravitational dual [37], although exceptions to this have now been found [38, 39]. The entropy and stress-energy tensors of the field theory at finite temperature are also simple to compute, and the results match with those of the Stefan-Boltzmann limit of the conformal field theory, except for a factor of 3/4, which is a strong coupling correction. While the simplicity of this correction may feel surprising, it is actually the conformal symmetry which constrains the correction to be no more than a constant of proportionality.

## 2.3 Holography and the renormalization group

From the field theory point of view the duality adds an extra dimension relative to the usual four spacetime dimensions. This dimension can be interpreted from the field theory point of view as a renormalization scale, and the radial evolution of the metric under the equations of motion as a renormalization group flow.

In the bulk space, one can identify the radial coordinate with an energy scale [40, 41]. Let $r$ be the radial coordinate in the string metric, where the boundary of AdS is at $r = \infty$. Given a local excitation on the boundary field theory, one can show from the solutions of the wave equation in the bulk that the excitation affects a transversal area of size

$$\delta x_{\parallel} \sim \frac{\sqrt{N} g_s}{r}, \tag{2.6}$$

at the spacetime volume at $r$. This corresponds to energy $E \sim r$. Consider now a four-volume at large, but finite $r_0$. This is also a valid surface where to set the boundary conditions, and according to Eq. (2.6), the relation between boundary conditions of the field theory at $r = \infty$ and at $r_0$ is that point like excitations at $r = \infty$ spread out over a region of size $\sim \frac{1}{r_0}$. Therefore the evolution of the solution along the radial direction, according to the bulk equations of motion, corresponds to a coarse graining of the full solution. This is precisely the philosophy in the Wilsonian view of renormalization, recast in a holographic form.

In the Maldacena duality the boundary field theory is conformal at the full quantum level, so the beta functions are all identically zero. In the bulk theory, this is borne out by the fact that the dilaton scalar, which is dual to the 't Hooft coupling, is a constant. In theories where the geometry in the bulk differs from pure AdS, of which we will say more in the next section, in general the bulk dilaton field is not constant in the radial direction. At radial position $r$, we can consider the dilaton field as sourcing the same operator it sources in the boundary, but at energy scale $E \sim r$. Therefore the value of the field at that position can be considered as the coupling of the operator at that energy scale. This allows identifying the beta function as

$$\beta(\lambda) = \mu \frac{\mathrm{d}\lambda}{\mathrm{d}\mu} = r \frac{\mathrm{d}\lambda}{\mathrm{d}r}, \tag{2.7}$$

where the renormalization scale $\mu$ has been identified with the coordinate $r$. In general this identification is only up to a multiplicative constant, and for non-conformal theories it is unambiguous only in the UV, although see [42] for work on clarifying this in the case of general Einstein-Dilaton theories.

As all fields in the bulk are sources for operators, their values at a finite $r$ can be considered as factors multiplying the operator at that energy scale, *i.e.* renormalizations of the coupling of that operator. Therefore to each bulk field there corresponds a beta function, with a similar definition as in Eq. (2.7).

The analogy between renormalization in field theories and holographic dualities extends even further. Just as in a field theory there are divergences related to the UV integrals, which must be regulated and renormalized, there are also similar divergences in the corresponding gravity duals. In the gravity theories, the divergences come from the boundary of the AdS, where the metric diverges. For example, in order to extract the energy-momentum tensor, which is sourced by the metric, one must subtract the divergent background metric first. This holographic renormalization can be matched one-to-one to renormalization in field theory, although the exact renormalization scheme this leads to is, unsurprisingly, not any of the usual field theory schemes. Computing the beta functions from this renormalization is of course consistent with results from the formula above.

The connection between gauge/gravity dualities and renormalization group flow in quantum field theories is of great importance both theoretically and from the point of view of constructing bottom-up models in holography, and will be a recurring theme in the following discussion of the papers [III, I, II, IV].

As a final point before getting to the core matter of this thesis, we will briefly review the ways in which the Maldacena duality can and has been extended for modeling more realistic theories.

## 2.4 Holographic modeling: top down, bottom up and middle across

While many interesting general results can be derived from the $\mathrm{AdS}_5$/CFT correspondence, and some other results can be argued to apply at certain situations for more realistic theories, from a phenomenological point of view it would be even more interesting to have a gravity dual for QCD, or QCD-like theories in the case of beyond the standard model physics. The arguments behind the AdS/CFT correspondence are however heavily based on symmetry and especially supersymmetry, so any kind of rigorous formulation for a QCD gravity dual seems to be out of reach at the moment. This has lead to holographic modeling, which attempts to embed as many features of QCD as possible in the context of holography.

In addition to interest stemming from strongly coupled field theories in particle physics, in the recent years a lively field of modeling condensed matter problems by holographic dualities has sprung up, see for example [43–45]. As the interest in holographic QCD methods has also moved toward finite baryon density, the gravity descriptions in these two applications are actually closely related, and there has been significant dialogue between these fields.

In trying to build holographic models closer to the desired phenomenology, whether that is QCD, BSM models or condensed matter, there are two main approaches. The conceptually simpler, although technically more demanding, approach is to start from what we know. That is, take the Maldacena duality as given, and generalize it only minimally by assuming that in general, string theory on an asymptotically AdS space is dual to a field theory on its boundary. Then one can try to modify the string theory background and analyze the resulting operators on the field theory boundary in order to find out the modifications that result in the boundary theory. This is known as the top-down approach.

The major downside of the top-down method is its difficulty. The analysis of various backgrounds in string theory or supergravity is technically very demanding, and typically only asymptotic results can be easily achieved. Evaluating the correlators on the boundary theory from Eq. (2.1) requires solving the equations of motion for general boundary data, which is quite difficult for the full non-linear equations of motion. Usually a large group of supersymmetries is needed in order to make the bulk theory tractable, which makes it difficult to match to boundary theories which typically only have gauge symmetry and Poincaré symmetry. All of this of course also serves to make the top-down models more predictive. See for example [46, 47] for reviews on top-down models for QCD, and [48, 49] for some examples of top-down models for technicolor theories.

The other method is to generalize the AdS/CFT duality maximally, that is, to say that any theory of gravity in the bulk is dual to a boundary theory via the master equation, Eq. (2.1). In this way we get to select the bulk theory such that it has tractable classical solutions and, by using the holographic dictionary in reverse, that the duals of the fields correspond to operators that we want to model in the boundary theory. This is called the bottom-up method.

This is of course a much bolder suggestion than the generalization needed for top-down methods. There is however some evidence that the generalization may not be entirely unfounded [22, 23, 44]. For one, the holographic principle comes only from very generic considerations of semi-classical gravity and quantum field theory. Taking this argument seriously, and further assuming that holography appears similarly as it does in $\mathrm{AdS}_5$/CFT, one is tempted to claim that any theory of classical gravity and quantum fields on an asymptotically $\mathrm{AdS}_D$ space, for which there is a completion to a consistent theory of quantum gravity, is dual to a $D-1$ dimensional quantum field theory. This would include at least any model that is a low-energy approximation to any asymptotically AdS vacuum in the string theory landscape. If there are other theories of quantum gravitation that are consistent and fulfill the holographic principle, they would generate more holographic duals for field theories.

Unless the completion of the bulk theory to quantum gravity is known, the duality is expected to break down at the UV limit of the bulk theory. In the usual application of holographic dualities the bulk theory is taken to be classical and at that limit explicit bulk quantum gravity is not needed. In the $\mathrm{AdS}_5$/CFT -case, this is manifest as a duality between classical string theory and large $N_c$ SYM -theory. A plausible generalization of this to the general case would then be that the classical limit of the bulk theory, which is well defined at the bulk UV -limit, is dual to a field theory which has an infinite number of local degrees of freedom. From a more explicit point of view, Marolf has recently argued [22] that holography can be found from essentially the quantum generalization of Gauss's law in the bulk.

Many actual bottom-up models take the approximations further, such as setting a fixed background. In this case they are not even classical theories of gravitation, and are therefore certainly not exactly dual to any field theory. The hope is however that some essential features of a field theory will be modeled.

Even if we accept the above argument about the wide-reaching generality of gauge/gravity -dualities, there are several challenges facing the bottom-up program. The first one is encountered in building the model, since the holographic dictionary only fixes the boundary asymptotics, and therefore only the near boundary action, of the bulk fields when the desired boundary operators are given. The IR part of the action must be fixed based on arguments of simplicity, desired IR behavior of the dual theory and whatever other information is available. This is of course an ambiguous procedure, and usually after all possible general knowledge has been used, we are still left with a number of free constants or even functions of the fields. The procedure then continues by computing a number of observables with known values (from other calculational methods, such as lattice or effective field theories, or experiment) and matching their values by adjusting the free parameters. After the parameters are so fixed, all new results are then predictions of the model.

The bottom-up method defines the action in the bulk, and defines the dual theory via Eq. (2.1), which gives the operators of the theory. Determining them would require a solution of the full bulk theory with arbitrary sources, and even if we could do that, we would not directly get the fields on the boundary. Therefore in bottom-up methods we usually cannot write down the action or the equations of motion of the boundary theory, and due to this they are not in any sense controlled approximations. Rather we construct the model, compute with it, and then judge its validity based on the results.

One more way to construct gauge/gravity models is a mixture of these two, which one could call a middle across method, or string-inspired bottom-up. In this case, take the overall attitude of the bottom-up -program, in that the goal is simply to construct whatever bulk action produces the correct results (by any suitable definition of correctness), but then take heed of lessons that can be learned from string theory constructions. For example, whereas most bottom-up models implement fermions by linear terms in the action, dual to boundary fermionic operators, a string inspired choice might be to instead take the DBI -action for a space-filling brane in flat space. Then simply assume that this form can be used, with some of the couplings promoted to potentials for the fields, also in curved space-time. The difference to top-down models is that the correct action for a space-filling brane in curved backgrounds is not known, and it is this ignorance which is encoded in the potentials. The primary examples of the string-inspired holographic method are Improved Holographic QCD [50–53] and its extension, holographic Veneziano QCD [54–56] [II, IV]. These models are central to papers [I, II, IV] of this thesis.

Next we will review one family of bottom-up models, called AdS/QCD[2], of which one representative is the one considered in [III] of this paper.

## 2.5 AdS/QCD Models

The original AdS/QCD model [57], see also [58–61], is a bottom-up hard wall model of low-energy QCD. We will first describe that model in some detail, and then describe the dynamic AdS/QCD model considered in [III], which elaborates on the original model by introducing a running quark anomalous dimension and a dynamic generation of the confining IR geometry.

### 2.5.1 Hard wall AdS/QCD

The basic setup is Einstein gravity with a positive cosmological constant in five dimensions, leading to the $\mathrm{AdS}_5$ background solution. The metric is then taken as a fixed background, that is, the back-reaction of the matter fields on the geometry is not considered. The operators to be modeled are the left- and right-handed chiral currents $\bar{q}_{\mathrm{L,R}}\gamma^\mu t^a q_{\mathrm{L,R}}$, and the quark condensate $\bar{q}^i_R q^j_L$, which is the chiral order parameter. Here $t^a$ is a generator of the adjoint representation of the flavor symmetry, and the indices $i, j$ denote the flavors themselves. In the 2-flavor model considered in [57], this means that $a = 1 \dots 3$ and $i, j = 1 \dots 2$.

The background metric is

$$\mathrm{d}s^2 = \frac{1}{z^2}(-\mathrm{d}z^2 + \mathrm{d}x^\mu \mathrm{d}x_\mu), \tag{2.8}$$

where $z$ is the extra-dimensional coordinate, and $x^\mu$ are the four-dimensional spacetime coordinates.

[2]This name is sometimes also used for the whole program of trying to model QCD by gauge/gravity dualities, but here we use it exclusively for the family of models deriving more or less directly from the original hard wall $\mathrm{AdS}_5$ -model [57].

According to the holographic dictionary, global symmetries on the boundary become corresponding gauge symmetries in the bulk, so the symmetry of the chiral currents is a dual to a $SU(N_\mathrm{f})$ gauge field $A^a_{L,R\mu}$ in the bulk. The kinetic term for a gauge field is, as usual, $\mathrm{Tr}F^2 = \frac{1}{4}F^a_{\mu\nu}F^{a,\mu\nu}$, and we add one for both the left- and the right-handed current. The gauge fields are then sources of the chiral current operators.

The field dual to the quark condensates must have indices $i, j$ in the fundamental representation of flavor in order for the product of the source and operator terms in the effective action to transform as a scalar. The energy dimension of the quark operator is three, whereas a massive field with a dimensionless coupling in $\mathrm{AdS}_5$ is dimensionless (the metric determinant in the action cancels the dimensions of $\mathrm{d}^D x$ in curved space). Since the boundary action is four-dimensional, the dimension of the product of the source and the operator must equal four and therefore the source should have dimension 1. Based on this, the actual source must be $\frac{2}{z}X^{ij}$, where $X^{ij}$ is a $N_\mathrm{f} \times N_\mathrm{f}$ dimensionless matrix field. The factor 2 is simply conventional.

The scaling dimension $\Delta$ of the operator dual to a $p$-form field in the bulk is related to the bulk fields mass by [33]

$$(\Delta - p)(\Delta + p - 4) = m_5^2, \tag{2.9}$$

where $m_5^2$ is the bulk mass of the field. We see that $m_X^2 = -3, m_{A^2_{\mathrm{L,R}}} = 0$ gives the boundary operators the dimensions corresponding their naive tree-level scaling dimension.

Giving the field $X$ the usual kinetic and mass terms leads to the bulk action

$$S = \int \mathrm{d}^5 x \sqrt{g}\ \mathrm{tr}\left[ |DX|^2 + 3\,|X|^2 - \frac{1}{4g_5^2}(F_L^2 + F_R^2) \right], \tag{2.10}$$

where $g$ is the determinant of the metric tensor, $D$ is the gauge covariant derivative $D_\mu = \partial_\mu - iA_{\mathrm{L}\mu} + iA_{\mathrm{R}\mu}$, and the absolute value squared is $|X|^2 = XX^\dagger$. The gauge coupling $g_5$ is a free parameter to be determined.

The remaining ingredient is modeling confinement, which requires breaking conformal symmetry. This can be achieved most simply by imposing a hard wall: the AdS space is considered to reach only up to some $z_m$ in the bulk, where boundary conditions for the fields are to be determined. The hard wall is also called the IR brane. This introduces confinement, since now the energy density related to a string with both endpoints on the boundary, long enough to touch the hard wall, scales linearly with the distance of the string endpoints. These endpoints are dual to the $SU(N_\mathrm{c})$ charges, and the linearly growing energy density signifies that they cannot be separated without creating new string endpoints, *i.e.* charges. The hard wall also breaks the conformal symmetry in the bulk, since now any conformal transformation would move the hard wall. This also breaks the conformal symmetry of the boundary theory.

The classical solutions of the field equations then determine the boundary theory observables. First, the $X$ -field determines the quark mass matrix $M^{ij}$

and the matrix of quark condensate expectation values $\Sigma^{ij}$ via its UV expansion,

$$\lim_{z\to 0} X^{ij} = \frac{1}{2}M^{ij} + \frac{1}{2}\Sigma^{ij}z^3. \tag{2.11}$$

The simplest model is achieved by setting both matrices proportional to the unit matrix $\mathbb{I}$, $M = m_q\mathbb{I}$ and $\Sigma = \sigma\mathbb{I}$. The boundary conditions on the IR brane then determine $m_q$ and $\sigma$. In this model, it is however the condensate and mass which are taken as inputs, and therefore remain free parameters. Once the quark mass and condensate are set, the IR boundary condition for $X$ is fixed.

The boundary condition for the gauge fields is $F_{z\mu} = 0$. This is not set from a priori principles, but the computations show that the results are not very sensitive to this term either.

The free parameters of the theory are then $m_q, \sigma, z_m$ and $g_5$. The gauge coupling can be fixed by matching the vector current two-point function at large momentums to the perturbative QCD result. That leads to the identification $g_5 = \frac{24\pi}{N_c}$, where $N_c$ is the number of colors. Note that since this is the only place where the rank of the 4D gauge group appears, we are free to pick $N_c = 3$, and hope that the gravity approximation is at least qualitatively reliable at $1/N_c = 1/3$.

Now that the action is fully fixed, one can derive the equations of motion and solve them. The four dimensional bound state masses come from eigenvalue equations for five dimensional linearized fluctuations around the background. More specifically, consider a plane wave solution of the 4D theory, say a scalar field, $\phi(x) = e^{ik_\mu x^\mu}$, where $k^\mu, x^\mu$ are the 4-momentum and position. We lift this to five dimensions as

$$\phi(x^\mu, z) = e^{ik_\mu x^\mu}\phi(z). \tag{2.12}$$

Inserting this as a fluctuation ansatz into the bulk action, and assuming $\phi(z) \ll 1$ leads to a linearized fluctuation equation. Given an IR boundary condition, in the case of this model $\phi(z_m) = 0$, the other boundary condition is just a normalization. The kinetic term in the action produces a term $\sim k^2\phi(z)$ in the equations of motion, and requiring that the wavefunction is normalizable, *i.e.* finite on the boundary, makes the equation an eigenvalue equation for $k^2 = m^2$. Solving this equation then gives the dispersion relation in the 4D theory, or by setting $k^{1,2,3} = 0$, the rest masses of the particles. There is an infinite tower of solutions to these equations, corresponding to an infinite number of four dimensional bound states. Taking a more general ansatz

$$\phi(x^\mu, z) = \int \mathrm{d}^4k\,\phi_0(k)e^{ik_\mu x^\mu}\phi(z), \tag{2.13}$$

*i.e.* representing the 4D field by its Fourier transform, we can also extract correlators, and therefore decay constants and more from the duality. At finite temperature, reached by considering black hole solutions in the bulk, the correlators become thermal and thermodynamic variables can be computed. [62, 63]

In the AdS/QCD -model, the gauge invariant fluctuations corresponding to the tower of $\rho$ -mesons come from vector fluctuations of the gauge fields, $V_\mu = (A_{L,\mu} + A_{R,\mu})/2$, and the axial mesons, $a_1$ and $\pi$, come from the axial vectors $A_\mu = (A_{L,\mu} - A_{R,\mu})/2$. Computing the correlators from these gives the masses and decay constants of the hadrons as a function of the free parameters $m_z, \sigma$ and $m_q$. It is also possible to extract the meson interaction couplings, for example $g_{\rho\pi\pi}$. Fixing the parameters from pion and rho masses and the pion decay constant gives column A of table 2.1. A best fit to all seven observables computed gives column B.

| Observable | Measured [MeV] | AdS A [MeV] | AdS B [MeV] |
|---|---|---|---|
| $m_\pi$ | 139.6±0.0004[9] | 139.6* | 141 |
| $m_\rho$ | 775.8±0.5 | 775.8* | 832 |
| $m_{a_1}$ | 1230±40 | 1363 | 1220 |
| $f_\pi$ | 92.4±0.35 | 92.4* | 84.0 |
| $F_\rho^{1/2}$ | 345±8 | 329 | 353 |
| $F_{a_1}^{1/2}$ | 433±13 | 486 | 440 |
| $g_{\phi\pi\pi}$ | 6.03±0.07 | 4.48 | 5.29 |

Table 2.1: Several meson variables from the AdS/QCD model described in the text. AdS A is the best fit to the starred observables, whereas AdS B is the best fit to all seven.[57]

### 2.5.2 Dynamic AdS/QCD

Dynamic AdS/QCD expands the hard wall AdS/QCD of the previous section by making the metric a simplified version of the brane metric stemming from D3/probe-D7 top down models [46, 64–67], and adding a running anomalous dimension for the quark mass. First of all, the metric is now

$$\mathrm{d}s^2 = \frac{\mathrm{d}\rho^2}{\rho^2 + |X|^2} + (\rho^2 + |X|^2)\mathrm{d}x^2, \tag{2.14}$$

where $X$ is again the field sourcing the quark mass and condensate. This specific form comes from the D3/D7 model. The action is

$$S = \int \mathrm{d}^4x\ \mathrm{d}\rho\, \mathrm{Tr}\, \rho^3 \left[\frac{1}{\rho^2 + |X|^2}|DX|^2 + \frac{\Delta m^2}{\rho^2}|X|^2 + \frac{1}{2\kappa^2}(F_V^2 + F_A^2)\right]. \tag{2.15}$$

Here $\Delta m$ is related to the conformal dimension of the quark condensate in a way that we will soon elaborate, and $F_V, F_A$ are field strengths written here directly

[9]The peculiar expression $139.6 \pm 0.0004$ comes from the results column of the table in the original text [57]. The error limit which is much smaller than the least significant digit of the measured value apparently serves the purpose of denoting that the measured value is coarsely 139.6, but that it is actually known to an error margin of 0.0004.

for the vector and axial combinations of the gauge fields. The $\sqrt{-g}$ -term is written as $\rho^3$, as in the D3/D7 -model, instead of $(\rho^2+|X|^2)^{\frac{3}{2}}$. This will turn out to be crucial to the generation of the dynamic wall.

As in the previous section, if the field dual to the quark operators has 5D mass $-3$, it will describe a field theory with no quark anomalous dimension. However, the action in Eq. (2.15) is not in the form where $X$ is a canonical scalar, so this does not apply directly. Writing $L=|X|$ and $L=\rho\phi$, and integrating the kinetic term by parts, we find that the field $\phi$ is a canonical scalar with mass $m_\phi^2=-3+\Delta m^2$. We see then that $\Delta m$ controls the anomalous dimension by

$$m_\phi^2 = \Delta(\Delta-4) = -3+\Delta m^2 \tag{2.16}$$

$$\Rightarrow \Delta_\pm = 2\pm\sqrt{1+\Delta m^2} \approx 2\pm(1+\frac{1}{2}\Delta m^2) \tag{2.17}$$

and therefore the anomalous dimension of the condensate, which corresponds to the positive root, is

$$\gamma = -\frac{1}{2}\Delta m^2 \tag{2.18}$$

for small $\Delta m^2$. The value $\Delta m^2=-1$, where the square root becomes imaginary, is the Breitenlohner-Freedman (BF) bound $m_\phi=-4$ [68, 69].

Now we take $\Delta m^2$ to be a function of the holographic energy scale $\mu$. The scale is identified from the metric Eq. (2.14) as the overall scale of the 4D part of the metric, which is $\sqrt{\rho^2+|X|^2}$. The energy dependence is conjured from perturbation theory, where the one-loop result is

$$\gamma = \frac{3(N_\mathrm{c}^2-1)}{4N_\mathrm{c}\pi}\alpha \tag{2.19}$$

and therefore

$$\Delta m^2 = -\frac{3(N_\mathrm{c}^2-1)}{2N_\mathrm{c}\pi}\alpha. \tag{2.20}$$

The running of the coupling $\alpha$ as a function of the energy scale $\mu$ is solved from the perturbative two-loop beta function. After plugging that in, with $\mu=\sqrt{\rho^2+|X|^2}$, the form of $\Delta m^2$ as a function of the fields is fixed. Notice that we do this at the level of equations of motion, not of the action. This is because otherwise a term $\propto \Delta m^{2\prime}(L)$ would be generated, while there is no corresponding term in gauge theory. Also in the region near the conformal window, where this model is most interesting, the derivative is anyway not large, and therefore ignoring it does not make a significant difference.

At this point, we have fully defined the action and the classical equations of motion. The next step is to find a background solution to the full equations of motion. As usual, $F_V=F_A=0$ is a solution. $L=0$ is also a solution, but if the mass squared of the scalar violates the BF bound, we expect that solution to be unstable. With the above fixing of $\Delta m^2$, the bound is violated when $N_\mathrm{f}\lesssim 12$.

This matches well with the results from a perturbative Schwinger-Dyson -analysis, which is not surprising as we have taken the running of the anomalous dimension from perturbation theory as input. In the region where the $L = 0$ solution is unstable, we look for non-zero solutions to the $L$ equation of motion, which describes dynamic breaking of chiral symmetry. The IR boundary condition is $L'(0) = 0$, as in the D3/D7 model. Applying the holographic dictionary to the canonical scalar $\phi = L/\rho$, the UV asymptote of $L$ is

$$L \sim m_q + \sigma\rho^{-2}. \tag{2.21}$$

This allows us to set the quark mass, in practice numerically by using the shooting method [70] to determine $L(0)$ such that the desired $m_q$ is reached in the UV. We have then found a stable background solution of the theory.

In order to study the fluctuations around this background, the remaining parameter to fix is $\kappa$. This can be fixed by requiring the correct masses for vector and axial mesons in QCD. Since we will be working with massless quarks, we take into account the two light flavors of QCD and require the match at $N_\mathrm{f} = 2, N_\mathrm{c} = 3$. Requiring also a smooth restoration of chiral symmetry at $N_\mathrm{f}^\mathrm{c} \sim 12$, we find that the simplest ansatz for $N_\mathrm{f}$ dependent $\kappa$ is[3]

$$\kappa^2 = 3.6(N_\mathrm{f}^\mathrm{c} - N_\mathrm{f}). \tag{2.22}$$

With these we are ready to compute the meson spectrum and decay constants from the fluctuations. The fluctuation ansatz is

$$V^\mu = \epsilon^\mu K_V(\rho)e^{-iq\cdot x}, \quad A^\mu = \epsilon^\mu K_A(\rho)e^{-iq\cdot x}, \quad L = L_0(\rho) + \delta(\rho)e^{-iq\cdot x}, \tag{2.23}$$

where $L_0$ is the background solution to the equations of motion. Inserting these into to the action leads to the fluctuation equations for the mesons,

$$\partial_\rho\left[\rho^2\partial_\rho K\right] - \frac{q^2}{\rho}K = 0 \tag{2.24}$$

$$\partial_\rho(\rho^3\partial_\rho\delta) - \Delta m^2\rho\delta - \rho L_0\delta\left.\frac{\partial\Delta m^2}{\partial L}\right|_{L_0} = 0, \tag{2.25}$$

where $K$ is $K_A$ or $K_V$. We find the masses and decay constants from numerical solutions to these.

Let us review the results. First of all, the nature of the model allows us to consider fractional values of $N_\mathrm{f}$, and we take advantage of that freedom in order to see behavior of the observables as we approach the conformal window. This can also be considered to reflect, to some extent, the behavior of the model at large $N_\mathrm{c}$, $N_\mathrm{f}$, where their ratio becomes a continuous parameter.

In the model, the onset of the conformal window is at $N_\mathrm{f}^\mathrm{c} \sim 12$. This is exhibited in Fig. 2.1 in the form of Miransky, or BKT, scaling of the masses

[3] There is a sign error in the corresponding formula in [III].

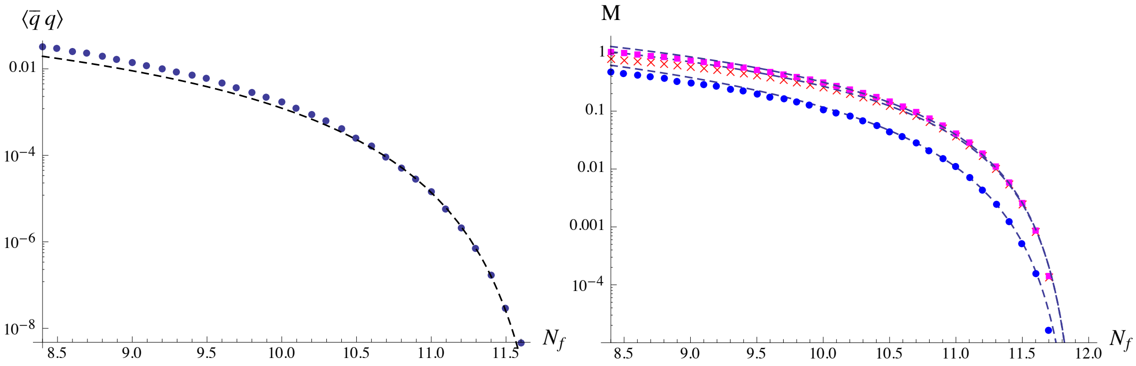


Figure 2.1: Left: The dots show numerical results for the quark condensate as a function of $N_f$. The dashed line is the BKT fit $a\exp(-3b/(N_f^c - N_f)^{1/2})$ with parameters $a = 63.090$ and $b = 5.111$. Right: Masses of scalar (dots), vector (crosses) and axial (squares) mesons as a function of $N_f$. The dashed lines show fits of the form $M_i = (N_f^c - N_f)^{p_i} L(0)$, where $p_S = 1.27$, $p_V = 0.77$ and $p_A = 0.85$.

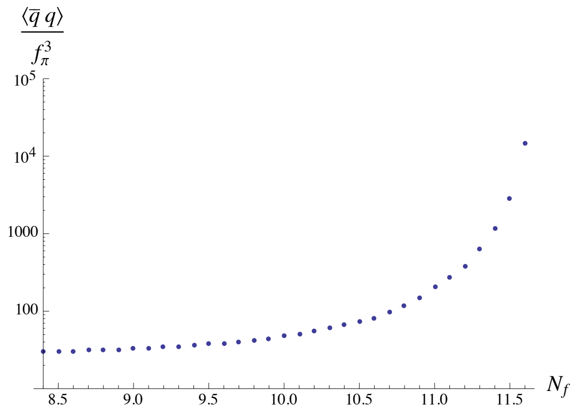


Figure 2.2: The quark condensate normalized by $f_\pi^3$ vs $N_f$.

[71]. The quark condensate, determined as the factor multiplying the $\rho^{-2}$ -term in the UV solution limit of $L_0(\rho)$, displays the scaling in a pure form, and the masses of the mesons display the exponential scaling corrected by slight power law deviations, as predicted in [72]. In addition, while the dimensionful observables go to zero when approaching the conformal window, the dimensionless ratio $\langle\bar{q}q\rangle/f_\pi^3$ is enhanced, as shown in Fig. 2.2. This is a key prediction of walking technicolor that allows a generation of sufficiently large fermion masses without generating too large flavor changing neutral currents.

Another part of technicolor lore reproduced neatly here is the appearance of a parametrically light dilaton. As shown in Fig. 2.3, the mass of the lightest excitation dual to the scalar $X$ becomes light compared to all other masses. This is interpreted in the field theory as due to the fact that when we approach the onset of the conformal window, the amount of explicit conformal symmetry breaking decreases and a light Goldstone boson corresponding to the spontaneous

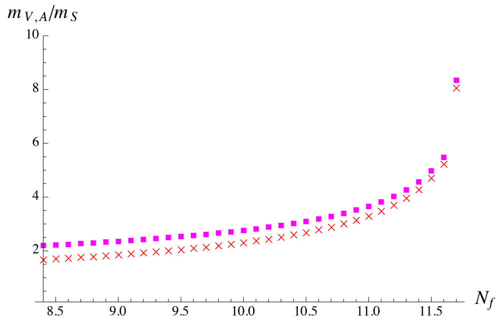


Figure 2.3: Mass ratios: the crosses and squares show the ratio of the $V$ and $A$ masses to the $S$ mass, respectively, revealing the Goldstone-like nature of the scalar meson as one approaches the critical point.

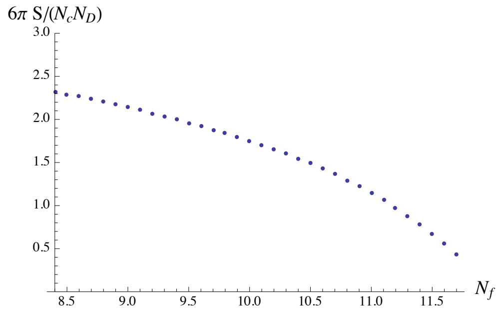


Figure 2.4: The contribution to the S parameter from the lightest vector and axial mesons (normalized by the number of techni-doublets and to the perturbative value from a single mass degenerate doublet).

symmetry breaking is generated. It becomes arbitrarily light as we approach the lower limit of the conformal window, as the theory approaches an IR fixed point.

The most important constraint for walking technicolor is the $S$-parameter, which has to be sufficiently small. The $S$ -parameter is defined as

$$S = 4\pi(\Pi'_{AA}(0) - \Pi'_{VV}(0)), \tag{2.26}$$

where $\Pi_{AA}(0), \Pi_{VV}$ are the axial axial and vector vector correlators, respectively. The contribution from the lightest vector and axial mesons to the S -parameter is shown in Fig. 2.4, where we see that it indeed goes to zero as we approach the lower boundary of the conformal window, as expected in walking technicolor.

In summary, the model is fixed by the quark mass, the value of the coupling $\kappa$, and most importantly, by inputting the running of the quark anomalous dimension from perturbation theory. This produces predictions for the correlators of scalar, vector and axial mesons of the theory, and allows computing the meson masses and their decay constants, among other observables.

The results of the computation reproduce all the expected features of walking technicolor theories near the conformal window. This suggests that, to the extent that this model is reliable, all of the generally desirable features of technicolor are already encoded in the running of the anomalous dimension of the quark condensate.

# Chapter 3

# Thermodynamics of IHQCD and VQCD

*"Of our studies it is impossible to speak, since they held so slight a connection with anything of the world as living men conceive it."*

–H.P. Lovecraft, "Hypnos", 1922–

In this chapter we will review what constitutes the main body of work done for this thesis, first published in papers [I, II, IV]. The work is based on Improved Holographic QCD (IHQCD) [50–53], based on which a model of quasi-conformal field theory has been built in [73–75]. In paper [I] we have computed the thermodynamics and finite temperature mass spectrum of the model, exhibiting the melting of bound states as the system undergoes a phase transition.

In [II] we exhibited the first computations of the phase diagram and various other thermodynamic quantities in a model extended from IHQCD. This Veneziano QCD (VQCD) model [54] adds explicit quark degrees of freedom to the IHQCD description of gauge theory dynamics, producing a bottom-up model with full backreaction between the fermionic and gauge degrees of freedom. In paper [IV] we extended these computations to finite quark density.

## 3.1 IHQCD

In the Maldacena duality, there is no running of the gauge coupling constant, but its constant value is controlled by the expectation value of the dilaton in the supergravity theory. It is therefore natural that when the conformality of the gauge theory is broken, leading to a running coupling, the dual to the coupling becomes a non-constant dilaton in the bulk. Modeling a non-conformal pure gauge theory holographically is then up to prescribing the dynamics of the bulk dilaton. A generic way to achieve that is to give the dilaton a canonical kinetic term and a potential. This leads to a wide class of bottom-up models, where various methods of fixing the potential are prescribed, varying from string theory considerations to perturbative matching and purely phenomenological potentials. A sampling of such models can be found in [76–84]

### 3.1.1 A string inspired model for glue

Improved Holographic QCD is an Einstein-Dilaton model, and therefore its action is

$$S = \frac{1}{16\pi G_5}\int d^5x\,\sqrt{-g}\left[R - \tfrac{4}{3}(\partial_\mu\phi)^2 + V(\phi)\right]. \tag{3.1}$$

This is the action of a scalar field $\phi$ coupled to Einstein gravity in five dimensions, with a potential $V(\phi)$ for the scalar. The peculiar normalization factor $\frac{4}{3}$ originates from the Weyl transformation from the string frame to the Einstein frame.

The task is to then define the potential $V(\phi)$. Gursoy and Kiritsis argue in [50, 51] that the form Eq. (3.1) can be justified as an approximation to string theory, in which case the potential is a resummation of unknown higher $\alpha'$ corrections. Using the duality, we can then fix the potential to results from perturbation theory. The first step is deriving the equations of motion. A sufficiently general metric ansatz compatible with the vacuum solution, *i.e.* four dimensional translation invariance, is

$$\mathrm{d}s^2 = e^{2A(r)}\left(\mathrm{d}r^2 + \eta_{\mu\nu}\mathrm{d}x^\mu \mathrm{d}x^\nu\right), \tag{3.2}$$

where $r$ is the five dimensional coordinate. This is known as the conformal coordinate system. If $A(r) = -\log(r)$ this describes an $\mathrm{AdS}_5$ spacetime with the boundary at $r = 0$. Here it is however expected that $A(r)$ will be only asymptotically AdS, since the running of the coupling constant should generate breaking of the conformal invariance. Inserting Eq. (3.2) to the action leads to the equations of motion

$$\begin{aligned}
12\dot{A}^2 - \frac{4}{3}\dot{\phi}^2 - e^{2A}V &= 0 && (3.3)\\
6\ddot{A} + 6\dot{A}^2 + \frac{4}{3}\dot{\phi}^2 - e^{2A}V &= 0 && (3.4)\\
\ddot{\phi} + 3\dot{A}\dot{\phi} + \frac{3}{8}e^{2A}\frac{\mathrm{d}V}{\mathrm{d}\phi} &= 0, && (3.5)
\end{aligned}$$

where $A$ and $\phi$ are taken to be functions of $r$ only due to 4D Poincaré symmetry, and the overdot denotes a derivative with respect to $r$.

The form of the potential in the UV can then be fixed by considerations of the running of the renormalization scale. First, since QCD is conformal in the UV, we expect the gravity dual to be also asymptotically $\mathrm{AdS}_5$, as mentioned above. Let us fix a part of the radial reparametrization invariance by fixing the position of the UV AdS at $r = 0$. We therefore expect that $A(r) = -\log(r) + \text{corrections}$. We identify the dilaton field $\phi$ with the logarithm of the 't Hooft coupling, as in the usual AdS/CFT duality:

$$\lambda = \lambda(r) = N_c e^\phi. \tag{3.6}$$

From here on, we suppress the factor $N_c$, which amounts to just a redefinition of the potential. Then we need to consider the identification of the energy scale. In the UV, where the metric is near AdS, the identification of the energy scale with the conformal factor of the metric holds based on the same arguments as in the Maldacena duality, that is

$$E = e^{A(r)}. \tag{3.7}$$

This identification is taken to hold at all scales, not just in the UV. With these identifications, the holographic beta function is

$$\beta(\lambda) = \frac{\mathrm{d}\lambda}{\mathrm{d}\log(E)} = \frac{\mathrm{d}\lambda}{\mathrm{d}A} = \frac{1}{\frac{\mathrm{d}A}{\mathrm{d}r}}\frac{\mathrm{d}\lambda}{\mathrm{d}r}. \tag{3.8}$$

This is identified with the perturbative beta function, which in an asymptotically free gauge theory is

$$\beta(\lambda) = -b_0\lambda^2 - b_1\lambda^3 + \mathcal{O}\left(\lambda^4\right), \tag{3.9}$$

where $b_0 > 0$. This is our constraint for the potential in the UV.

Defining a variable $X = \frac{\dot{\phi}}{3\dot{A}}$, it is possible to rewrite the equations of motion Eq's. (3.3, 3.4, 3.5) as a first order system. Especially we have

$$\frac{\mathrm{d}X}{\mathrm{d}\phi} = \left(8X + 3\frac{\mathrm{d}\log(V)}{\mathrm{d}\phi}\right)\frac{X^2-1}{6X}. \tag{3.10}$$

This is a differential equation for $V(\phi)$ as a function of $X$. From the definition of $X$, we see that it is closely related to the beta function:

$$X = \frac{\beta(\lambda)}{3\lambda}, \tag{3.11}$$

and this allows us to express the dilaton potential as a function of the beta function:

$$V(\phi) = V_0\left(1 - \frac{\beta(\lambda)^2}{9\lambda^2}\right)\exp\left(-\frac{8}{3}\int\limits_{-\infty}^{\phi}\frac{\beta(\lambda)}{3\lambda}\mathrm{d}\phi\right), \tag{3.12}$$

where we substitute $\lambda = e^{\phi}$. This will actually turn out to be very important in its own right, but for now, the main point to note is that the expansion of this with respect to $\lambda$, inserting the perturbative beta function, is

$$V(\lambda) = V_0(1 + V_1\lambda + V_2\lambda^2 + \ldots), \tag{3.13}$$

where $V_1 = \frac{8}{9}b_0$ and $V_2 = \frac{23b_0^2 - 36b_1}{81}$. In the zeroth order approximation, where $\lambda = \text{const.}$, this is just a constant potential, which generates an $\mathrm{AdS}_5$ -spacetime, as expected. The presence of a $\sim\lambda$ -term is the first correction. It generates logarithmic corrections to the AdS space, the conformal factor to the first non-trivial order being

$$e^{2A(r)} = \frac{L^2}{r^2}\left[1 + \frac{8}{9\log(r\Lambda)} + \mathcal{O}\left(\frac{\log(\log r\Lambda)}{(\log r\Lambda)^2}\right)\right], \tag{3.14}$$

where $L$ is the AdS -radius, related to $V_0$ by $V_0 = \frac{12}{L^2}$, and $\Lambda$ is an integration constant which essentially gives the scale where the conformal invariance is broken.

In this way the UV potential of the model can be related to the perturbative beta function. The 't Hooft coupling displays the logarithmic running $\lambda \propto \frac{1}{\log E}$ expected for an asymptotically free theory.

From the string theory point of view, the $\lambda$ -term in the potential, which is crucial in matching to asymptotically free field theories, seems puzzling. In a system of D3 -branes, the dilaton potential is a constant plus a $\lambda^2$ -term. However, as analyzed in detail in [50], the series of higher $\alpha'$ -corrections to the leading order string theory picture generates a full series of $\lambda$ -corrections, producing precisely the form of the regular expansion in Eq. (3.13). In many other ways, this construction passes some very non-trivial consistency checks, and has several features which suggest that it may be a better approximation than it superficially seems to be. The beta function coefficients only appear in combinations $b_{i+1}/b_i^2(b_0\lambda)^n$, which are independent of rescalings of the 't Hooft coupling. Stringy corrections to the identification of 't Hooft coupling from brane physics only enter together with $b_i, i \geq 1$. This means that the two lowest order coefficients of the beta function, which are renormalization scheme independent, do not get corrections, and a similar result holds for the identification of the energy scale. This suggests that it may be valid to take these identifications as exact by fixing the renormalization scheme such that the stringy corrections are canceled. This is called the holographic renormalization scheme. Encouraged by this, the model is treated as an approximate dual to QCD not only in the strong coupling regime, but over the full range of energies.

Confinement is a key input for fixing the IR form of the potential. Still following the detailed analysis in [50], it turns out that the model is confining if the asymptotic behavior of the conformal factor is

$$A = -Cr^{\alpha} + \ldots, \tag{3.15}$$

where $C$ is a constant and $\alpha > 1$. Using Eq. (3.12), this leads to the asymptotic form of the potential

$$V(\lambda) \propto \lambda^{\frac{4}{3}}(\log\lambda)^{\frac{\alpha-1}{\alpha}}. \tag{3.16}$$

This general form of the IR potential produces confinement and a discrete glueball spectrum. As shown in [53], choosing further $\alpha = 2$ gives a linear spectrum, and this is therefore the relevant value for modeling QCD-like theories.

Given the aforementioned UV and IR asymptotics, IHQCD reproduces most of the qualitative expectations for a pure Yang-Mills -theory [52]. Confinement in the vacuum solution turns out to be in one-to-one correspondence with the existence of a finite temperature Hawking-Page -transition between a compactified vacuum solution and a black hole phase. The requirement for the potential to confine is also the same condition that leads to the existence of a mass gap [51].The UV asymptotics of thermodynamic observables at high temperatures approach logarithmically the Stefan-Boltzmann -limit, up to a normalization which can be used to fix the 5D gravitational constant, *i.e.* the holographic model actually reproduces the correct thermodynamics at the limit of *weak coupling*, in support of taking the model seriously at all values of $\lambda$.

An explicit model giving the interpolation between the IR and UV limits is presented in [53]. The model describes $N_c = 3$ lattice data quantitatively after matching two free parameters to a subset of the data.

In summary, the IHQCD model is string inspired, in the sense that the overall structure can be tentatively related to string theory origins, but it is also a phenomenological model, since the potentials are not computed from any string theory construction, but rather fixed to observables. It is also not a controlled approximation, since the stringy arguments used in its construction do not offer a way to compute corrections to the results or to estimate their magnitude. The model is therefore between the top-down and bottom-up approaches, although certainly closer to the bottom-up approach.

After having now presented the original IHQCD model, we are ready to start discussing paper [I], which uses essentially that model, but with a different philosophy for fixing the potentials.

### 3.1.2 A quasi-conformal beta function

The overall method of the IHQCD -model is applicable, at least phenomenologically, to other gauge theories than QCD. The actual input that determines the specific gauge theory is the perturbative beta function. In addition, the original brane construction relies heavily on the fact that a stack of N D$p$ -branes produces, at the low energy limit, an $SU(N)$ gauge theory with the fermions in the fundamental representation. Since the model only considers the pure gauge part, the fermion representation is not expected to appear at this level of approximation, and therefore it is plausible to model any $SU(N)$ -theory by IHQCD, given the beta function of the gauge theory. Stretching the model even further, one can take the beta function of a theory with fermions, and use that as an input to the IHQCD -construction. This is in essence a quenched approximation, where the fermions are treated as non-dynamical degrees of freedom.

In [73–75], the IHQCD -approach is applied to study the thermodynamics of a class of field theories which have an IR fixed point, or which come close to an IR fixed point, *i.e.* are quasi-conformal. While this is not true for any pure gauge theory at the two-loop level, it is believed that the most important effects of the fermions can be captured in their effect on the renormalization group flow. The approach also differs from the original IHQCD in the sense that instead of matching the scheme-independent coefficients only, the full beta function is used to determine the potential via Eq. (3.12). This typically does not have the correct IR asymptotics, for which a further correction has to be implemented. Since, at finite temperature above the deconfinement transition, the horizon of the black hole hides the deep IR, the effect of this is reasonably small as long as the black hole horizon is not at too large $r$. As it turns out, black holes with a horizon in the deep IR correspond to a thermodynamically unstable branch of solutions, so such solutions are not relevant for the thermodynamics.

In [75] the approach is specifically applied to the beta function

$$\beta(\lambda) = -c\lambda^2 \frac{(1-\lambda)^2 + e}{1 + a\lambda^3} \tag{3.17}$$

which has the form needed for walking technicolor at small to intermediate $\lambda$, when $e$ is small but non-zero. The parameters $e$ and $c$ allow tuning to potential in order to see the effects of approach to conformality, whereas the ratio $\frac{c}{a}$ is fixed by confinement. The specific analytical form is inspired by the exact beta function ansatze of Ryttov and Sannino, and Antipin and Tuominen [85, 86].

In [I], we continue from [75], and compute, in addition to the thermodynamics, the mass spectrum at finite temperature.

**Thermodynamics in holography**

We consider with some detail the way thermodynamics are computed from a holographic dual in the context of IHQCD -like models, using the setup in [I] as an example. There are two classes of bulk configurations which are dual to thermodynamic states. The simplest is the vacuum configuration of the bulk theory, which can be Euclinianized and compactified to any temperature. This state is essentially dual to a thermal gas of hadrons [52]. The more interesting class of bulk configurations are those with a black hole. The black hole has an associated Hawking temperature and a Bekenstein-Hawking entropy. Since the states of the bulk and boundary theories are assumed to be in one-to-one correspondence, the boundary theory must have this same temperature and entropy. It is necessary then to find the vacuum and all black hole solutions to the theory, and for any set of solutions that correspond to the same temperature, compute their free energies in order to find the thermodynamically preferred state. This gives the phase diagram, and allows computing thermodynamic variables as a function of temperature.

For black hole solutions, the metric ansatz is generalized to

$$\mathrm{d}s^2 = b^2(r)\left[-f(r)\mathrm{d}t^2 + \mathrm{d}\vec{x}^2 + \frac{\mathrm{d}r^2}{f(r)}\right], \tag{3.18}$$

where $b(r) = e^{A(r)}$. The function $f$ allows for inhomogenous scaling between the timelike and the radial directions. If $f$ has a zero at $r_h$, then there is a horizon

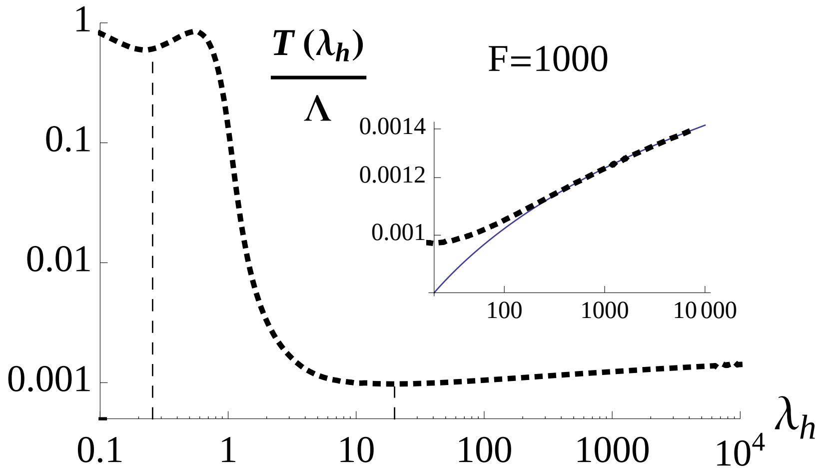


Figure 3.1: An example of temperature as a function of the value of the 't Hooft coupling at the horizon. $\Lambda$ is a constant which determines the units of energy in the boundary theory.

at $r_h$, and this is a black hole solution. In order to have the usual Minkowski metric at $r = 0$, we require the boundary condition $f(r = 0) = 1$.

With this setup, the equations of motion become

$$6\frac{\dot{b}^2}{b^2} + 3\frac{\ddot{b}}{b} + 3\frac{\dot{b}}{b}\frac{\dot{f}}{f} = \frac{b^2}{f}V(\phi), \tag{3.19}$$

$$6\frac{\dot{b}^2}{b^2} - 3\frac{\ddot{b}}{b} = \frac{4}{3}\dot{\phi}^2, \tag{3.20}$$

$$\frac{\ddot{f}}{\dot{f}} + 3\frac{\dot{b}}{b} = 0. \tag{3.21}$$

This is a set of three second order differential equations, so there are six independent boundary conditions. After fixing horizon regularity and coordinate symmetries, the only physically significant boundary condition is $\lambda_h$. Therefore $\lambda_h$ determines a family of physically inequivalent black hole solutions, and also all such solutions are found by going through all possible values of $\lambda_h$.

In practice then, for all but the most simplified potentials, the solutions are generated numerically. The boundary conditions are set at the black hole horizon and the solution is evolved in the radial direction by a numeric integrator. For each solution we can then compute the temperature and entropy, which are given as a function of $\lambda_h$. The temperature plot, such as in Fig. 3.1, then shows the potential phase structure of the theory. In the example, at large temperatures there are two black hole phases, one at small $\lambda_h$ and the other at large $\lambda_h$. At a temperature range $T \sim 0.6 \ldots 1.0$ there are three black hole phases. There are no black hole phases at temperatures below $\sim 0.001$.

In order to decipher the thermodynamically dominant phases, it is necessary

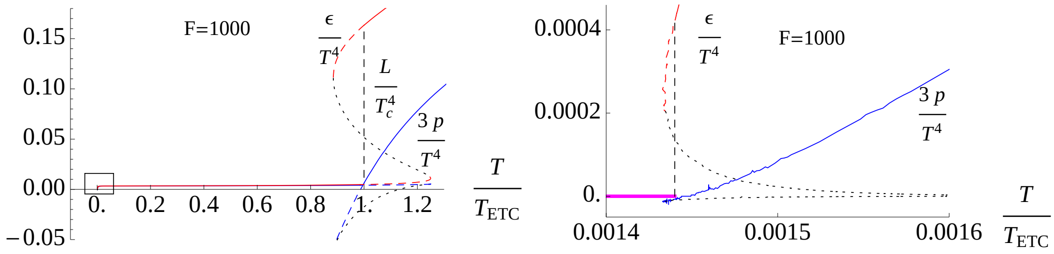


Figure 3.2: Pressure as a function of temperature. The energy density $\epsilon$ is also plotted. The right panel shows an enlargement of the boxed area in the left panel.

to compute the free energy or, equivalently, the pressure of each solution. This could be done in principle directly by evaluating the value of the action at the solution, but that is technically demanding, due to the need to subtract the vacuum background and the slow convergence of the logarithmic corrections to the AdS -background. A more practical method is to define the zero of energy in the boundary theory such that the vacuum solution has zero pressure or free energy. Then the pressure of the black hole solutions can be computed by integrating

$$\mathrm{d}p = s\mathrm{d}T \Rightarrow \frac{\mathrm{d}p}{\mathrm{d}\lambda_h} = s(\lambda_h)\frac{\mathrm{d}T}{\mathrm{d}\lambda_h} \tag{3.22}$$

over a range of black hole solutions. The resulting plot of pressure vs. temperature in the example case is displayed in Fig. 3.2. The phase diagram can be read from the plot as follows. At temperatures which the pressure curve does not reach, or where it is negative, the vacuum phase, dual to a thermal gas, is dominant. Where the pressure function is positive, the black hole solutions are dominant, and if it is multi-valued, the branch with the largest temperature is dominant.

In the case at hand, this leads to the following phase structure

- At low temperature, $T < 0.000977$, there are no corresponding black hole phases, and the theory is in the thermal gas phase.
- At temperatures between $T = 0.000977$ and $T = 0.653$, the theory is in a deconfined black hole phase, with the smaller $\lambda_h$ of the two possible black hole branches. The branch with larger $\lambda_h$ black holes is mechanically unstable.
- At $T = 0.653$, there is a first order transition to another branch of black hole solutions.

### 3.1.3 Glueball spectrum

As a baseline for understanding the spectrum at finite temperature, we need to compute the spectrum at $T = 0$. The principle is the same as described in section

2.5, although since the metric in this model is dynamical, not only the dilaton but also the metric fluctuates around its vacuum solution. There are actually two kinds of gauge invariant fluctuations: the tensor fluctuations of the metric, $g_{\mu\nu} = g^{(0)}_{\mu\nu} + h_{\mu\nu}$, where $g^{(0)}_{\mu\nu}$ is the background solution and the fluctuation $h_{\mu\nu}$ satisfies

$$\partial_\mu h^{\mu\nu} = 0, h^\mu_\mu = 0. \tag{3.23}$$

These are dual to tensor glueball states in the boundary theory. The other gauge invariant fluctuation is that of the dilaton, which mixes with those components of the metric that is not involved in the tensor fluctuations. These are scalar glueball states in the boundary theory.

The fluctuation equations can be cast in the form of a Schrödinger-like equation,

$$-\psi''(u) + V^{(i)}(u)\psi(u) = \omega^2\psi(u), \tag{3.24}$$

where $u$ is a specific reparametrization of the radial coordinate,

$$u(r) = \int_0^r \frac{\mathrm{d}\tilde{r}}{f(\tilde{r})}. \tag{3.25}$$

At $T = 0$, $u = z$. The potentials $V^{(i)}$, $i = S, T$ are determined from the background solutions and correspond to the scalar and tensor fluctuations, respectively. The advantage of this form is that it allows applying intuition concerning bound states in quantum mechanics to the spectrum of particles in the boundary theory. The potentials for the scalar and tensor modes at $T = 0$ is shown in Fig. 3.3.

Solving the background and the fluctuation equations numerically gives the lowest lying spectrum as

$$\begin{aligned} m^{(S)} &= (0.005948, 0.0077235, 0.0090700, 0.010182) \\ m^{(T)} &= (0.0069018, 0.0083675, 0.0095586, 0.010581), \end{aligned} \tag{3.26}$$

where $m^{(S)}$ are the scalar glueball masses and $m^{(T)}$ are the tensor glueball masses. The pattern continues as an infinite tower of excitations, well approximated by taking an approximate potential which can be solved analytically to yield a spectrum $m_n^2 = \Xi^2(n + 2)$, where $\Xi$ is an energy scaled determined from the numerical solutions. This type of spectrum is generically produced by holographic Einstein-Dilaton -models. Especially note that the two peaks at small-$z$ only act to slightly perturb the linear spectrum of states near the energies where the two metastable states exist.

### 3.1.4 Melting the glueballs at the technicolor phase transition

Interesting things happen when we take this setup to finite temperature. The Schrödinger -like equation keeps the same form, but now the reparametrization

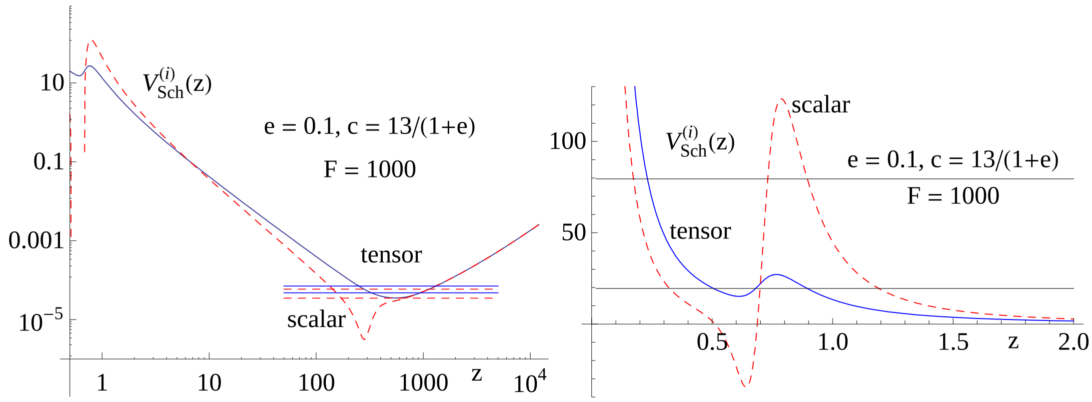


Figure 3.3: Left: The scalar (dashed line) and tensor (continuous line) potentials in the IR large-$z$ region. Two lowest scalar and tensor excitations are plotted, the ordering is $E_0^{(S)} < E_0^{(T)} < E_1^{(S)} < E_1^{(T)}$. The corresponding mass values, $m^2 \equiv E$, are in 3.26. Right: the same potentials in the small-$z$ -region, where the near-conformality generates an extra peak in both potentials. The horizontal lines show the energies of the two states that the scalar potential binds in the approximation where there is a hard wall just beyond the right peak, at $z = 1.2$.

Eq. (3.25) is non-trivial, and the potentials change slightly. Especially note that the horizon $r_h$ maps to $u = \infty$, so only the solution up to the horizon now affects the fluctuations. The boundary condition for the fluctuation is the in-falling condition at the horizon, which translates in $u$ simply to a requirement that the solution is asymptotically a plane wave propagating towards $u = \infty$. Then the requirement that the solution is normalizable at the boundary leads to solutions where $\omega^2$ is complex. These correspond to quasi-normal modes in the boundary theory propagators, with the real part of $\omega$ the mass and its imaginary part the decay constant.

Now the role of the peaks in the potentials at small-$z$, which are related to quasi-conformality, become clear. Just above the lowest $T$ deconfining transition, the horizon is already to the left of the bottom of the potential around $z \approx 500$ in Fig. 3.3. The right side of the potential wall is therefore dissolved, and no bound states in the theory are stable. There are, however, metastable states, which now correspond to the previously near-invisible perturbations caused by the small-$z$ peaks. Fig. 3.4 shows the disappearance of the peaks in the scalar potential as the horizon moves closer to the boundary, *i.e.* the temperature is increased. The right panel shows the corresponding movement of the quasi-normal modes away from the real axis, which signifies that the states are becoming increasingly short-lived.

Taking into account the phase structure presented above, the thermodynamically preferred phases when heating the theory go as follows:

- Below the deconfinement transition, there is an infinite tower of bound

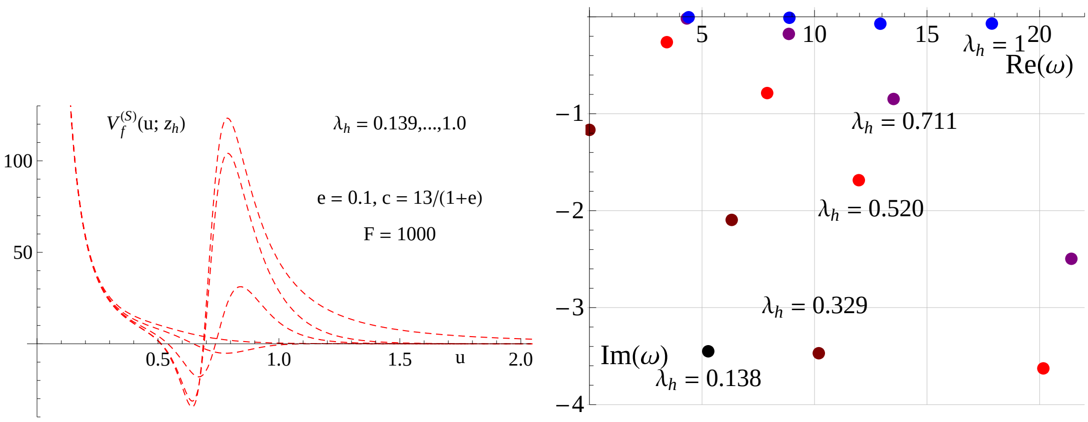


Figure 3.4: Left panel: The dependence of the scalar potential $V^{(S)}(u, z_h)$ for $\lambda_h = 0.138,\ 0.329,\ 0.520,\ 0.711,\ 1.0$, $z_h = 0.57, 0.719, 0.724, 0.92, 2.96$, $\pi T \approx 1/z_h$. The peak disappears when one is approaching the UV, where $\lambda_h$ decreases. Right: Corresponding quasinormal modes. Each color corresponds to a single value of $\lambda_h$, or equivalently, fixed temperature.

states. In terms of an interpretation as a technicolor model, these would be the technimesons, represented here by glueballs. For a realistic theory of technicolor, the lowest state would have to be a light dilaton playing the role of the Standard Model Higgs, but we do not see that here.

- At the deconfinement transition, this infinite tower disappear as the technicolor degrees of freedom become deconfined. However, there are now metastable states, which are heavy compared to the low-lying states of the vacuum spectrum, and they are long-lived at this temperature. In terms of the technicolor interpretation, these would be extended technimesons. This remains the case across the wide range of energy scales corresponding to the walking regime.

- At the upper first order transition, the theory jumps to the branch of black hole solutions with a smaller $\lambda_h$. This transition actually skips over most of the region where the small-$z$ -peak in the potential dissolves, so it is actually a transition from a state where the extended technimeson states are very long lived to a state where they are very short lived. Increasing the temperature further quickly washes out any trace of bound states.

This holographic model then shows an explicit and quantitative implementation of the intuitive picture when there are two strongly coupled theories with widely separated energy scales, and they have two separate deconfining transitions. As a novel feature, the model shows that above the first deconfining transition, the bound states corresponding to the theory with the higher energy scale already become metastable, albeit only slightly.

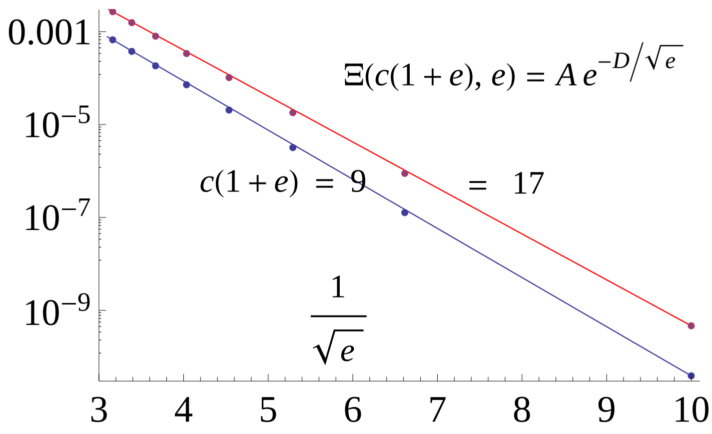


Figure 3.5: The dependence of the IR scale $\Xi$ on $e$ at two values of the parameter $c$ in the beta function, $c(1+e) = 9$ and $c(1+e) = 17$, with a fit to Miransky scaling shown by the solid line. All masses and dimensional observables behave very similarly.

### 3.1.5 Miransky scaling

In addition to the finite temperature quasi-normal spectrum, we verified in [I] that as the theory approaches conformality, *i.e.* the parameter $e \to 0$, the model exhibits Miransky scaling. This is shown most clearly by the behavior of the IR scale which appears in the vacuum mass spectrum. The scale as a function of $e$ is plotted in Fig. 3.5.

## 3.2 VQCD

In the quest for a more realistic holographic model for QCD-like theories, introduction of dynamical fermions is the next logical step forward from IHQCD. As in the case of IHQCD, we can look for inspiration from string theory and top-down holographic models. The reason for non-existence of quarks at the string theory level is that as long as there is only a single stack of branes, all open strings go from the stack to itself. Therefore they each carry two indices denoting which brane their attached to, which in the duality become indices in the adjoint representation of the gauge group. A setup is needed where only one end of a string is attached to the D3-brane. This can be achieved by introducing more branes, so called flavor branes, such that the local $U(N_\mathrm{f})$ gauge symmetry on these branes becomes dual to the global flavor symmetry of the dual field theory. These flavor branes are often introduced in the 10-dimensional superstring theory set up, where all but four spacetime dimensions and the single radial direction are then compactified. [46]

For building a bottom-up -model, it is necessary to find an expression of the flavor branes in the 5 -dimensional context. This is most easily achieved by considering space-filling branes in five dimensions. Such an extension is presented by Järvinen and Kiritsis in [54], where they introduce a bottom-up holographic model called Veneziano QCD (VQCD).

### 3.2.1 Fermions from a brane-antibrane system

At the string theory level, the fermions can be introduced by adding a space-filling $D4-\bar{D}4$ brane-antibrane pair. The lowest lying excitation on this pair of branes is a tachyon, and tachyon condensation corresponds to annihilation of the branes. The action for the tachyon is the Dirac-Born-Infeld (DBI) action. These branes then carry the flavor dynamics in the bulk, and fermions become open strings whose one end is on the space filling branes and other end on the stack of D3 -branes that generates the $U(N_c)$ gauge group, and where the boundary theory lives on.

As a bottom-up model of this scenario, Järvinen and Kiritsis introduce the DBI -action of the brane pair in the holographic bulk space of IHQCD [54]:

$$S_f = -x_f M^3 N_c^2 \int d^5x V_f(\lambda, T)\sqrt{\det(g_{\mu\nu} + \kappa(\lambda, T)D_\mu T D_\nu T^\dagger + \omega(\lambda, T)F_{\mu\nu})}. \tag{3.27}$$

This is schematically the flat space DBI -action, but the potentials $V_f, \kappa$ which depend on the matter fields have been introduced. The potential $V_f$ in flat space at large $T$ is $V_f(\lambda, T) = \frac{1}{\lambda}e^{-T^2}$, and $\kappa$ is a factor coming from the transformation from string frame to the Einstein frame, which in flat space is $\lambda^{-4/3}$. Their form is expected to change in a curved background, and we will fix them based on phenomenological arguments. The field $T$ is the tachyon, which is a bifundamental field dual to the quark bilinears. For everything that follows, we consider the fermion flavors to be degenerate and therefore $T = \tau \mathbb{I}_{N_f \times N_f}$, where $\tau$ is a scalar field. The derivatives are covariant with respect to the gauge fields living on the brane, and $F_{\mu\nu}$ is the field strength for the gauge fields. The relative number of flavors is determined by $x_f \equiv \frac{N_f}{N_c}$.

The total action of the model is then the DBI action combined with the action of IHQCD,

$$S = S_{\text{IQHCD}} + S_f. \tag{3.28}$$

The model is to be considered on the Veneziano limit

$$N_c \to \infty,\ N_f \to \infty,\ x_f = \frac{N_f}{N_c} \text{ finite},\ \lambda = N_c g_{\text{YM}}^2 \text{ finite}. \tag{3.29}$$

We immediately see that at the 't Hooft limit $x_f \to 0$ and the model reduces to IHQCD. An appropriate metric ansatz for studying the background solutions is the same as in Eq. (3.18),

$$ds^2 = e^{2A(r)}\left[-f(r)dt^2 + d\vec{x}^2 + \frac{dr^2}{f(t)}\right], \tag{3.30}$$

where $f = 1$ for the zero temperature background. At zero density, the background solution for the gauge fields is $F_{\mu\nu} = 0$.

After deriving the equations of motion, we find that $\tau(r) = 0$ is one solution to them. When $\tau(r) = 0$, the model reduces to IHQCD with the potential

$$V(\lambda) = V_{\mathrm{g}} - x_{\mathrm{f}} V_{\mathrm{f}}(\lambda, 0), \tag{3.31}$$

where $V_{\mathrm{g}}$ is the potential appearing in $S_{\mathrm{IHQCD}}$ in Eq. (3.28). We can then fix this combination of potentials in the UV from the beta function as in IHQCD. Considering the beta function as a function of $x_{\mathrm{f}}$, the fixing can be done for both parts separately. This is immediately an important result: in the context of this model, the $\tau = 0$ -phase of the theory can be modeled with just the gravity dual to the gauge degrees of freedom, and the quarks enter only through the dilaton potential or the beta function. This actually post-justifies, at least to some extent, our approach in section 3.1.2.

The tachyon dependence in the potential $V_{\mathrm{f}}$ is assumed to enter the same way as in the flat space DBI action,

$$V_{\mathrm{f}}(\lambda, \tau) = V_{\mathrm{f},0}(\lambda) e^{-a(\lambda)\tau^2}, \tag{3.32}$$

where $a(\lambda)$ is related to the anomalous dimension of the quark condensate. The function $\kappa$ is chosen in such a way that $\kappa \propto \lambda^{-4/3}$ in the IR, and $\kappa = 1$ in the UV, and that it has a smooth series expansion at small $\lambda$.

All this leaves open a number of choices on concerning the interpolation of $a$ and $\kappa$ between their IR and UV asymptotes, and how to distribute the confinement factor between $V_{\mathrm{g}}$ and $V_{\mathrm{f}}$. These choices will need to be determined by how well they reproduce expected features of the dual field theory.

Once a potential has been fixed, the next step is to solve the background configuration of the model. As mentioned above, $\tau = 0$ is a solution, which leads to the same solutions for the other fields as in IHQCD, with the potential Eq. (3.31). There are also solutions with a non-zero tachyon. The requirement that the IR singularity is "good" (see [54] for definition) fixes one of the two boundary conditions, and the other determines the quark mass via the relation

$$\tau(r) = m_q r \log(r)^a + \sigma r^3 \log(r)^{-a} \text{ at small } r, \tag{3.33}$$

where the coefficient of the $r^3$ term is the chiral condensate. Fixing the quark mass then leads to a unique solution, with a non-zero $\sigma$. We also see that the $\tau = 0$ solutions are all solutions with zero-mass quarks. The $\tau \neq 0$ solution represents chiral symmetry breaking, explicit when $m_q \neq 0$, and spontaneous when $m_q = 0$. We will concentrate on the zero mass case from here on.

Comparing the free energies between the two $m_q = 0$ vacuum solutions, it turns out that the chiral symmetry breaking solution is dominant below a critical value $x_{\mathrm{f}} = x_{\mathrm{c}}$. At larger $x_{\mathrm{f}}$, the chiral symmetry breaking solution does not exist at all, and the system evolves to an IR fixed point. This is the conformal window, which the model therefore predicts. The model also predicts the walking form of the beta function at just below $x_{\mathrm{c}}$, even though the two-loop beta function, which is used as input in determining the potentials, is never of the walking form. In this model, the walking beta function comes directly as a consequence of the non-perturbative dynamics of the fermions, as seen in Fig. 3.6.

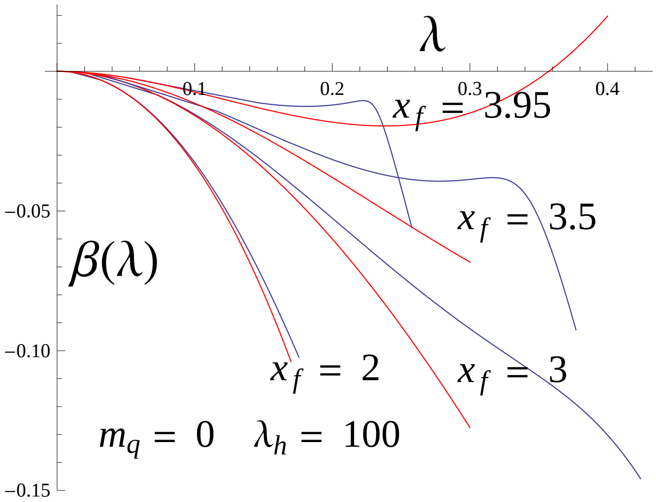


Figure 3.6: The beta function as a function of $x_\mathrm{f}$. The red curves are the input beta functions used in determining the potentials, whereas the blue curves are the beta functions computed from the holographic model.

### 3.2.2 The general phase structure

In [II], we computed the phase diagram, and a few other select thermodynamic variables in the VQCD model, in order to try to pin down the most reasonable potentials. The principles behind the thermodynamics are the same as in IHQCD, but there are now a few added complications. At zero quark density, we can take $F_{\mu\nu} = 0$. Then the only new field is the tachyon $\tau$. Both the $\tau = 0$ and non-trivial $\tau(r)$ solutions persist in the finite temperature background, with the IR singularity constraint replaced by horizon regularity for the $\tau$-field. Our analysis of the boundary conditions for IHQCD still holds for the other fields, so at fixed $m_q$ there are no new free parameters. At $m_q = 0$ there are then two separate branches of black hole solutions, both parametrized by $\lambda_h$, among which we must select the thermodynamically preferred one for each temperature. At $m_q \neq 0$, the tachyon field must be non-trivial, and there is only one branch of black hole solutions.

In order to compute the tachyonic solutions, we must determine the value $\tau_h$ of the tachyon at the black hole horizon from the desired quark mass. Defining a function $m_q(\tau_h; \lambda_h)$, we must find roots of $m_q(\tau_h; \lambda_h) = m_q$ and therefore construct a function $\tau_h(\lambda_h; m_q)$. At non-zero $m_q$, this always has a solution. When $m_q = 0$ it turns out that at small $\lambda_h$ there are no roots. Above a certain limiting value, $\lambda_\mathrm{end}$, there is one solution. As $\lambda_h$ increases, more roots appear[1]. However, all but the one at largest $\tau_h$ are unstable Efimov vacuums, and therefore the procedure of computing the tachyonic solutions at $m_q = 0$ involves specifically finding the largest root of $m_q(\tau_h) = 0$.

[1] This also happens at small $m_q$, but at large enough $m_q$ the solution is unique.

In order to define the pressure by integrating the thermodynamic relation, as in IHQCD, we have to fix the relative integration constants between the solutions. This can be done at a special value of $\lambda_{\rm end}$, since there the function $\tau_h(\lambda_h; m_q)$ goes continuously to zero as $\lambda_h \to \lambda_{\rm end}$. This means that the tachyonic background configuration also goes continuously, although not with continuous derivatives, to the non-tachyonic background configuration. Therefore all observables which depend on the solution locally, *i.e.* can be determined without computing derivatives with respect to $\lambda_h$, must also be the same for the two solutions at that point. This includes the pressure, so we have the pressure matching condition

$$p_b(\lambda_{\rm end}) = p_u(\lambda_{\rm end}), \tag{3.34}$$

where $p_b$ is the pressure in the tachyonic phase, where $b$ stands for broken chiral symmetry, and $p_u$ is the pressure in the non-tachyonic phase, where $u$ stands for unbroken chiral symmetry.

Computing the temperature as a function of $\lambda_h$ and pressure as a function of $T$, we get, for a certain choice of potentials, Fig. 3.7. This shows us the generic phase structure of the theory:

- At low temperatures, the vacuum hadron gas solution dominates, since the pressure of all phases is negative there.

- At a certain temperature $T_h$ there is a first order transition to the tachyonic chiral symmetry breaking black hole phase. This is a deconfined, but chiral symmetry breaking phase in the bulk theory. For some other potentials than the example shown here, the pressure of the chiral symmetry breaking black hole phase may also be everywhere less than that of the chirally symmetric black hole phase, in which case the transition is directly to the chirally symmetric phase.

- When the previous transition is to the chiral symmetry breaking phase, as in the example case, there is another transition at $T_{\rm end}$, where the theory transitions to the chirally symmetric black hole phase. This happens at the configuration corresponding to $\lambda_{\rm end}$, and as described above, all observables are continuous across this transition, while their derivatives are not, making this a second order transition.

- At a much higher temperature, of order $10^2 T_{\rm end}$, there is a peak in the interaction measure, which can be interpreted to correspond to a crossover from a walking near-conformal phase to the UV conformal phase. This transition is generic in this class of models, and the crossover becomes stronger when approaching the lower edge of the conformal window, that is, $x_{\rm f} \to x_{\rm c}$. This model therefore displays behaviors expected of a quasi-conformal theory without any fine tuning.

Being able to compute the thermodynamics given the set of potentials $V_{\rm f}, V_{\rm g}, \kappa$, the question is then how to fix the details of the potentials. In [II], we computed

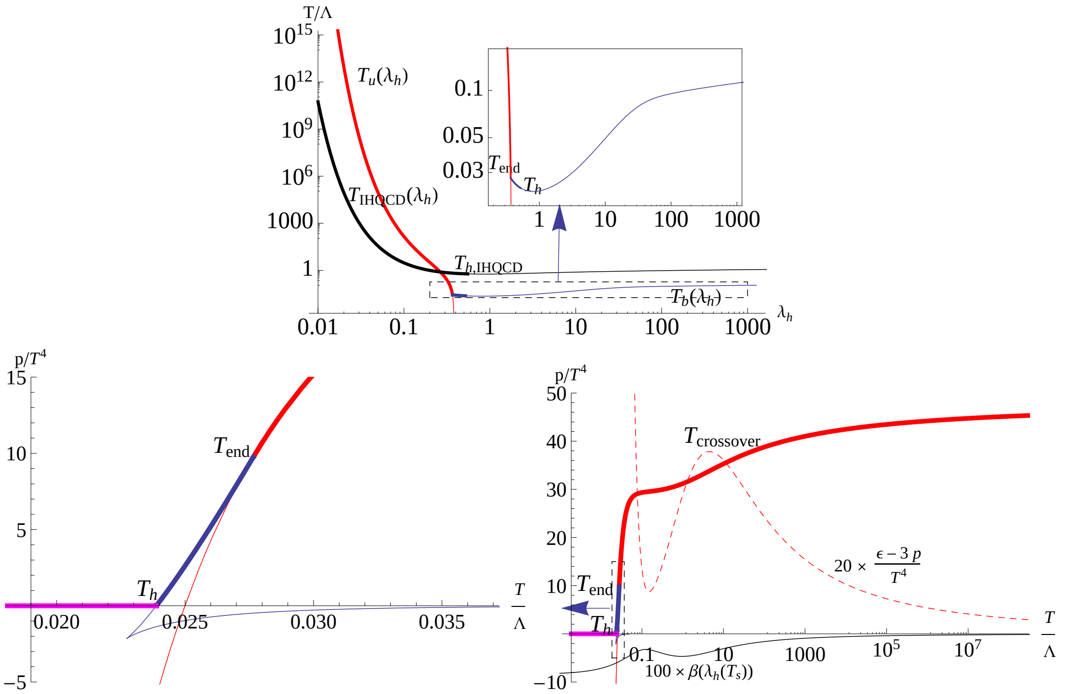


Figure 3.7: Examples of the $T_{\rm end}$, $T_h$ and $T_{\rm crossover}$ transitions with the potential termed potential II SB in [II], with $x_{\rm f} = 3$. The red curves correspond to the phases with zero tachyon, and the blue curves to phases with non-zero tachyon. *Upper*: The temperature $T(\lambda_h)$ . The inset shows the minimum of $T_b(\lambda_h)$, which causes $p_b$ to be positive between $T_h$ and $T_{\rm end}$. For comparison, we also plot $T(\lambda_h)$ for IHQCD with $x_f = 0$. *Lower left*: $p/T^4$ in a close-up around the region of the $T_h$ and $T_{\rm end}$ -transitions. *Lower right*: an overview of the pressure in the same case, also showing the interaction measure, the peak of which determines the position of $T_{\rm crossover}$. The black curve shows the vacuum beta function, scaled to fit, as a function of temperature in the symmetric phase, so that $\beta(T) = \beta(\lambda_u(T))$, where $\lambda_u(T)$ is the inverse function of $T_u(\lambda_h)$. The walking maximum of the beta function clearly coincides with the plateau related to $T_{\rm crossover}$, confirming that the $p/T^4 \sim$ constant phase below $T_{\rm crossover}$ is indeed the quasi-conformal phase related to walking dynamics.

the phase diagram in the $x_{\rm f}, T$-plane for 11 combinations of the various choices. We found that the behaviors outlined above are generic, with the main variation being whether the chiral symmetry breaking deconfined phase is stable at any temperature and $x_{\rm f}$. As a function of $x_{\rm f}$, we find a smooth variation of the transition temperatures, and at the edge of the conformal window, the temperatures scale according to Miransky scaling for all potentials. Also at certain extremes of potentials and $x_{\rm f}$ ranges, some extra phases appear, which are likely to be artifacts of a poorly chosen potential (although studying how these phases appear may be an interesting exercise in holography). We were able to narrow in on a single potential, which's phase diagram is displayed in Fig. 3.8, out of those studied in the paper, as a best candidate for modeling QCD-like theories on the Veneziano limit. Some further refinements to the potentials have been suggested

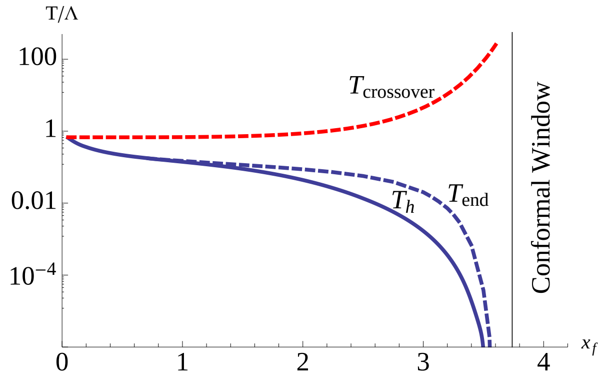


Figure 3.8: The phase diagram for the best candidate potential found in [II].

in [56].

### 3.2.3 Finite chemical potential

A net charge, *i.e.* quark density, in the boundary theory is dual to a non-zero charge in the bulk theory. The charge lives on the flavor branes, so the corresponding gauge field is the field in the DBI action. The charge needs a source, and an appropriate object sourcing the charge is a black hole. Using the charged black hole solutions, we studied VQCD at finite temperature and chemical potential in [IV]. Our goal in that paper was to develop the methods and technology to the computations, so we concentrated on the example case of the preferred potential found in the previous paper, $m_q = 0$ and $x_f = 1$.

The first step is then turning on the gauge field $F_{\mu\nu}$ in Eq. (3.27). Since we are considering degenerate quark flavors, this is a $U(1)$ gauge field, which allows us to use gauge transformations to set all other components of the bulk gauge field except $A_0$ to zero, and to decouple it from the covariant derivative. From the computational point of view then, the only change is the introduction of a new field $A_0$, and it also generates a new boundary condition. This boundary condition can be exchanged for an integration constant $\tilde{n}$, which is essentially the charge of the black hole. We can then generate solutions corresponding to pairs $\lambda_h, \tilde{n}$, and for each solution the values of the thermodynamic observables can be computed.

The ranges of the parameter $\lambda_h, \tilde{n}$ are not unlimited. In the $\mu = 0$ case we had the lower limit $\lambda_{\mathrm{end}}$ for the existence of the tachyonic solution, and an upper limit $\lambda_*$ for the existence of the non-tachyonic solution. Now both of these become regions in the $\lambda_h, \tilde{n}$ -plane, plotted in Fig. 3.9.

This diagram of the physical region has some very interesting features. At a generic point in the plane, where a solution exists, the solution has both a finite temperature and a finite chemical potential. All of the solutions corresponding to zero temperature but finite chemical potential live at the upper of two points marked $\mathrm{AdS}_2$. At that point, the horizon of the black hole becomes non-analytic,

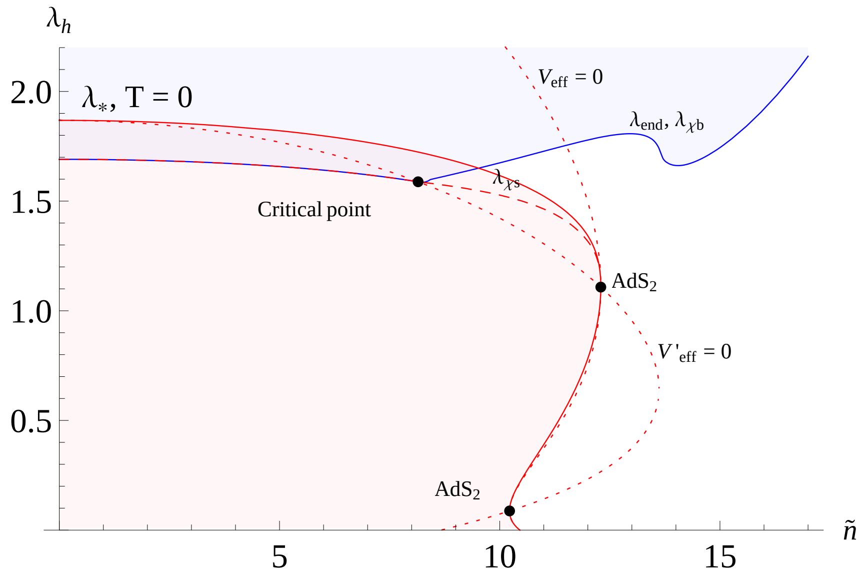


Figure 3.9: The physical region on the $\lambda_h, \tilde{n}$ plane for chirally symmetric (red region) and chirally broken (blue region, unbounded above) solutions. The chirally symmetric region is bounded from above by the curve $\lambda_*(\tilde{n})$ along which $T = 0$ up to the point $\text{AdS}_2$ at $\tilde{n} = 12.295$, $\lambda_h = 1.108$, then from the right by a segment of the curve $V_{\text{eff}} = 0$ up to the second $\text{AdS}_2$ point at $\tilde{n} = 10.223$, $\lambda_h = 0.0873$ and finally by a segment to $\tilde{n} = 10.457$, $\lambda_h = 0$. Tachyonic chiral symmetry breaking solutions exist only above the blue curve $\lambda_{\text{end}}(\tilde{n})$. This curve has a discontinuity at $\tilde{n} = \tilde{n}_{\text{cr}} \approx 8$ at which it breaks into two branches, $\lambda_{\text{end}} \equiv \lambda_{\chi b}$ and $\lambda_{\chi s}$. Below $\tilde{n}_{\text{cr}}$ the symmetric and broken phases are in thermal equilibrium along $\lambda_{\text{end}}$, above $\tilde{n}_{\text{cr}}$ the states on $\lambda_{\chi b}$ and $\lambda_{\chi s}$ are in equilibrium.

and a new degree of freedom, which translates to the value of the chemical potential in the boundary theory and the size of the non-analytic terms in the bulk, emerges. The curve bounding the existence of the chirally symmetric phase has $T = 0$, $\mu = 0$ at all points above the $\text{AdS}_2$ point, $T = 0, \mu = \infty$ below that point, and again $T = 0, \mu = 0$ below the lower $\text{AdS}_2$ point. Also note how the lower boundary of the tachyonic region has a derivative discontinuity: this point maps to a critical point on the phase diagram.

### 3.2.4 Phase diagram as a function of $T, \mu$

The phase diagram Fig. 3.10 that results from computing the dominant solutions at each $T, \mu$ shows a first order deconfining transition at small $\mu$, which goes from the hadron gas phase to the deconfined, but chiral symmetry breaking phase. Slightly above that, there is a second order transition to the chirally symmetric UV phase. Interesting things happen at slightly larger $\mu$: the second order chiral symmetry restoring transition becomes a first order transition. This is visible also in the parameter plane in Fig. 3.9, where the $\lambda_{\text{end}}$ splits into two. The higher branch in the chirally broken phase turns upward, and another branch, denoted $\lambda_{\chi s}$ in the figure, emerges and continues to the chirally symmetric phase. Between these two curves there is an equilibrium, that is, given a point with a certain temperature and chemical potential on one curve, there is a point with the same temperature and chemical potential on the other, and these two points also have the same pressure. The model produces a first order transition in a very non-trivial way. At finite but small quark mass, the phase diagram is essentially the same, except that the second order chiral symmetry restoring transition becomes first order. The critical point at $m_q = 0$ is therefore also a tricritical point.

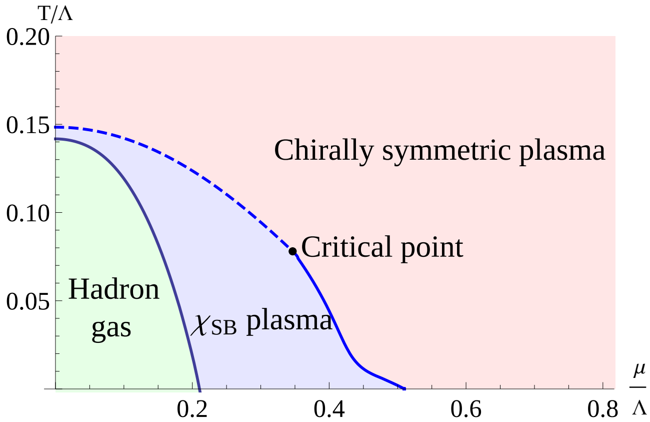


Figure 3.10: Chemical potential dependence of transition temperatures of the deconfining ($T_h(\mu)$) and chiral ($T_\chi(\mu)$) transitions at $m_q = 0$. The dashed line corresponds to a second order phase transition while the solid lines correspond to first order ones. The critical point shown in the figure is tricritical. If finite quark mass is turned on, the second order transitions become smooth crossovers and the tricritical point becomes a critical endpoint of the line of first order transitions. The $T = 0$ lines in the $\chi_{SB}$ plasma phase as well as the chirally symmetric phase correspond to a new quantum critical semilocal phase at finite density.

The $T = 0$ axis is also very interesting. As mentioned above, the $T = 0$ chirally symmetric solutions all map to a single point in the parameter space. A new parameter emerges, which describes the non-analytic near-horizon structure of the black hole, and controls the chemical potential. This is dual to a new quantum critical phase at finite density. For the chiral symmetry breaking phase, we have not yet been able to construct the exact $T = 0$ -solutions, but there also the structure of the finite temperature solutions strongly suggests the existence of a similar, but chiral symmetry breaking, phase. A closer study of these finite density but zero temperature phases is currently underway.

The chiral symmetry restoring transition in our phase diagram is broadly in agreement with the modern expectations from other methods[2] [87]. The fact that our chiral symmetry restoration becomes a crossover at finite quark mass

[2]Although modern agreement overall on the QCD phase diagram is on the level where most people in the field agree only on that there is a $T$ axis and a $\mu$ axis!

is what is expected for QCD at physical quark mass, and similarly we find a critical point at a certain chemical potential. The exact structure of the phases at $T = 0$ in our model requires further study, and it will be very interesting to compare those phases to physical observables such as dense stars.

The deconfining transition that the model produces is somewhat suspect in terms of a physical interpretation. When quarks in the fundamental representation are involved, the center symmetry of $SU(N)$ gauge theories is lost, and there is no order parameter for deconfinement. Also, the pressure in the hadron gas phase should, based on simple field theory arguments, scale with the number of degrees of freedom, which is $N_\mathrm{f}^2$. In the holographic model, pressure is constant in the hadron gas phase. In the deconfined phase, the field theory number of degrees of freedom is $2N_\mathrm{c}^2 + \frac{7}{2}N_\mathrm{f}N_\mathrm{c}$, and the ratio of this to the field theory expectation in the hadron gas is

$$\frac{x_\mathrm{f}^2}{2 + \frac{7}{2}x_\mathrm{f}}. \tag{3.35}$$

The same ratio is 0 in the model. At $x_\mathrm{f} = 1$, the field theory value is $\frac{2}{11}$, but at $x_\mathrm{f} = 4$, around the beginning of the conformal window, it is 1. This means that either the holographic model of the hadron gas phase breaks down near the conformal window, or there are some very novel features emerging from strong coupling and quasi-conformality.

# Chapter 4

# Summary and outlook

> *"The dream-narratives and cuttings collected by the professor were, of course, strong corroboration; but the rationalism of my mind and the extravagance of the whole subject led me to adopt what I thought the most sensible conclusions."*
>
> –H.P. Lovecraft, "The Call of Cthulhu", 1926–

Holographic dualities are a new way to study quantum field theories at strong coupling, originating from string theory constructions where a superconformal gauge theory living on a D3-brane is found to be dual to string theory in the near-brane $AdS_5$ -background. The two main approaches for modeling gauge theories with less symmetry are top-down and bottom-up. The top-down approach attempts to find string theory constructions whose dual is as close as possible to the desired theory, whereas the bottom-up approach is based on constructing directly an action and field content in the higher dimensional bulk that hopefully captures phenomenologically some salient features of the theory to be modeled. Neither method produces controlled approximations to the dual field theory. The top-down duals are not controlled approximations because the differences between the dual theory and the theory to be modeled are typically qualitative, such as the existence of a Kaluza-Klein -tower of excitations not present in the desired field theory. The bottom-up models are not controlled because the theory is defined in the bulk, and exact correspondence between the bulk and the boundary theory is not typically known.

In this thesis, we have considered a number of holographic models. We presented Dynamical AdS/QCD, which gives a model of chiral symmetry breaking in QCD-like models, given the running of the anomalous dimension of the quark condensate as input. At the limit where running is near-conformal, *i.e.* the model is walking, the model corroborates all the main expectations accrued over the years, during the study of walking technicolor models, concerning quasi-conformal field theories.

We exhibited the Improved Holographic QCD model, which is a string inspired bottom-up model. Here the main input from the field theory side is the running of its gauge coupling constant, which we used again to study the effects of quasi-conformality, this time in finite temperature. We found a pattern of phase transitions, with a detailed description of the melting of bound and quasi-normal states during these transitions.

An extension of the IHQCD model, Veneziano QCD, which includes an explicit model for the quark degrees of freedom, was studied at finite temperature and chemical potential, and as a function of $x_\mathrm{f}$. As a function of $x_\mathrm{f}$ in the quasi-conformal region, we again find many of the features expected to be present in such theories. Interestingly though, a light dilaton nor a vanishing $S$-parameter is not found in this limit [55]. Overall, we find a pattern of phase transitions matching well with field theory expectations. At finite chemical potential, we find a phase diagram with a second order chiral symmetry restoring transition at small $\mu$, and a first order transition at larger $\mu$. There is a tricritical point connecting these two transition curves.

The zero temperature phases of Veneziano QCD are a subject which we did not give too much attention to yet. A further study of these is likely to give interesting insights into the behavior of holographic dense matter at strong coupling. A detailed matching of the potentials to QCD and lattice results will also allow deriving quantitative predictions from the holographic models.

The Dynamic AdS/QCD -model is simpler than VQCD, and has less free parameters, which makes it more predictive and easier to compute, although perhaps less realistic. It will be interesting to put this model also to finite temperature and finite chemical potential.

# References


[1] S. Weinberg, “A Model of Leptons”, Phys.Rev.Lett. **19**, 1264–1266 (1967).

[2] P. Ade et al., “Planck intermediate results. XVI. Profile likelihoods for cosmological parameters”, (2013), arXiv:1311.1657 [astro-ph.CO].

[3] S. E. Rugh and H. Zinkernagel, “The Quantum vacuum and the cosmological constant problem”, Stud.Hist.Philos.Mod.Phys. **33**, 663–705 (2002), arXiv:hep-th/0012253 [hep-th].

[4] M. Parker, “Beyond the Standard Model”, Phys.Scripta **T158**, 014015 (2013).

[5] T. Appelquist, M. Piai, and R. Shrock, “Fermion masses and mixing in extended technicolor models”, Phys.Rev. **D69**, 015002 (2004), arXiv:hep-ph/0308061 [hep-ph].

[6] T. Appelquist, N. D. Christensen, M. Piai, and R. Shrock, “Flavor-changing processes in extended technicolor”, Phys.Rev. **D70**, 093010 (2004), arXiv:hep-ph/0409035 [hep-ph].

[7] F. Sannino and K. Tuominen, “Orientifold theory dynamics and symmetry breaking”, Phys.Rev. **D71**, 051901 (2005), arXiv:hep-ph/0405209 [hep-ph].

[8] T. Appelquist, G. T. Fleming, and E. T. Neil, “Lattice Study of Conformal Behavior in SU(3) Yang-Mills Theories”, Phys.Rev. **D79**, 076010 (2009), arXiv:0901.3766 [hep-ph].

[9] T. Appelquist, G. T. Fleming, and E. T. Neil, “Lattice study of the conformal window in QCD-like theories”, Phys.Rev.Lett. **100**, 171607 (2008), arXiv:0712.0609 [hep-ph].

[10] A. Deuzeman, M. Lombardo, and E. Pallante, “Evidence for a conformal phase in SU(N) gauge theories”, Phys.Rev. **D82**, 074503 (2010), arXiv:0904.4662 [hep-ph].

[11] Y. Shamir, B. Svetitsky, and T. DeGrand, “Zero of the discrete beta function in SU(3) lattice gauge theory with color sextet fermions”, Phys.Rev. **D78**, 031502 (2008), arXiv:0803.1707 [hep-lat].

[12] Y. Iwasaki, K. Kanaya, S. Kaya, S. Sakai, and T. Yoshie, “Phase structure of lattice QCD for general number of flavors”, Phys.Rev. **D69**, 014507 (2004), arXiv:hep-lat/0309159 [hep-lat].

[13] S. Catterall and F. Sannino, “Minimal walking on the lattice”, Phys.Rev. **D76**, 034504 (2007), arXiv:0705.1664 [hep-lat].

[14] S. Catterall, L. Del Debbio, J. Giedt, and L. Keegan, “MCRG Minimal Walking Technicolor”, Phys.Rev. **D85**, 094501 (2012), arXiv:1108.3794 [hep-ph].

[15] A. Hietanen, J. Rantaharju, K. Rummukainen, and K. Tuominen, “Minimal technicolor on the lattice”, Nucl.Phys. **A820**, 191C–194C (2009).

[16] A. J. Hietanen, K. Rummukainen, and K. Tuominen, “Evolution of the coupling constant in SU(2) lattice gauge theory with two adjoint fermions”, Phys.Rev. **D80**, 094504 (2009), arXiv:0904.0864 [hep-lat].

[17] T. Karavirta, J. Rantaharju, K. Rummukainen, and K. Tuominen, “Determining the conformal window: SU(2) gauge theory with $N_f = 4$, 6 and 10 fermion flavours”, JHEP **1205**, 003 (2012), arXiv:1111.4104 [hep-lat].

[18] J. Rantaharju, K. Rummukainen, and K. Tuominen, “Running coupling in SU(2) with adjoint fermions”, (2013), arXiv:1301.2373 [hep-lat].

[19] S. Hawking and R. Penrose, “The Singularities of gravitational collapse and cosmology”, Proc.Roy.Soc.Lond. **A314**, 529–548 (1970).

[20] R. L. Arnowitt, S. Deser, and C. W. Misner, “The Dynamics of general relativity”, Gen.Rel.Grav. **40**, 1997–2027 (2008), arXiv:gr-qc/0405109 [gr-qc].

[21] J. M. Maldacena, “The Large N limit of superconformal field theories and supergravity”, AIP Conf.Proc. **484**, 51–63 (1999).

[22] D. Marolf, “Holography without strings?”, Class.Quant.Grav. **31**, 015008 (2013), arXiv:1308.1977 [hep-th].

[23] D. D. Dietrich, “Hadrons on the worldline, holography, and Wilson flow”, (2013), arXiv:1312.5718 [hep-ph].

[24] E. Kiritsis, *String theory in a nutshell* (Princeton University Press, 2007).

[25] H. Nastase, “Introduction to AdS-CFT”, (2007), arXiv:0712.0689 [hep-th].

[26] G. ’t Hooft, “The Holographic principle: Opening lecture”, 72–86 (1999), arXiv:hep-th/0003004 [hep-th].

[27] S. Hawking, “Black hole explosions”, Nature **248**, 30–31 (1974).

[28] S. Hawking, “Particle Creation by Black Holes”, Commun.Math.Phys. **43**, 199–220 (1975).

[29] S. Hawking, “Breakdown of Predictability in Gravitational Collapse”, Phys.Rev. **D14**, 2460–2473 (1976).

[30] J. Bekenstein, “Statistical Black Hole Thermodynamics”, Phys.Rev. **D12**, 3077–3085 (1975).

[31] S. Hawking, "Black Holes and Thermodynamics", Phys.Rev. **D13**, 191–197 (1976).

[32] L. Susskind, "String theory and the principles of black hole complementarity", Phys.Rev.Lett. **71**, 2367–2368 (1993), arXiv:hep-th/9307168 [hep-th].

[33] S. Gubser, I. R. Klebanov, and A. M. Polyakov, "Gauge theory correlators from noncritical string theory", Phys.Lett. **B428**, 105–114 (1998), arXiv:hep-th/9802109 [hep-th].

[34] E. Witten, "Anti-de Sitter space and holography", Adv.Theor.Math.Phys. **2**, 253–291 (1998), arXiv:hep-th/9802150 [hep-th].

[35] N. Beisert, C. Ahn, L. F. Alday, Z. Bajnok, J. M. Drummond, et al., "Review of AdS/CFT Integrability: An Overview", Lett.Math.Phys. **99**, 3–32 (2012), arXiv:1012.3982 [hep-th].

[36] J. A. Minahan, "Review of AdS/CFT Integrability, Chapter I.1: Spin Chains in N=4 Super Yang-Mills", Lett.Math.Phys. **99**, 33–58 (2012), arXiv:1012.3983 [hep-th].

[37] P. Kovtun, D. Son, and A. Starinets, "Viscosity in strongly interacting quantum field theories from black hole physics", Phys.Rev.Lett. **94**, 111601 (2005), arXiv:hep-th/0405231 [hep-th].

[38] A. Rebhan and D. Steineder, "Violation of the Holographic Viscosity Bound in a Strongly Coupled Anisotropic Plasma", Phys.Rev.Lett. **108**, 021601 (2012), arXiv:1110.6825 [hep-th].

[39] S. Cremonini, "The Shear Viscosity to Entropy Ratio: A Status Report", Mod.Phys.Lett. **B25**, 1867–1888 (2011), arXiv:1108.0677 [hep-th].

[40] L. Susskind and E. Witten, "The Holographic bound in anti-de Sitter space", (1998), arXiv:hep-th/9805114 [hep-th].

[41] A. W. Peet and J. Polchinski, "UV / IR relations in AdS dynamics", Phys.Rev. **D59**, 065011 (1999), arXiv:hep-th/9809022 [hep-th].

[42] E. Kiritsis, W. Li, and F. Nitti, "Holographic RG flow and the Quantum Effective Action", (2014), arXiv:1401.0888 [hep-th].

[43] S. A. Hartnoll, "Horizons, holography and condensed matter", (2011), arXiv:1106.4324 [hep-th].

[44] J. McGreevy, "Holographic duality with a view toward many-body physics", Adv.High Energy Phys. **2010**, 723105 (2010), arXiv:0909.0518 [hep-th].

[45] G. T. Horowitz, "Introduction to Holographic Superconductors", Lect.Notes Phys. **828**, 313–347 (2011), arXiv:1002.1722 [hep-th].

[46] J. Erdmenger, N. Evans, I. Kirsch, and E. Threlfall, "Mesons in Gauge/Gravity Duals - A Review", Eur.Phys.J. **A35**, 81–133 (2008), arXiv:0711.4467 [hep-th].

[47] Y. Kim, I. J. Shin, and T. Tsukioka, "Holographic QCD: Past, Present, and Future", Prog.Part.Nucl.Phys. **68**, 55–112 (2013), arXiv:1205.4852 [hep-ph].

[48] L. Anguelova, P. Suranyi, and L. Wijewardhana, "Holographic Walking Technicolor from D-branes", Nucl.Phys. **B852**, 39–60 (2011), arXiv:1105.4185 [hep-th].

[49] C. Nunez, I. Papadimitriou, and M. Piai, "Walking Dynamics from String Duals", Int.J.Mod.Phys. **A25**, 2837–2865 (2010), arXiv:0812.3655 [hep-th].

[50] U. Gursoy and E. Kiritsis, "Exploring improved holographic theories for QCD: Part I", JHEP **0802**, 032 (2008), arXiv:0707.1324 [hep-th].

[51] U. Gursoy, E. Kiritsis, and F. Nitti, "Exploring improved holographic theories for QCD: Part II", JHEP **0802**, 019 (2008), arXiv:0707.1349 [hep-th].

[52] U. Gursoy, E. Kiritsis, L. Mazzanti, and F. Nitti, "Holography and Thermodynamics of 5D Dilaton-gravity", JHEP **0905**, 033 (2009), arXiv:0812.0792 [hep-th].

[53] U. Gursoy, E. Kiritsis, L. Mazzanti, and F. Nitti, "Improved Holographic Yang-Mills at Finite Temperature: Comparison with Data", Nucl.Phys. **B820**, 148–177 (2009), arXiv:0903.2859 [hep-th].

[54] M. Järvinen and E. Kiritsis, "Holographic Models for QCD in the Veneziano Limit", JHEP **1203**, 002 (2012), arXiv:1112.1261 [hep-ph].

[55] D. Arean, I. Iatrakis, M. Järvinen, and E. Kiritsis, "V-QCD: Spectra, the dilaton and the S-parameter", Phys.Lett. **B720**, 219–223 (2013), arXiv:1211.6125 [hep-ph].

[56] D. Arean, I. Iatrakis, and M. Järvinen, "The spectrum of (h)QCD in the Veneziano limit", PoS **Corfu2012**, 129 (2013), arXiv:1305.6294 [hep-ph].

[57] J. Erlich, E. Katz, D. T. Son, and M. A. Stephanov, "QCD and a holographic model of hadrons", Phys.Rev.Lett. **95**, 261602 (2005), arXiv:hep-ph/0501128 [hep-ph].

[58] J. Polchinski and M. J. Strassler, "The String dual of a confining four-dimensional gauge theory", (2000), arXiv:hep-th/0003136 [hep-th].

[59] A. Karch, E. Katz, and N. Weiner, "Hadron masses and screening from AdS Wilson loops", Phys.Rev.Lett. **90**, 091601 (2003), arXiv:hep-th/0211107 [hep-th].

[60] S. Hong, S. Yoon, and M. J. Strassler, "On the couplings of vector mesons in AdS / QCD", JHEP **0604**, 003 (2006), arXiv:hep-th/0409118 [hep-th].

[61] L. Da Rold and A. Pomarol, "Chiral symmetry breaking from five dimensional spaces", Nucl.Phys. **B721**, 79–97 (2005), arXiv:hep-ph/0501218 [hep-ph].

[62] D. T. Son and A. O. Starinets, "Minkowski space correlators in AdS / CFT correspondence: Recipe and applications", JHEP **0209**, 042 (2002), arXiv:`hep-th/0205051 [hep-th]`.

[63] P. K. Kovtun and A. O. Starinets, "Quasinormal modes and holography", Phys.Rev. **D72**, 086009 (2005), arXiv:`hep-th/0506184 [hep-th]`.

[64] M. Bertolini, P. Di Vecchia, M. Frau, A. Lerda, and R. Marotta, "N=2 gauge theories on systems of fractional D3/D7 branes", Nucl.Phys. **B621**, 157–178 (2002), arXiv:`hep-th/0107057 [hep-th]`.

[65] A. Karch and E. Katz, "Adding flavor to AdS / CFT", JHEP **0206**, 043 (2002), arXiv:`hep-th/0205236 [hep-th]`.

[66] M. Grana and J. Polchinski, "Gauge / gravity duals with holomorphic dilaton", Phys.Rev. **D65**, 126005 (2002), arXiv:`hep-th/0106014 [hep-th]`.

[67] M. Kruczenski, D. Mateos, R. C. Myers, and D. J. Winters, "Meson spectroscopy in AdS / CFT with flavor", JHEP **0307**, 049 (2003), arXiv:`hep-th/0304032 [hep-th]`.

[68] P. Breitenlohner and D. Z. Freedman, "Stability in gauged extended supergravity.", Annals of Physics **144**, 249– (1982).

[69] P. Breitenlohner and D. Z. Freedman, "Positive energy in anti-de Sitter backgrounds and gauged extended supergravity", Physics Letters B **115**, 197–201 (1982).

[70] W. H. Press, S. A. Teukolsky, W. T. Vetterling, and B. P. Flannery, *Numerical recipes 3rd edition: the art of scientific computing*, 3rd ed. (Cambridge University Press, New York, NY, USA, 2007).

[71] D. B. Kaplan, J.-W. Lee, D. T. Son, and M. A. Stephanov, "Conformality Lost", Phys.Rev. **D80**, 125005 (2009), arXiv:`0905.4752 [hep-th]`.

[72] J. Braun, C. S. Fischer, and H. Gies, "Beyond Miransky Scaling", Phys.Rev. **D84**, 034045 (2011), arXiv:`1012.4279 [hep-ph]`.

[73] J. Alanen and K. Kajantie, "Thermodynamics of a field theory with infrared fixed point from gauge/gravity duality", Phys.Rev. **D81**, 046003 (2010), arXiv:`0912.4128 [hep-ph]`.

[74] J. Alanen, K. Kajantie, and V. Suur-Uski, "A gauge/gravity duality model for gauge theory thermodynamics", Phys.Rev. **D80**, 126008 (2009), arXiv:`0911.2114 [hep-ph]`.

[75] J. Alanen, K. Kajantie, and K. Tuominen, "Thermodynamics of Quasi Conformal Theories From Gauge/Gravity Duality", Phys.Rev. **D82**, 055024 (2010), arXiv:`1003.5499 [hep-ph]`.

[76] I. R. Klebanov and J. M. Maldacena, "Superconformal gauge theories and non-critical superstrings", Int.J.Mod.Phys. **A19**, 5003–5016 (2004), arXiv:`hep-th/0409133 [hep-th]`.

[77] S. Kuperstein and J. Sonnenschein, "Non-critical, near extremal AdS(6) background as a holographic laboratory of four dimensional YM theory", JHEP **0411**, 026 (2004), arXiv:`hep-th/0411009 [hep-th]`.

[78] C. Csaki and M. Reece, "Toward a systematic holographic QCD: A Braneless approach", JHEP **0705**, 062 (2007), arXiv:`hep-ph/0608266 [hep-ph]`.

[79] A. Karch, E. Katz, D. T. Son, and M. A. Stephanov, "Linear confinement and AdS/QCD", Phys.Rev. **D74**, 015005 (2006), arXiv:`hep-ph/0602229 [hep-ph]`.

[80] S. S. Gubser and A. Nellore, "Mimicking the QCD equation of state with a dual black hole", Phys.Rev. **D78**, 086007 (2008), arXiv:`0804.0434 [hep-th]`.

[81] B. Batell and T. Gherghetta, "Dynamical Soft-Wall AdS/QCD", Phys.Rev. **D78**, 026002 (2008), arXiv:`0801.4383 [hep-ph]`.

[82] W. de Paula, T. Frederico, H. Forkel, and M. Beyer, "Dynamical AdS/QCD with area-law confinement and linear Regge trajectories", Phys.Rev. **D79**, 075019 (2009), arXiv:`0806.3830 [hep-ph]`.

[83] J. Kapusta and T. Springer, "Potentials for soft wall AdS/QCD", Phys.Rev. **D81**, 086009 (2010), arXiv:`1001.4799 [hep-ph]`.

[84] J. L. Goity and R. C. Trinchero, "Holographic models and the QCD trace anomaly", Phys.Rev. **D86**, 034033 (2012), arXiv:`1204.6327 [hep-ph]`.

[85] T. A. Ryttov and F. Sannino, "Supersymmetry inspired QCD beta function", Phys.Rev. **D78**, 065001 (2008), arXiv:`0711.3745 [hep-th]`.

[86] O. Antipin and K. Tuominen, "Resizing the Conformal Window: A beta function Ansatz", Phys.Rev. **D81**, 076011 (2010), arXiv:`0909.4879 [hep-ph]`.

[87] M. Stephanov, "QCD phase diagram: An Overview", PoS **LAT2006**, 024 (2006), arXiv:`hep-lat/0701002 [hep-lat]`.

# Paper I

J. Alanen, T. Alho, K. Kajantie, and K. Tuominen, "Mass spectrum and thermodynamics of quasi-conformal gauge theories from gauge/gravity duality", Phys.Rev. **D84**, 086007 (2011), arXiv:`1107.3362 [hep-th]`

# Paper II

T. Alho, M. Järvinen, K. Kajantie, E. Kiritsis, and K. Tuominen, "On finite-temperature holographic QCD in the Veneziano limit", JHEP **1301**, 093 (2013), arXiv:`1210.4516 [hep-ph]`

# Paper III

T. Alho, N. Evans, and K. Tuominen, “Dynamic AdS/QCD and the Spectrum of Walking Gauge Theories”, Phys.Rev. **D88**, 105016 (2013), arXiv:`1307.4896 [hep-ph]`

# Paper IV

T. Alho, M. Järvinen, K. Kajantie, E. Kiritsis, C. Rosen, and K. Tuominen, “A holographic model for QCD in the Veneziano limit at finite temperature and density”, (2013), arXiv:1312.5199 [hep-ph], to appear in JHEP