Author(s): Eyvindson, Kyle; Hartikainen, Markus; Kurttila, Mikko

Title: Towards constructing a Pareto front approximation for use in interactive forest management planning

Year: 2013

Version:

Please cite the original version:

Eyvindson, K., Hartikainen, M., & Kurttila, M. (2013). Towards constructing a Pareto front approximation for use in interactive forest management planning. In J. Tucek, R. Smrecek, A. Majlingová, & J. Garcia-Gonzalo (Eds.), Implementation of DSS Tools into the Forestry Practice: Reviewed Conference Proceedings (pp. 83-91). Technical University in Zvolen.

Towards constructing a Pareto front approximation for use in interactive forest management planning

K. Eyvindson[1*], M. Hartikainen[2**] & M. Kurttila[3]
[1]University of Helsinki, Department of Forest Sciences, Finland [2]University of Jyväskylä, Department of Mathematical Information Technology, Finland
[3]Finnish Forest Research Institute, Finland
**Supported by the Academy of Finland (grant number 127681)*
***Partly supported by the Academy of Finland (grant number 128495) and the Jenny and Antti Wihuri Foundation*

Abstract

The selection of an appropriate multi-objective forest management plan can be a difficult task due to the vast number of alternatives available to the decision maker (DM). The complexity of the task depends e.g. on how clear the preferences of the DM are. For those DMs who do not have clear preferences, interactive methods of forest planning could assist in clarifying preferences and guiding the selection in an efficient fashion. Interactive planning methods are useful when the DM needs to consider a wide range of efficient solutions quickly. With large forest holdings or with complicated forest management goals, the development of new forest plans can become a rather computationally demanding task. The time taken to generate a new alternative limits the number of forest management plans the DM is able to or willing to consider prior to making a decision. To ease the computational demands and to limit the required waiting time, the aim of this study was to develop an approach which is able to approximate the Pareto front of the planning situation problem. The use of the approach allows the DM(s) to quickly consider new approximate solutions, which are representations of potential optimized solutions. A method called PAINT, which uses a predetermined number of Pareto optimal solutions to develop the approximation, was be used for this purpose. The suitability of the method for forest planning purposes was highlighted through the development of a Pareto front approximation for a large forest holding.

Keywords: Interactive Decision Making, Optimization, Forest management planning

1 Introduction

A common feature of modern forest management plans is the requirement to meet or exceed a range of often conflicting requirements. Multi-objective forest planning attempts to coordinate, manage and illustrate the interdependencies of these requirements. Finally, it suggests a schedule of actions which will direct the management of the forest resources to meet the needs of the owners in a best possible way. There are several significant challenges when conducting multi-objective forest planning. The initial challenge is to describe the potential range of alternatives to the decision maker (DM). The second challenge is to elicit accurate preferences from the DM, and if there are multiple DMs aggregate these preferences in a systematic and equitable fashion.

Developing a forest management plan which corresponds directly to the preferences and demands of the DM can be done in a variety of different fashions. If the DM can describe the requirements of the plan in a specific mathematical context the plan can be generated in a straight forward fashion. If the DM cannot state his/her preferences in a specific fashion, then an interactive process may be required to assist the DM in clarifying their preferences and finding a plan which is feasible according to the production possibilities of the forest. During the iterative process, the DM can have his/her preferences elicited through a variety of different methods, and alternative management options for the planning area can be developed in optimization calculations with respect to these preferences. For DMs with a well-defined understanding of their personal preferences, the iterative process should be rather short and straight forward. For those DMs with only vaguely defined preferences and a poor understanding of the potential resources available from the forest, the understanding on the preferences and forest use may develop and they may change during the iterative planning process. This can make the process longer and more complicated. For both of these cases, in order to maintain interest and promote repeated iterations, the iterative process should be smooth and the time taken to compute alternative management plans should not cause excessive delay.

The use of interactive optimization processes to develop forest plans has a history spanning over three decades. As a generalization, the process requires the DM to provide either weights (Pykäläinen [1]) or targets (Mykkänen [2]) for each criterion under analysis. The process is then iterated until the DM finds an acceptable solution or does not wish to continue (Korhonen et al. [3]). In one of the first applications for forest management Steuer and Schuler [4] developed a method which elicited the DM's choice from a variety of alternatives and created updated plans based on the DM's selection. More recently, interactive methods were used to support biodiversity protection decisions (Pykäläinen and Kurttila [5]) and to generate forest plans directly suited to the forest owner based on both a priori and a posteriori preference information (Eyvindson et al. [6]).

A common feature of interactive planning is the use of iterative methods to clarify and specify the preferences of the DM. In order to promote the successful application of the interactive processes the iterative process should be easy to use, and generate alternatives rapidly. The amount of time that a DM is willing to allocate to a specific decision making process is finite, and the DM needs to believe that the process is providing them with valuable guidance for decision making. If too much time is taken in generating the alternative solutions, the DM may be unwilling conduct the iterative process multiple times (Korhonen et al. [3]).

As a way to limit the amount of time taken during the interactive process dedicated to generating alternative forest plans, it could be beneficial to utilize an approximation of the Pareto front for the criteria under consideration (Hartikainen et al. [7]). The primary advantage of using a Pareto approximation is the significant reduction in the time required to computing the values of new alternatives. This is extremely important and evident in large-scale participatory processes, where the participants are not willing to gather to several meetings that concentrate only on examination and modification of alternatives. In addition, with datasets having e.g. 100 000 stands and a lot of treatment alternatives for individual stands, creation of new alternatives can take a lot of time. . For example, the simulation of new set of treatment alternatives for e.g. 100 000 stands can take several hours and the time demand increases if e.g. additional preceding GIS operations are needed before simulations. While there are other methods available which generate the Pareto front approximation, the method developed by Hartikainen et al. [8] is well suited to forest management planning because it can be used in conjunction with any interactive method. The use of this method allows the DM to quickly progress through a series of iterations, allowing him/her to adjust their preferences in a fluent fashion.

The aim of this paper is to become familiar with and gain insight from using the Pareto front approximation in a rather typical and computationally not very demanding forest management planning problem. The Pareto front approximation method is demonstrated with a three objective Pareto front approximation, which is created based on a set of systematically generated forest management plans.

2 Materials and Methods

2.1 Generating alternative forest plans

This case study will apply the Pareto front approximation method to a forest holding in Finland. This means that the input variables for the creation of the Pareto front approximation are Pareto efficient forest plan alternatives created for this holding. The forest holding used in this analysis is a subset of a larger forest holding managed by Metsähallitus used for research and teaching. For this study, the forest tract under analysis is 721.4 ha, consisting of a total of 423 stands ranging in size from less than 0.1 ha to over 21 ha, with the most recent inventory conducted in March 2008. This forest holding is significantly larger than the average private forest holding (Finnish Statistical Yearbook of Forestry [9]), and has been selected to highlight the potential for problems with a large amount of Pareto optimal forest plan alternatives. The planning horizon was set to 10-year plans, which corresponds to the current standard practice in Finland.

Prior to generating any forest management plans, a set of relevant criteria were selected for the analysis. In order to highlight the key features of multi-objective forest planning, to be able to examine the key features of the method and to allow visualization of the Pareto front, a set of three criteria was used. The criteria variables used in the analysis were:

1. Net income (in €) – for the entire 10-year period. Total income received from harvesting operations, less costs related to tending the forest.
2. Total standing wood volume at the end of the 10-year period (in m3) – includes pulp and saw logs. Can be thought of as total future cutting opportunities.
3. Mature forest at the end of the 10 year period (in hectares) – the area of forests over 80 years old. This variable can mean that improvements are made to recreational activities, to the forest landscape, to biodiversity and ecological values.

In order to create alternative forest plans which provides a reasonable description of the entire production frontier, a systematic approach was adopted to generate variety between the criteria values. The treatment schedules for the stands were simulated by MELA planning software (Redsven et al. [10]) and the forest management plans were created through the use of linear programming of the J software (Lappi [11]). The J program solves a model-I problem, which selects an optimal set of treatment schedules for the stands based on the stated objective function and constraints (Lappi [11]).

The approach used to generate the alternative plans began by maximizing (or minimizing) one criterion. In the following problem formulations, the other criteria values were constrained in a systematic fashion to either 1/3 or 2/3 of the difference from the minimum possible value to the maximum possible value plus the minimum possible value or to give the criteria no constraints (Table 1). Through the use of a systematic approach the specified set of alternatives generated will provide some representation over the entire production possibility frontier. In order to test accuracy of the Pareto approximation, a larger set of 88 plans (a small subset of these plans is provided in Table 2) was created as a way to provide a comparison between the solution from the Pareto approximation and the true Pareto solution.

2.2 Generating the Pareto front approximation

The Pareto front approximation of this paper was constructed with the PAINT method (Hartikainen et al. [8]). PAINT method is designed for interpolating between a given set of Pareto optimal solutions to a multiobjective optimization problem. The method is specially developed in a way that it will not produce an interpolation containing approximate solutions that would either dominate or be dominated with any of the given solutions. As shown e.g., by Hartikainen et al. [12], general interpolation approaches may generate interpolations that contain dominated or dominating approximate solutions and such interpolations may mislead the DM.

	Lower bounds for the criteria, “free” if no bound and “max” for the optimized criteria			Actual criteria values		
	Net Income	Volume	Old Growth	Net Income	Volume	Old Growth
Plan 1	max	free	free	367 745	47 395	61.9
Plan 2	free	max	free	-4 590	114 413	84.1
Plan 3	free	free	max	131 206	83 192	92.2
Plan 4	max	free	72.0	366 237	47 553	73.2
Plan 5	max	69 734	free	288 755	69 734	62.7
Plan 6	max	free	82.1	357 832	48 812	83.3
Plan 7	max	92 073	free	156 687	92 073	69.9
Plan 8	max	92 073	72.0	156 602	92 073	73.2
Plan 9	max	69 734	82.1	286 220	69 734	83.3
Plan 10	max	92 073	82.1	155 429	92 073	83.3
Plan 11	max	69 734	72.0	288 178	69 734	73.2
Plan 12	243 633	max	free	243 633	77 869	63.3
Plan 13	243 633	max	72.0	243 633	77 821	73.2
Plan 14	243 633	max	82.1	243 633	77 543	83.3
Plan 15	119 521	max	free	119 521	97 650	71.1
Plan 16	119 521	max	82.1	119 521	97 479	83.3
Plan 17	119 521	max	72.0	119 521	97 623	73.2
Plan 18	free	max	72.0	-4 590	114 413	84.1
Plan 19	free	max	82.1	-4 590	114 413	84.1
Plan 20	243 633	92 073	max	243 633	77 869	63.3
Plan 21	119 521	92 073	max	119 520	92 073	91.0
Plan 22	free	92 073	max	92 032	92 515	91.0
Plan 23	243 633	69 734	max	243 633	69 870	91.0
Plan 24	119 521	69 734	max	131 205	83 192	92.2
Plan 25	free	69 734	max	131 206	83 192	92.2
Plan 26	243 633	free	max	243 633	66 001	91.0
Plan 27	119 521	free	max	243 633	66 001	91.0

Table 1. On the left side of the table, the lower bounds for criteria in optimization and the optimized criteria. On the right side of the table, the corresponding criteria values for the generated plan.

	Criteria values		
	Net Income	Volume	Old Growth
Test Plan 4	45 000	108 281	81.4
Test Plan 19	175 000	88 742	90.2
Test Plan 25	0	113 663	90.2
Test Plan 29	288 754	69 734	62.7
Test Plan 35	288 178	69 734	73.2
Test Plan 44	131 205	83 192	92.2
Test Plan 60	347 408	56 643	64.9
Test Plan 66	17 005	112 052	81.4
Test Plan 71	70 994	104 698	72.2
Test Plan 77	308 544	55 957	91.0
Test Plan 83	245 052	77 620	62.2

Table 2. A selection of the 88 plans generated for testing the accuracy of approximation.

One of the unique properties of the PAINT method is that it takes a representation of Pareto optimal solutions as given and then constructs a Pareto front approximation based on this general representation. In contrast, many other Pareto front approximation methods (surveyed e.g., in Ruzika and Wiecek [13]) either generate the Pareto optimal representation at the same time or assume some properties from this representation. Because of this property, the PAINT method can be used modularly with any problem to which we can generate a set of Pareto optimal solutions. This is also how the PAINT method was used in this paper – a set of Pareto optimal solutions was generated with the systematic approach explained in the previous section and then this set was given to the PAINT method

The PAINT method operates by first constructing the Delaunay triangulation of the given representation using the QHULL implementation of the Quickhull algorithm (Barber et al. [14]). After this PAINT starts systemically inspecting dominance relations between the polytopes in the Delaunay triangulation and the solutions in the representation and also between two polytopes in the Delaunay triangulation and removing polytopes whenever necessary. The polytopes that are not removed in this procedure are accepted into the Pareto front approximation and it is guaranteed that this approximation does not contain approximate solutions that dominate or are dominated by any of the given Pareto optimal solutions.

The Pareto front approximation constructed with PAINT can then be used in decision making through the implied multiobjective mixed integer linear problem that can be seen as a surrogate problem for the original problem. The implied surrogate problem can be efficiently used with interactive methods instead of the original (computationally expensive) problem, because many scalarizations used by interactive methods (see e.g., Miettinen and Mäkelä [15] for a survey) maintain linearity of the surrogate problem and, thus, the scalarizations can be efficiently solved with powerful solvers e.g., CPLEX. In this way, approximate Pareto optimal solutions can be provided to the DM without delay within the interactive method, and, thus, using the interactive method is smooth. Once a preferred approximate solution can be found, the closest actual Pareto optimal solution can be found by solving e.g., a reference point based achievement scalarizing problem from Wierzbicki [16].

3 Results

An approximation of the Pareto front spanned three forest management criteria and provides estimated values for the example private forest holding. Through the generation of the Pareto front approximation the forest owner has the opportunity to analyze and compare a wide range of potential forest management plans. This particular Pareto front approximation provided a comprehensive overview of the potential available from a forest management perspective. The inclusion of the extreme maximum alternatives (those plans with no constraints for the other criteria) allow the forest owner to learn about the sacrifices or trade-offs required to obtain the results at the extreme limits.

Taking a look at the generated plans (Table 1), one could first notice that some solutions appear multiple times in the data set (e.g., Plan 2 is the same as Plan 19 and Plan 3 is the same as Plan 25). Once these multiple occurrences were removed from the set of plans, 22 plans remained. In addition, some of the plans were dominated and were thus removed from consideration. For instance Plan 27 is dominated by Plan 23, because they have similar net income and mature forest area, but the total wood volume for the Plan 23 is higher than that for Plan 27. In addition, Plan 24 is dominated by Plan 25, and thus it was also removed from consideration. This left us with 20 unique mutually nondominated plans. This set of 20 plans was taken as the given set of Pareto optimal solutions for the PAINT method. The range of the Pareto optimal solutions in the set given to PAINT is for the net income from -4590 euros to 367745 euros, for the wood volume from 47395 cubic meters to 114413 cubic meters and for the mature forest area from 61.9 hectares to 92.2 hectares.

The Pareto front approximation constructed with the PAINT method contained 27 2-polytopes i.e., triangles in the three dimensional objective space. The vertices of these triangles are Pareto optimal plans from Table 1 and they are given in Table 3. This approximation is also shown in Figure 1. The Delaunay triangulation included 50 3-polytopes (i.e., 3-dimensional convex polyhedral sets), 118 2-polytopes (i.e., triangles) and 87 1-polytopes (i.e., line segments). The largest (by inclusion) Pareto front approximation constructed from these polytopes was the union of 27 2-polytopes shown in Figure 1.

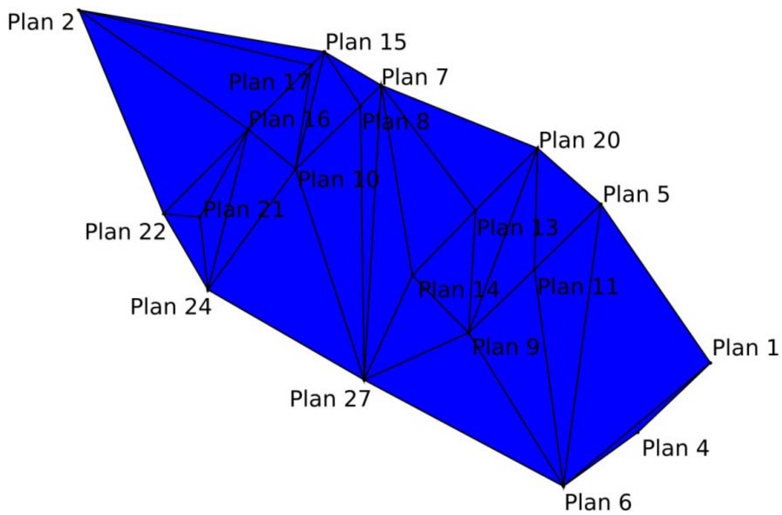

Figure 1. A figure of the approximation constructed with the PAINT method in the 3-dimensional objective space.

It is possible to calculate how well the test set (subset given in Table 3) is approximated with the Pareto front approximation constructed with PAINT. For this, we use the scaled Chebyshev distance, where the values of criteria are scaled with their respective ranges. The calculation is formulated measuring the distance of two vectors *z* and *y* in the objective space:

$$max_{i=1,2,3}\left\{\frac{|z_i - y_i|}{maximum_i - minimum_i}\right\}, \qquad (1)$$

where $minimum_i$ and $maximum_i$ *are respectively the minimum and maximum o*f objective i. When comparing, the set of Pareto optimal solutions given to PAINT with the test set, one can quickly see that the smaller set does not approximate the larger set very well. For example, Test Plan 77 (net income of 308 544€, 55 957 m^3 of wood volume and old growth forest area of 91 ha), is closest in the metric to Plan 27, where the scaled distance is approximately 0.22, or the distance for the closest point is about 22 per cent of the range of objective. The Pareto front approximation constructed with PAINT can approximate Test Plan 77 with much smaller distance of 0.09 with Polytope 26. Test plan 25 is the furthest away from the Pareto front approximation constructed with PAINT and the distance is only 0.11. This highlights that the approximation constructed with PAINT is more accurate than the closest Pareto optimal solutions.

The approximation constructed with PAINT implies a surrogate multiobjective optimization problem for the original problem, as explained in Hartikainen et al. [8]. Solving this surrogate problem corresponds to finding a preferred vector on the approximation shown in Figure 1. This surrogate problem is linear and has both integer and linear variables. Mixed integer optimization

	Vertices		
	Vertex 1	Vertex 2	Vertex 3
Polytope 1	Plan 5	Plan 6	Plan 1
Polytope 2	Plan 6	Plan 4	Plan 1
Polytope 3	Plan 7	Plan 13	Plan 20
Polytope 4	Plan 7	Plan 14	Plan 13
Polytope 5	Plan 7	Plan 27	Plan 14
Polytope 6	Plan 8	Plan 7	Plan 27
Polytope 6	Plan 9	Plan 11	Plan 6
Polytope 7	Plan 10	Plan 8	Plan 27
Polytope 8	Plan 11	Plan 5	Plan 6
Polytope 9	Plan 13	Plan 20	Plan 9
Polytope 10	Plan 14	Plan 13	Plan 9
Polytope 11	Plan 15	Plan 8	Plan 7
Polytope 12	Plan 15	Plan 10	Plan 8
Polytope 13	Plan 16	Plan 17	Plan 10
Polytope 14	Plan 16	Plan 24	Plan 10
Polytope 15	Plan 17	Plan 15	Plan 10
Polytope 16	Plan 19	Plan 16	Plan 17
Polytope 17	Plan 19	Plan 17	Plan 15
Polytope 18	Plan 19	Plan 22	Plan 16
Polytope 19	Plan 19	Plan 22	Plan 16
Polytope 20	Plan 20	Plan 9	Plan 11
Polytope 21	Plan 20	Plan 11	Plan 5
Polytope 22	Plan 21	Plan 16	Plan 24
Polytope 23	Plan 22	Plan 21	Plan 16
Polytope 24	Plan 22	Plan 21	Plan 24
Polytope 25	Plan 24	Plan 10	Plan 27
Polytope 26	Plan 27	Plan 9	Plan 6
Polytope 27	Plan 27	Plan 14	Plan 9

Table 3. Vertices of the polytopes in the Pareto front approximation.

problems may be potentially computationally hard, but the surrogate problem implied with PAINT can be solved efficiently with powerful solvers e.g., CPLEX. In this case, solving e.g., any of the scalarizations used in the synchronous NIMBUS method (Miettinen and Mäkelä [17]) took only milliseconds with CPLEX. In this way, the decision maker would not have had to wait while solving the surrogate problem with the NIMBUS method.

4 Discussion

This paper has applied a Pareto front approximation to a three criteria multi-objective forest management planning problem. The primary benefit of this method is that it provides a surrogate problem which is not computationally demanding. This provides the forest owner an opportunity to consider a wide range of outcomes for different alternative forest management plans without the requirement to actually generate the specific alternatives. This could be used to empower the forest owner to select a plan which accurately reflects his/her preferences, and thus results in more commitment to follow the plan that has been created based on his actual goals. The presented method is particularly useful for large-scale strategic planning calculations, where the participating stakeholder group can express wishes or what-if questions when exploring the alternative space of the planning situation at hand. The time savings for the participants would be evident, although the planning expert needs to create the Pareto front approximation before the actual stakeholder meeting.

The example used to highlight the process for forest management planning developed the Pareto front approximation was not so complicated and it utilized only three criteria. This was done to ensure clarity for the problem and the method. The criteria set should be dependent on the requirements and interest of the DM, i.e. the owner(s) or the participants. A quick superficial survey of scientific articles which compare forest plan alternatives through several criteria revealed that the normal range of criteria used in the analysis has been between 3 and 9 (e.g. Nordström et al. [18]; Diaz-Baltiero and Romero, [19]; Sheppard and Meitner, [20]). This method can be used to generate Pareto front approximations for problems with even larger criteria sets, as there is no restriction to the dimensionality of the problem. Thus, while this example was conducted for descriptive purposes with a limited number of criteria, an analysis with a larger set of criteria is possible and it may be desired by some DMs and planning situations.

When generating a Pareto front approximation with a larger set of criteria, a greater number of optimized forest plan alternatives will be required. This will increase the computational complexity when actually generating the Pareto front approximation, however once it has been created, the computational effort used to find alternatives on the approximation will be comparable to the approximation of a lower dimensionality. The computational burden in the generation of the Pareto front approximation could be greatly decreased through automatic alternative generation which utilizes e.g. constraints which are calculated from the range of the feasible region. Then, when incorporated with an interactive planning support tool, the DM will be able to scan the approximation for an alternative which matches his/her preferences for a large number of criteria.

Another interesting possibility of the Pareto front approximation method used is that there is a possibility to describe the maximum potential error related to the values of the criteria variables of the approximation. The error estimate could be calculated either prior to, or after the approximation has been generated (Hartikainen et al. [7]). As a result, it would be possible to develop a Pareto front approximation with a known maximum error. If either the DM or the analyst is believes that the error level is too large, a larger number of alternatives need to be created as an attempt to decrease the error level. For this process, a tool which generates optimum alternatives in a random fashion would be beneficial.

In this example, the alternative forest management plans used in developing the Pareto front approximation were created through a systematic approach. The systematic approach was used to ensure that the distribution of the alternatives covered the entire decision making space. The disadvantage of using a systematic approach is that there will be large regions of decision space without alternatives. Thus the use of other approaches to generate an appropriate set of alternatives should be explored. For instance, an approach which includes randomness into the optimization process may be useful. The approach could utilize linear programming and set a single criterion to be either maximized or minimized with the other criteria constrained randomly to a value set between the minimum and maximum for that specific criterion. This process would require a greater number of alternatives to be produced, and would lead to some sets of constraints being unfeasible. The introduction of randomness would allow for the possibility that the production frontier could be represented in equitably.

This paper introduces a potential shift in forest management planning, away from allowing the DM to compare only specific predetermined alternative plans to providing the DM an opportunity to analyze an entire spectrum of potential solutions. This shift from a discrete set of alternatives to a full continuum may make the decision process more complicated and challenging, but at the same time should improve the legitimacy of decision and more accurately reflect the DM's preferences.

5 References

[1] Pykäläinen, J. 2000. Defining forest owner's forest management goals by means of a thematic interview in interactive forest planning problems. Silva Fenn 39:67-80.

[2] Mykkänen, R. 1994. Aspiration-based utility functions in a planning model for timber flow management. Acta For Fenn 245:66.

[3] Korhonen, P., Mosokwitz, H. & Wallenius, J. 1990. Choice behaviour in interactive multiple criteria decision making. Annals of Operations Research 23:161-179.

[4] Steuer, R. and Schuler, A. 1978. An interactive multiple-objective linear programming approach to a problem in forest management. Oper Res 26:254-269.

[5] Pykäläinen, J. and Kurttila, M. 2009. Interactive method for supporting forest owners in biodiversity protection decisions. Small-scale Forestry. 8:337-348.

[6] Eyvindson, K., Hujala, T., Kurttila, M. and Kangas, A. 2012. Interactive preference elicitation incorporating a priori and a posteriori methods. Manuscript.

[7] Hartikainen, M., Miettinen, K. and Wiecek, M. M. 2011. Constructing a Pareto front approximation for decision making. Math Meth Oper Res 73:209-234.

[8] Hartikainen, M., Miettinen, K. and Wiecek, M. M. 2011. PAINT: Pareto front interpolation for nonlinear multiobjective optimization. Comput Optim Appl. doi:10.1007/s10589-011-9441-z.

[9] Finnish Statistical Yearbook of Forestry. 2011. Finnish Forest Research Institute. http://www.metla.fi/metinfo/tilasto/julkaisut/vsk/2011/vsk11_01.pdf (accessed 15.3.12.).

[10] Redsven, V., Hirvelä, H., Härkönen, K., Salminen, O., Siitonen, M., 2011. MELA2009 Reference Manual (2nd edition). The Finnish Forest Research Institute. 664 p. ISBN: 978-951-40-2283-3 (PDF).

[11] Lappi, J. 2004. J-users' guide. Version 0.9.4. Finnish Forest Research Institute, Suonenjoki Research Station. 102pp.

[12] Hartikainen, M., Miettinen, K. and Wiecek, M. M. 2011. Decision making on Pareto front approximations with inherent non-dominance. in Y. Shi and S. Wang and G. Kou and J. Wallenius (editors) New State of MCDM in the 21st Century, Selected Papers of the 20th International Conference on Multiple Criteria Decision Making 2009, 35-46

[13] Ruzika, S. and Wiecek, M. M. 2005. Approximation Methods in Multiobjective Programming. J Optim Theor Appl 126:473-501
Volume 126, Number 3, 473-501, DOI: 10.1007/s10957-005-5494-4

[14] Barber, C.B., Dobkin, D.P. and Huhdanpää, H.T. 1996 The Quickhull algorithm for convex hulls, ACM Trans. Math. Software. 22:469-483

[15] Miettinen, K. and Mäkelä, M. M. 2002. On Scalarizing Functions in Multiobjective Optimization. OR Spectrum, 24:193–213.

[16] Wierzbicki, A. P. 1986. On the completeness and constructiveness of parametric characterizations to vector optimization problems. OR Spectrum 8:73–87.

[17] Miettinen, K. and Mäkelä, M. M. 2006. Synchronous approach in interactive multiobjective optimization. European Journal of Operational Research, 170, 909–922.

[18] Nordström, E.-M., Romero, C., Eriksson, L.O. and Öhman, K. 2009. Aggregation of preferences in participatory forest planning with multiple criteria: an application to the urban forest in Lycksele, Sweden. Can. J. For. Res. 39(10): 1979–1992.

[19] Diaz-Balteiro L. and Romero, C. 2004. Sustainability of forest management plans: a discrete goal programming approach. J. Environ. Mange. 71, 351-359.

[20] Sheppard, S.R.J., Meitner, M. 2005. Using multi-criteria analysis and visualisation for sustainable forest management planning with stakeholder groups. For Ecol Manag 207:171–186.