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# TECHNICOLOR AND NEW MATTER GENERATIONS

BY  
MATTI HEIKINHEIMO

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# Preface

The work presented in this thesis has been carried out in the Department of Physics, University of Jyväskylä, between 2007 and 2010. The work was supervised by Doc. Kimmo Tuominen, to whom I am very grateful for the excellent guidance along these years. I also want to thank Prof. Kari Rummukainen and Prof. Katri Huitu, who carefully read the manuscript and provided valuable comments.

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# Abstract

This work consists of an overview part and three research papers. The subject of this work is a class of models for dynamical electroweak symmetry breaking, and new generations of fermionic matter. An introductory overview of the standard model of electroweak interactions is given, as well as an overview of some of the recent developments in the field of walking technicolor models.

We study some recently proposed models for dynamical electroweak symmetry breaking, namely the minimal walking technicolor (MWT) and next to minimal walking technicolor (NMWT) model. We show that, as a result of cancellation of the global and gauge anomalies associated with the technicolor sector, a non sequential SM-like matter generation may naturally arise. We study the effects of this new matter generation on electroweak and flavor observables and derive constraints for the masses of the new fermions. We show that the new fermions may have a significant impact on the physics of the composite Higgs boson of the technicolor theory. We present the resulting decay branching ratios and production cross sections of the composite Higgs boson. We also find that the fermions themselves should be visible in the LHC experiment, and outline basic search strategies.

We construct a model framework for the origin of fermion masses, in which a technicolor sector is accompanied by a scalar boson. In this bottom-up-approach the scalar represents the low energy spectrum of the yet unknown full gauge theory responsible for fermion masses. We construct a low energy effective Lagrangian and use electroweak and flavor precision observables, as well as direct detection limits, to constrain the parameters of the model. We find that the low energy particle spectrum of the model consists of one light and one heavy Higgs-like scalar, accompanied by three massive technipions.

We find that all of the models studied in this work are viable in the light of all existing electroweak and flavor precision data. The LHC experiment will be able to give crucial information on the subject, and possibly confirm or rule out some of the models studied in this work.



# List of publications

This thesis is based on the work contained within the following publications:

## I **Natural fourth generation of leptons**

O. Antipin, M. Heikinheimo and K. Tuominen, JHEP **0910** (2009) 018 [arXiv:0905.0622 [hep-ph]].

## II **Unnatural Origin of Fermion Masses for Technicolor**

M. Antola, M. Heikinheimo, F. Sannino and K. Tuominen, JHEP **1003** (2010) 050 [arXiv:0910.3681 [hep-ph]].

## III **The Next Generation**

O. Antipin, M. Heikinheimo and K. Tuominen, JHEP **1007** (2010) 052 [arXiv:1002.1872 [hep-ph]].

In the first paper [I] the author has participated in the construction and analysis of the model and numerically implemented the calculations concerning the oblique corrections and Higgs decay. The author has participated in the interpretation of all results.

The author has contributed to the construction of the low energy effective Lagrangian presented in the second paper [II], as well as to the derivations and interpretation of the results. The author is mainly responsible for the numerical implementation of the oblique corrections.

In the third paper [III] the author has participated in construction of the models and performed and numerically implemented the calculations concerning the oblique corrections and Higgs production and decay.

The author has participated in writing of all papers.





# Contents

<b>Preface</b>	<b>i</b>
<b>Abstract</b>	<b>iii</b>
<b>List of publications</b>	<b>v</b>
<b>1 Introduction: The Standard Model</b>	<b>1</b>
1.1 Elementary particles and interactions . . . . .	1
1.2 Electroweak symmetry breaking . . . . .	2
1.3 Naturality, hierarchy and triviality . . . . .	4
1.4 Unification . . . . .	5
1.5 Dark and bright matter . . . . .	5
<b>2 Model building</b>	<b>7</b>
2.1 Technicolor . . . . .	7
2.1.1 Dynamical symmetry breaking . . . . .	7
2.1.2 Extended technicolor . . . . .	8
2.1.3 Flavor changing neutral currents . . . . .	10
2.1.4 Walking technicolor . . . . .	11
2.2 A natural next generation . . . . .	17
2.2.1 Anomaly cancellation . . . . .	17
2.2.2 Minimal walking technicolor . . . . .	20
2.2.3 Next to minimal walking technicolor . . . . .	20
2.3 Unnatural origin of fermion masses . . . . .	21
2.3.1 The model . . . . .	21
2.3.2 Particle spectrum . . . . .	23
<b>3 Constraints</b>	<b>25</b>
3.1 Detecting new physics . . . . .	25
3.2 Oblique corrections . . . . .	26
3.3 Flavor changing neutral currents . . . . .	29
<b>4 Results</b>	<b>31</b>
4.1 Fourth generation fermions . . . . .	31
4.1.1 MWT and fourth generation of leptons . . . . .	31

4.1.2	MWT and fourth generation of QCD quarks . . . . .	37
4.1.3	NMWT and two new lepton generations . . . . .	40
4.1.4	Unification . . . . .	41
4.2	Bosonic technicolor . . . . .	42
4.2.1	Oblique corrections and FCNCs . . . . .	42
4.2.2	Results . . . . .	44
<b>5</b>	<b>Conclusions</b>	<b>47</b>
5.1	Technicolor and fourth generation of fermionic matter . . . . .	47
5.2	The origin of fermion masses . . . . .	48
5.3	Outlook . . . . .	48
	<b>References</b>	<b>51</b>
	<b>Papers I-III</b>	<b>55</b>

# Chapter 1

## Introduction: The Standard Model

### 1.1 Elementary particles and interactions

The elementary matter particles and their interactions, excluding gravity, are described by a quantum field theoretic model, the Standard Model (SM), schematically represented in fig 1.1. These interactions consist of electroweak (EW) and strong interactions. All known particles interact via the electroweak interaction. In addition to that, quarks and gluons feel the strong interaction, described by quantum chromodynamics (QCD).

In quantum field theory all interactions are due to the gauge principle: The fundamental forces are manifestations of the local symmetries of the Lagrangian of the theory, conveniently described with corresponding gauge groups. A gauge group is the group that consists of the transformations under which the Lagrangian is locally invariant. The gauge group related to the electroweak interaction is  $SU(2) \times U(1)$ , and the one corresponding to the strong interaction is  $SU(3)$ . There is a spin-1 gauge boson related to each generator of the corresponding gauge group.

The particle content of the SM consists of the gauge bosons, responsible for mediating the EW and strong interactions, and matter particles, which are spin- $\frac{1}{2}$  fermions. Each matter particle is accompanied by a corresponding antiparticle. Mass and spin of the antiparticle are equal to those of the particle, but the charges under the different gauge groups of the SM are opposite, as well as the charges associated with the global symmetries, such as baryon number. So e.g. the electron has electric charge of minus one and lepton number of one, whereas the anti-electron, the positron, has electric charge of plus one and lepton number of minus one. The matter fields are arranged into three generations, the first of which is the lightest and composes all stable matter found in nature. Second and third generation quarks and leptons are heavier and unstable, decaying to the first generation particles. The gluon and the photon are massless but the weak gauge bosons  $W$  and  $Z$  are massive.

Three Generations of Matter (Fermions)				
	I	II	III	
mass→	2.4 MeV	1.27 GeV	171.2 GeV	0
charge→	$\frac{2}{3}$	$\frac{2}{3}$	$\frac{2}{3}$	0
spin→	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1
name→	<b>u</b> up	<b>c</b> charm	<b>t</b> top	<b>γ</b> photon
Quarks	4.8 MeV	104 MeV	4.2 GeV	0
	$-\frac{1}{3}$	$-\frac{1}{3}$	$-\frac{1}{3}$	0
	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1
	<b>d</b> down	<b>s</b> strange	<b>b</b> bottom	<b>g</b> gluon
Leptons	<2.2 eV	<0.17 MeV	<15.5 MeV	91.2 GeV
	0	0	0	0
	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1
	<b>ν<sub>e</sub></b> electron neutrino	<b>ν<sub>μ</sub></b> muon neutrino	<b>ν<sub>τ</sub></b> tau neutrino	<b>Z</b> weak force
	0.511 MeV	105.7 MeV	1.777 GeV	80.4 GeV
	-1	-1	-1	$\pm 1$
	$\frac{1}{2}$	$\frac{1}{2}$	$\frac{1}{2}$	1
	<b>e</b> electron	<b>μ</b> muon	<b>τ</b> tau	<b>W</b> weak force

Figure 1.1: The particle content of the Standard Model.

## 1.2 Electroweak symmetry breaking

The gauge boson mass terms of the form  $m_A^2 A^\dagger A$  are not gauge invariant and thus can not be simply included in the Lagrangian. Instead the gauge boson masses are generated by the process of spontaneous symmetry breaking. To achieve this symmetry breaking, a scalar doublet field  $\phi$  is added to the Lagrangian, with a potential of the form<sup>1</sup>

$$V(\phi) = -\mu^2 \phi^\dagger \phi + \lambda (\phi^\dagger \phi)^2, \quad \phi = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ H \end{pmatrix}, \quad (1.1)$$

where  $\mu^2 > 0$ . Classically, this potential has a minimum at

$$H = \frac{\mu}{\sqrt{\lambda}}. \quad (1.2)$$

In quantum field theory this means that  $H$  acquires a vacuum expectation value  $\langle H \rangle = v = \frac{\mu}{\sqrt{\lambda}}$ . We then rewrite the field  $H$  as  $H(x) = v + h(x)$ , where  $h$  is a scalar field with zero vacuum expectation value  $\langle h \rangle = 0$ . The excitation of this

<sup>1</sup>Generally a scalar doublet is written as  $\phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$ , where  $\phi^+$  and  $\phi^0$  are complex fields. However, we may use the  $SU(2)$  symmetry to rotate the field  $\phi$  into the form of equation (1.1). This corresponds to choosing a gauge, in this case the unitary gauge.

field then becomes a physical massive scalar, the Higgs boson, with a potential

$$V(\phi) = \frac{1}{2}m_h^2 h^2 + \sqrt{\frac{\lambda}{2}}m_h h^3 + \frac{1}{4}\lambda h^4, \quad (1.3)$$

where  $m_h = \sqrt{2}\mu$  is the Higgs boson mass. The field  $\phi$  is coupled to the electroweak  $SU(2) \times U(1)$ -gauge fields via the covariant derivative

$$D_\mu\phi = (\partial_\mu - igA_\mu^a\tau^a - i\frac{g'}{2}B_\mu)\phi, \quad (1.4)$$

where  $\tau^a = \frac{1}{2}\sigma^a$  are the generators of  $SU(2)$ . At the minimum of the potential  $H = v$  and the kinetic term of the field  $\phi$  acquires the form

$$(D_\mu\phi)^\dagger(D^\mu\phi)\Big|_{H=v} = \frac{v^2}{8}\left(g^2(W_\mu^{+\dagger}W^{+\mu} + W_\mu^{-\dagger}W^{-\mu}) + (g^2 + g'^2)Z_\mu^\dagger Z^\mu\right) + \dots, \quad (1.5)$$

where  $W_\mu^\pm = \frac{1}{\sqrt{2}}(A_\mu^1 \mp iA_\mu^2)$  and  $Z_\mu = \frac{1}{\sqrt{g^2 + g'^2}}(gA_\mu^3 - g'B_\mu)$  are the massive weak gauge bosons and interaction terms have been neglected. The masses of the gauge bosons can be read from equation (1.5), yielding

$$m_W = \frac{gv}{2}, \quad m_Z = \sqrt{g^2 + g'^2}\frac{v}{2}. \quad (1.6)$$

The photon field  $A_\mu = \frac{1}{\sqrt{g^2 + g'^2}}(g'A_\mu^3 + gB_\mu)$  remains massless. The electron charge  $e$  and the Weinberg angle  $\theta_w$  are defined as

$$e = \frac{gg'}{\sqrt{g^2 + g'^2}}, \quad \sin\theta_w = \frac{g'}{\sqrt{g^2 + g'^2}}, \quad (1.7)$$

yielding the  $W$  and  $Z$  mass ratio

$$\frac{m_W}{m_Z} = \cos\theta_w. \quad (1.8)$$

The Higgs field is therefore needed to break the electroweak symmetry and generate masses for the weak gauge bosons. In the SM, the Higgs field is also responsible for generating fermion masses. The weak  $SU(2)$  gauge fields only couple to the left handed fermions. Thus left handed fermions are  $SU(2)$  doublets and right handed fermions are singlets.

$$E_L = \begin{pmatrix} e_L \\ \nu_{eL} \end{pmatrix}, \quad e_R, \quad Q_L = \begin{pmatrix} u_L \\ d_L \end{pmatrix}, \quad u_R, \quad d_R. \quad (1.9)$$

Since left and right handed fermions transform under different representations of the  $SU(2)$  gauge group, a mass term of the form  $m_f(\bar{f}_L f_R + \bar{f}_R f_L)$  is not gauge invariant. Instead the fermion masses originate from gauge invariant Yukawa couplings with the Higgs field:

$$\mathcal{L}_{m_q} = -\lambda_d \bar{Q}_L \phi d_R - \lambda_u \bar{Q}_L \tilde{\phi} u_R + h.c., \quad (1.10)$$

where  $\tilde{\phi} = i\sigma^2\phi^\dagger$ , yielding the quark mass terms

$$m_d = \frac{\lambda_d}{\sqrt{2}}v, \quad m_u = \frac{\lambda_u}{\sqrt{2}}v. \quad (1.11)$$

The neutrinos are considered to be massless, and thus only the left handed neutrinos are included in the SM.

### 1.3 Naturality, hierarchy and triviality

The SM has proven to be extremely accurate in describing high energy phenomena up to the scale of a couple hundred GeV, but there are reasons to believe that some form of new physics lies in the TeV scale. This is because the mathematical structure of the SM breaks down at high energies<sup>2</sup>, suggesting that the SM is not an UV-complete theory, but an effective low energy model of a more complete theory.

The problems of the SM arise from the existence of a fundamental scalar field, the Higgs field. The mass term of such a field is problematic, since the radiative corrections tend to drive it towards a very large value. The radiative corrections to the parameters of the Lagrangian of a given model usually include divergent integrals. Regulating these integrals yields a mass scale to the theory, which is interpreted as the cut-off scale above which the theory is no longer valid. The quantum effects of gravity are known to be significant at the Planck scale  $\Lambda_P \sim 10^{16}$  TeV. Since the SM does not contain a quantum theory of gravity, it is to be taken as an effective model that is only valid below the Planck scale. The radiative corrections to scalar mass are quadratically divergent, that is,  $\delta M_H \sim \Lambda^2$ . This means that in order to generate a Higgs mass of the order  $\sim 100$  GeV, the mass parameter has to be fine tuned order by order with a precision of  $M_H^2/\Lambda_P^2 \sim 10^{-34}$ . This kind of fine tuning is considered to be extremely unnatural. Also, the radiative corrections to the scalar mass are *additive*, that is,  $\delta M_H$  receives a term proportional to the mass of each particle in the theory. Alternatively this means that there is no symmetry protecting the scalar mass: even if set to zero, it will receive large radiative corrections. Given the possibility that there may exist a number of yet unknown heavy fermions (e.g. fourth generation quarks and leptons), it becomes excessively difficult to understand the low value of  $M_H$ .

The self-coupling  $\lambda$  of the Higgs field is also problematic. The renormalization group equation (RGE) implies a running of the Higgs self-coupling of the form [2]

$$\lambda(\mu) \simeq \frac{\lambda(\Lambda)}{1 + (24/16\pi^2)\lambda(\Lambda) \log(\Lambda/\mu)}. \quad (1.12)$$

---

<sup>2</sup>Actually, as discussed recently [1], the mathematical consistency of the SM may survive all the way up to the Planck scale. However, the problems related to fine tuning of the mass parameter still remain.

As the cut-off scale  $\Lambda$  is taken to infinity, the self-coupling vanishes. This problem is known as the triviality problem. To have a non-trivial Higgs sector we thus need a finite cut-off scale  $\Lambda_c$  above which the Higgs field -description of electroweak symmetry breaking is no longer valid. In other words, this implies that the scalar Higgs field is just a low energy effective model of a more complete high energy theory.

## 1.4 Unification

The SM Higgs sector is therefore interpreted as an incomplete description of the electroweak symmetry breaking. This motivates the agenda to build models beyond the SM, that are capable of breaking the electroweak symmetry without the problems associated with a SM-like elementary scalar field. In addition to breaking the electroweak symmetry one may wish that the new theory possesses some other desirable features. One such feature is the unification of the electroweak and strong interactions. Since the beginning of quantum field theory (and probably already long before that), physicists have pursued a '*theory of everything*'; a simple model that would describe all interactions of all particles. One step towards that goal would be a theory that combines the electroweak and strong gauge groups of the SM under one unified gauge group. To achieve this goal, one would need the electroweak and strong coupling constants to unify at some high scale. This means that the renormalization group equation (RGE) running of the couplings should drive them to a common value at that scale.

This does not happen in the SM, but it is possible to tweak the matter content of the SM so that this unification is achieved. One may wish that a beyond SM theory, designed to cure the problems of the electroweak sector, would at the same time drive unification of the electroweak and strong interactions. As a matter of fact, this happens if a new symmetry between fermions and bosons, called supersymmetry, is invoked [3]. Partly based on this observation, supersymmetric theories have for long been the prime candidate for beyond SM physics, but these models will not be considered here. For a review of supersymmetry, see for example [4].

## 1.5 Dark and bright matter

In addition to the problems in the mathematical structure of the Higgs sector, there are also some phenomenological problems in the SM. First, the neutrinos are massless in the SM, but there now exists direct evidence of non-zero neutrino masses [5]. However, this is not a very severe fault in the SM, since the model can easily be extended to include right handed neutrinos. A much more problematic issue in the SM is the lack of a dark matter particle and a mechanism to generate excess of baryons over antibaryons, starting from a baryon-antibaryon symmetric initial state.

The evidence of the existence of dark matter comes from numerous different sources. The orbital velocity of stars and gas clouds in spiral galaxies is inconsistent with the predictions of classical gravity, if only the visible matter in the galaxy is taken in to account. But if one assumes that the galaxy is filled and surrounded by a halo of dark matter, the rotation curves can be made compatible with gravity. Another, more direct evidence comes from observing colliding galaxy clusters. In such collision the interstellar plasma that makes up most of the visible matter of the cluster collides with the plasma of the other cluster and slows down. The weakly interacting dark matter instead passes through and thus gets separated from the visible matter. Since dark matter makes up most of the mass of the galaxy cluster, the center of gravity of the cluster moves with the dark matter and is separated from the center of radiation. Such collisions have been observed, see e.g. [6], and this separation has been verified, giving direct evidence of the existence of weakly interacting dark matter. However, the SM includes no such particle that would be this matter. Therefore one wishes that an extension of the SM would include a particle that is stable and interacts weakly enough to be a possible candidate for dark matter.

Besides the weakly interacting massive particle (WIMP) paradigm, there have been other suggestions for the content of dark matter, such as massive compact halo objects (Machos) or microscopic black holes. However, the different sources of evidence seem to suggest that a WIMP is the most likely constituent of dark matter. For a modern review of dark matter candidates, see for example [7] and the references therein.

Also, the origin of baryonic matter, i.e. visible matter in the universe remains unknown on the basis of the SM. As all matter is believed to have been born as the radiation dominated universe cooled down just moments after the big bang, all matter particles must have been produced in processes where radiation turns into matter. But the SM, excluding weak interactions, obeys a symmetry under charge conjugation and parity transformation (CP), which prevents producing different amounts of matter and antimatter in these kinds of processes. Therefore there should be as much antimatter in the universe as there is matter, but this antimatter is nowhere to be found. The weak interactions violate the CP symmetry in the SM, but these CP-violations are much too weak to explain the origin of baryonic matter. Extending the SM to account for the problems described above, one may also wish to give rise to more CP violation and thus explain the origin of matter in the universe.



# Chapter 2

## Model building

### 2.1 Technicolor

#### 2.1.1 Dynamical symmetry breaking

Dynamical symmetry breaking is a natural way to break the electroweak symmetry. Indeed, the electroweak symmetry is dynamically broken in the SM already without the Higgs sector, by QCD. Let us examine a simple case of QCD dynamics, taking into account only two massless quarks

$$Q_L = \begin{pmatrix} U_L \\ D_L \end{pmatrix}, \quad U_R, \quad D_R, \quad (2.1)$$

and let's assume there is no Higgs field and thus the weak gauge bosons are massless. The formation of the condensate

$$\langle \bar{U}U + \bar{D}D \rangle \neq 0 \quad (2.2)$$

breaks the chiral  $SU(2)_L \times SU(2)_R$ -symmetry down to  $SU(2)_V$ . The resulting Goldstone bosons are the charged and neutral pions  $\pi^\pm$  and  $\pi^0$ . These bosons couple to the electroweak gauge bosons via the  $SU(2)_L$  currents  $J_\mu^\pm$ ,  $J_\mu^0$  and the hypercharge current  $J_\mu^Y$ . The strength of the coupling is given by the pion decay constant  $F_\pi$

$$\langle 0 | J_\mu^\pm | \pi^\pm \rangle = F_\pi q_\mu, \quad \langle 0 | J_\mu^0 | \pi^0 \rangle = F_\pi q_\mu, \quad \langle 0 | J_\mu^Y | \pi^0 \rangle = F_\pi q_\mu. \quad (2.3)$$

From this coupling, the  $W$  boson propagator receives a correction of the form

$$\Delta^{\mu\nu}(q^2) = -\frac{ig^{\mu\nu}}{q^2} \rightarrow -\frac{ig^{\mu\nu}}{q^2(1 - \Pi(q^2))}, \quad (2.4)$$

where  $\Pi(q^2)$  is the vacuum polarization of the  $W$ . The massless Goldstone boson coupling to the  $W$  gives rise to the vacuum polarization of the form

$$\Pi(q^2) = \frac{g^2 F_\Pi^2}{4q^2}, \quad (2.5)$$

shifting the pole in the  $W$ -propagator in (2.4) from zero to

$$m_W^2 = \frac{1}{4}g^2 F_\pi^2. \quad (2.6)$$

The neutral bosons mix with a mixing matrix

$$m^2 = \begin{pmatrix} g^2 & gg' \\ gg' & g'^2 \end{pmatrix} \frac{1}{4}F_\pi^2, \quad (2.7)$$

yielding the masses of the  $Z$  and the photon

$$m_Z^2 = \frac{1}{4}(g^2 + g'^2)F_\pi^2, \quad m_\gamma = 0. \quad (2.8)$$

As a result, the electroweak symmetry is broken and the gauge boson masses obey the correct ratio  $m_W = m_Z \cos \theta_w$ . The scale of the masses is given by the pion decay constant  $F_\pi$ , which in QCD is of the order of 100 MeV. Thus we see that dynamical electroweak symmetry breaking already occurs in the SM, but unfortunately the gauge boson masses generated by QCD are three orders of magnitude smaller than the observed values. The simplest idea that comes to mind would then be to introduce a '*scaled up QCD*', that is, a sector identical to QCD in all other aspects, but with chiral symmetry breaking occurring at  $10^3$  times higher scale. Models that incorporate a new strong interaction in order to break the electroweak symmetry are called technicolor models [8] [9].

### 2.1.2 Extended technicolor

Given the '*scaled up QCD*' model described above, the electroweak symmetry is correctly broken and the weak gauge bosons have their correct masses, all this without an elementary scalar and the problems associated with it. But one problem still remains: in the SM, fermion masses were generated by Yukawa-couplings to the Higgs field. By removing the Higgs field, we have also removed these couplings, rendering all fermions massless. Since fermions have masses, we need to introduce a new interaction capable of providing mass terms for fermions. This interaction is called extended technicolor interaction (ETC). ETC couples techniquarks, that is, the quarks of the new QCD-like sector, to the SM fermions. The full gauge symmetry of ETC is broken at some high scale  $\Lambda_{ETC}$ , and the ETC gauge bosons become massive. Therefore, at energy scales well below  $\Lambda_{ETC}$ , this coupling is effectively represented by a four fermion interaction. This is similar to what happens in, say, the muon decay: if the masses and energies of all the particles involved in the process are much below the mass of the mediating vector boson, the interaction is effectively represented by a four fermion interaction. Diagrammatically this can be understood as shown in figure 2.1.

The resulting effective four fermion interaction is given by the operator

$$\frac{g_{ETC}^2}{\Lambda_{ETC}^2} \bar{f}_L f_R \bar{Q}_L^{TC} Q_R^{TC}, \quad (2.9)$$

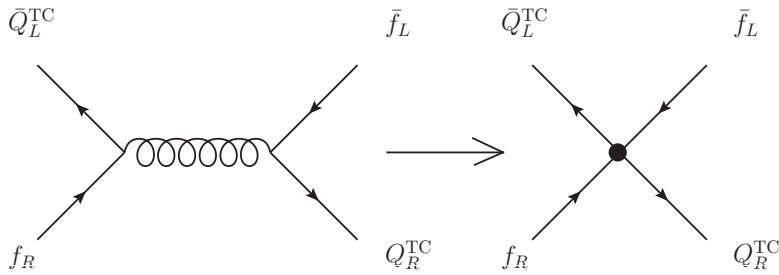


Figure 2.1: The effective four fermion interaction generated by the ETC coupling.  $f$  is a SM fermion and  $Q^{TC}$  is a techniquark. The curly line represents the ETC-vector boson with mass  $\sim \Lambda_{ETC}$ .

where  $g_{ETC}$  is the ETC-coupling constant. This operator implies fermion mass terms of the form

$$m_f \sim \frac{g_{ETC}^2}{\Lambda_{ETC}^2} \langle \bar{Q}_L^{TC} Q_R^{TC} \rangle_{ETC}, \quad (2.10)$$

where  $\langle \bar{Q}_L^{TC} Q_R^{TC} \rangle_{ETC}$  presents the vacuum expectation value of the techniquark condensate at the ETC-scale  $\Lambda_{ETC}$ . Based on RGE this can be related to the value of the condensate at the scale  $\Lambda_{TC}$  of the chiral symmetry breaking by the equation [2]

$$\langle \bar{Q}_L^{TC} Q_R^{TC} \rangle_{ETC} = \langle \bar{Q}_L^{TC} Q_R^{TC} \rangle_{TC} \exp \left( \int_{\Lambda_{TC}}^{\Lambda_{ETC}} \frac{d\mu}{\mu} \gamma_m(\mu) \right), \quad (2.11)$$

where  $\gamma_m$  is the anomalous dimension of the operator  $\bar{Q}^{TC} Q^{TC}$ . Given  $\gamma_m \ll 1$  in QCD-like dynamics, we get

$$\langle \bar{Q}_L^{TC} Q_R^{TC} \rangle_{ETC} \approx \langle \bar{Q}_L^{TC} Q_R^{TC} \rangle_{TC} \approx 4\pi F_{\Pi}^3 \quad (2.12)$$

This can be used to approximate the fermion mass term (2.10), yielding

$$m_f \sim \frac{4\pi g_{ETC}^2 F_{\Pi}^3}{\Lambda_{ETC}^2}. \quad (2.13)$$

To be able to generate the top quark mass we must have  $m_f \gtrsim m_t$ . To get correct masses for the weak gauge bosons the technipion decay constant must be of the order  $F_{\Pi} \sim 250$  GeV. These conditions can be translated into an upper limit for the ETC-scale, yielding

$$\frac{\Lambda_{ETC}}{g_{ETC}} \lesssim 1 \text{ TeV}. \quad (2.14)$$

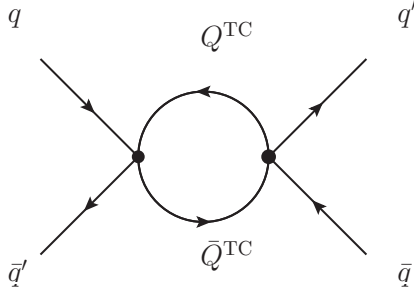


Figure 2.2: Flavor changing neutral current generated by the effective four fermion interaction.

### 2.1.3 Flavor changing neutral currents

Currents that mediate processes in which an up(down)-type quark of given generation changes into an up(down)-type quark of another generation are called flavor changing neutral currents (FCNC). That is, processes in which a quark changes flavor but not electric charge. These processes are absent in the tree level of the SM, and are experimentally known to be very rare. This is due to the GIM mechanism [10], which ensures that the neutral  $Z$  boson couples diagonally to the fermion flavors. A generic problem of the ETC-model is that the effective four fermion interaction required to build fermion masses inevitably generates flavor changing neutral currents via the diagram presented in figure 2.2.

This interaction can generally be described by the operator

$$\mathcal{L}_{FCNC} = \frac{g_{ETC}^2 V_{qq'}^2}{\Lambda_{ETC}^2} \bar{q}' \Gamma^\mu q \bar{q} \Gamma'_\mu q' + h.c., \quad (2.15)$$

where  $V_{qq'}$  is a possibly complex mixing angle factor, presumably of the order  $0.1 \lesssim |V_{ds}| \lesssim 1$ . The matrixes  $\Gamma_\mu, \Gamma'_\mu$  are the chirality matrixes associated with the ETC-vertex. We shall put  $\Gamma_\mu, \Gamma'_\mu = \frac{1}{2} \gamma_\mu (1 - \gamma_5)$  and double count the interaction to allow for any chirality arrangement. The most stringent constraint for the FCNC processes comes from the  $\Delta S = 2$  processes. We can approximate the contribution of interaction (2.15) to the neutral kaon mass difference  $\Delta M_K = m_{K_L^0} - m_{K_S^0}$  to constrain the ETC-scale  $\Lambda_{ETC}$ . The mass difference is given by

$$\Delta M_K = \frac{1}{M_K} \Re(\langle K^0 | \mathcal{L}_{FCNC} | \bar{K}^0 \rangle). \quad (2.16)$$

Putting  $q = d, q' = s$  and using the vacuum saturation approximation [11]  $\langle K^0 | \mathcal{L}_{FCNC} | \bar{K}^0 \rangle \approx |\langle 0 | \bar{d} \gamma_\mu \gamma_5 s | K^0 \rangle|^2$ , we get

$$\Delta M_K \approx \frac{g_{ETC}^2 \Re(V_{ds}^2)}{\Lambda_{ETC}^2} f_K^2 M_K, \quad (2.17)$$

where  $f_K \approx 110$  MeV is the kaon decay constant, defined by

$$\langle 0 | \bar{d} \gamma_\mu \gamma_5 s | K^0(p) \rangle = i\sqrt{2} f_K p_\mu. \quad (2.18)$$

This has to be below the measured value  $\Delta M_K = 3.48 \times 10^{-12}$  MeV [5], yielding a lower limit for the ETC-scale

$$\frac{\Lambda_{ETC}}{g_{ETC}} \gtrsim \sqrt{\Re(V_{ds}^2)} \times 1300 \text{ TeV}. \quad (2.19)$$

Putting  $\Re(V_{ds}^2) \sim \mathcal{O}(10^{-1})$ , this is clearly inconsistent with the upper limit obtained in equation (2.14). If  $V_{ds}$  is complex, we run into similar problems with the imaginary part of the  $K^0 - \bar{K}^0$  mass matrix. Scaled up QCD thus fails to produce large enough fermion masses without producing too large contributions to the flavor changing neutral current processes.

### 2.1.4 Walking technicolor

The discrepancy between the upper and lower limit for the ETC-scale from fermion masses and FCNCs can be resolved by altering the dynamics of the technicolor gauge coupling  $\alpha_{TC}$ . This means moving away from the simple scaled up QCD, into a class of models where the gauge coupling evolves very slowly, *walks* instead of running, over a certain range of energy scales.

In the evaluation of the fermion mass term (2.10) we used the approximation  $\gamma_m \ll 1$  for the anomalous dimension of the techniquark condensate, which holds well if the dynamics of the technicolor gauge theory are similar to that of QCD. But if we assume that the value of the  $\beta$ -function of the technicolor gauge theory is very small near the value of the technicolor coupling constant at the scale of the chiral symmetry breaking  $\Lambda_{TC}$ , then  $\gamma_m$  may be large in that energy range. This means that we assume that the coupling constant is near a critical value at the chiral symmetry breaking scale, that is  $\alpha_{TC}(\Lambda_{TC}) \lesssim \alpha_{TC}^*$ , where  $\alpha_{TC}^*$  is an infrared fixed point of the  $\beta$ -function,  $\beta(\alpha_{TC}^*) = 0$ . Following the discussion in [12] we conclude that near such fixed point the value of the anomalous dimension  $\gamma_m$  is close to unity. If we then make the assumption that  $\alpha_{TC}$  remains near its critical value over the range  $\Lambda_{TC}$  to  $\Lambda_{ETC}$ , we can substitute the approximation  $\gamma_m(\mu) \approx 1$  to equation (2.11), yielding

$$\langle \bar{Q}_L^{TC} Q_R^{TC} \rangle_{ETC} \approx \frac{\Lambda_{ETC}}{\Lambda_{TC}} \langle \bar{Q}_L^{TC} Q_R^{TC} \rangle_{TC}. \quad (2.20)$$

This enhances the fermion masses generated by the effective four fermion interaction by a factor of  $\Lambda_{ETC}/\Lambda_{TC}$ , which we assume to be of the order of  $10^2 - 10^3$ , therefore allowing for high enough value for the ETC-scale to suppress the FCNC interactions below the experimentally allowed values.

Walking behavior is thus needed to suppress the FCNCs while still being able to produce heavy fermion masses. To achieve this kind of behavior, the

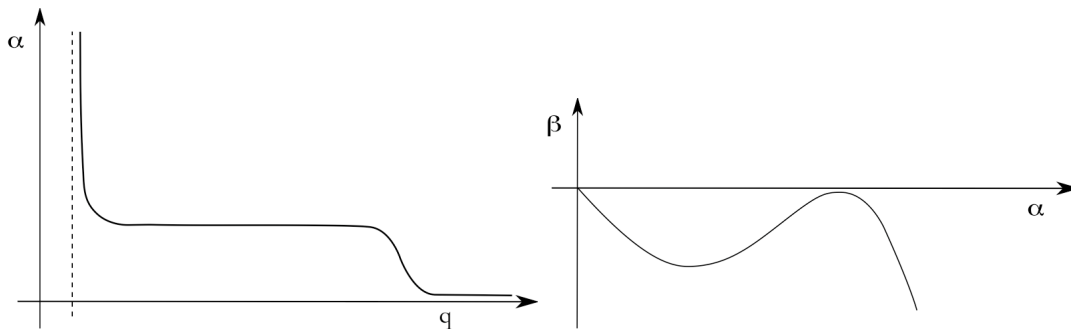


Figure 2.3: The walking behavior of the coupling constant as a function of energy, and the corresponding  $\beta$ -function.

$\beta$ -function of the technicolor sector has to be of a very specific form. In low energy range the theory must confine, so the coupling constant has to grow large in the low energy region. This means that the  $\beta$ -function has to get large negative values for large values of  $\alpha_{TC}$ . At the chiral symmetry breaking scale  $\Lambda_{TC}$  the coupling constant must freeze, and stay almost constant up to the ETC-scale  $\Lambda_{ETC}$ . This means that the  $\beta$ -function has to be almost zero at the corresponding value of  $\alpha_{TC}$ . Yet, the  $\beta$ -function must not be exactly zero, since that would freeze the coupling constant permanently and render the theory conformal in the infrared (IR). After the ETC-scale we wish to have a QCD-like, asymptotically free theory, so the  $\beta$ -function must be negative for small values of  $\alpha_{TC}$ . The walking behavior of  $\alpha_{TC}$  and the corresponding  $\beta$ -function are schematically presented in figure 2.3.

It is not possible to analytically calculate the exact form of the  $\beta$ -function in all orders of perturbation theory, except for some very special cases. One can, however, approximate the behavior of the  $\beta$ -function with respect to different properties of the gauge theory, such as the number of colors  $N$ , the number of flavors  $N_f$  and the representation under which the fermions transform. We shall now try to schematically describe the conditions that are required for a theory to express conformal or close to conformal dynamics. We will explicitly focus on the two loop  $\beta$ -function, but the same conditions must be true for the analytical all orders  $\beta$ -function as well for the model to work as desired.

### Phase diagram and the conformal window

As discussed above, to achieve asymptotic freedom the  $\beta$ -function must be negative for small values of  $\alpha_{TC}$ . This indicates that the lowest order term in perturbation theory has to be negative,  $\beta_0 < 0$ . For IR-conformal dynamics the theory must have an infrared fixed point at a given value of the coupling constant  $\alpha_{TC}$ , which we shall call  $\alpha_{TC}^*$ , at which the value of the  $\beta$ -function is zero. This implies that the graph of the  $\beta$ -function must turn upwards as shown in figure 2.3, which in turn implies  $\beta_1 > 0$ . On the other hand, there is a critical value

$\alpha_{TC}^c$  of the coupling constant, at which the chiral symmetry breaking occurs. If, in the RGE evolution of the coupling constant  $\alpha_{TC}$ , this value is achieved much before  $\alpha_{TC}^*$ , the chiral symmetry breaking will alter the dynamics of the gauge theory and drive the theory away from the fixed point, thereby preventing the walking or conformal behavior. If, on the other hand, the infrared fixed point is achieved before  $\alpha_{TC}^c$ , the coupling freezes and chiral symmetry breaking never occurs. Thus the Goldstone bosons do not appear and the electroweak symmetry is left unbroken. Therefore these two critical values of the coupling constant should be of the same magnitude and ordered as  $\alpha_{TC}^c \lesssim \alpha_{TC}^*$ .

Following the discussion in [13], we will now try to find the *conformal window*, an area in the  $(N, N_f)$ -plane in which the theory is expected to be conformal. The two-loop  $\beta$ -function of an  $SU(N)$  gauge theory reads

$$\beta(g) = \beta_0 \frac{g^3}{(4\pi)^2} + \beta_1 \frac{g^5}{(4\pi)^4}, \quad (2.21)$$

with  $\beta_0$  and  $\beta_1$  given by

$$\begin{aligned} 2N\beta_0 &= -\frac{11}{3}C_2(G) + \frac{4}{3}T(R), \\ (2N)^2\beta_1 &= -\frac{34}{3}(C_2(G))^2 + \frac{20}{3}C_2(G)T(R) + 4C_2(R)T(R). \end{aligned} \quad (2.22)$$

Here  $R$  refers to the representation under which the fermions transform and  $G$  refers to the adjoint representation.  $C_2(R)$  is the quadratic Casimir operator of the representation  $R$ ,

$$2NX_R^a X_R^a = C_2(R)\mathbb{I}, \quad (2.23)$$

where  $X_R^a$  are the generators of the group  $SU(N)$  in representation  $R$ . Correspondingly,  $C_2(G)$  is the quadratic Casimir operator of the adjoint representation  $G$ .  $T(R)$  is the trace normalization factor for the representation  $R$ , given by

$$N_f C_2(R) d(R) = T(R) d(G), \quad (2.24)$$

where  $d(R)$  is the dimension of the representation  $R$ , and  $d(G)$  the dimension of the adjoint representation  $G$ . Requiring  $\beta_0 < 0$  and substituting (2.24) into (2.22) yields an upper limit for the number of flavors

$$N_f < \frac{11d(G)C_2(G)}{4d(R)C_2(R)}, \quad (2.25)$$

above which the theory is no longer asymptotically free. On the other hand, we must require  $\beta_1 > 0$ , yielding a lower limit for the number of flavors

$$N_f > \left( \frac{d(G)C_2(G)}{d(R)C_2(R)} \right) \frac{17C_2(G)}{10C_2(G) + 6C_2(R)}, \quad (2.26)$$

below which there is no infrared fixed point. Using the ladder approximation of the Schwinger Dyson equation for the techniquark self energy [14], the critical coupling may be evaluated as

$$\alpha_{TC}^c = \frac{2\pi N}{3C_2(R)}. \quad (2.27)$$

The infrared fixed point is given by equation (2.21) as  $\alpha_{TC}^* = g^{*2}/(4\pi)$ , where  $g^*$  is the zero of the  $\beta$ -function, yielding  $\alpha_{TC}^* = -4\pi\beta_o/\beta_1$ . Requiring  $\alpha_{TC}^* = \alpha_{TC}^c$  then yields the critical value of flavors

$$N_f^c = \left( \frac{d(G)C_2(G)}{d(R)C_2(R)} \right) \frac{17C_2(G) + 66C_2(R)}{10C_2(G) + 30C_2(R)}, \quad (2.28)$$

for which conformal dynamics is expected.

Using these expressions we can construct the phase diagram of a given  $SU(N)$  gauge theory. For the fundamental representation  $F$  and the adjoint representation  $G$  of  $SU(N)$  we have

$$\begin{aligned} d(F) &= N, & C_2(F) &= N^2 - 1, \\ d(G) &= N^2 - 1, & C_2(G) &= 2N^2. \end{aligned} \quad (2.29)$$

Substituting these into equations (2.25), (2.26) and (2.28) then yields the phase diagram, presented in figure 2.4. Inside the conformal window the theory is expected to be IR-conformal. The walking behavior is expected to occur when  $N_f$  is just slightly below the conformal window, for there the  $\beta$ -function is expected to behave as presented in figure 2.3, that is, to get very close to zero without actually reaching the infrared fixed point.

It should be pointed out that there is no way to conclude with absolute certainty whether a given gauge theory exhibits walking behavior or not. The procedure presented above is a semi-analytical analysis, relying on perturbation theory, and thus is only able to give an estimate on the area of the parameter space where walking behavior is most likely to occur. There is ongoing research conducted with the help of lattice simulations [15], [16], [17] as well as with analytical methods [18], [19], [20] aimed at confronting the results obtained with the above analysis, and this far the results seem to be encouraging.

Looking at figure 2.4 we conclude that the required number of flavors to obtain walking behavior is quite large,  $N_f \gtrsim 8$  for  $SU(2)$  and  $N_f \gtrsim 12$  for  $SU(3)$ . Addition of that many new fermions coupled to the electroweak gauge bosons is likely to influence the results of the electroweak precision tests and contradict existing data. This will be discussed in more detail in chapter 3. For now we conclude that it would be desirable to obtain walking behavior with much less new fermions.

## Higher representations

Although all matter fields in the SM transform under the fundamental representation of the corresponding gauge group, there is no apparent reason why



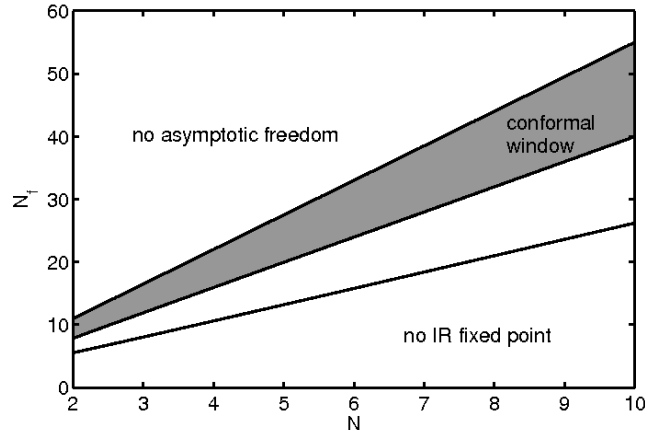


Figure 2.4: The phase diagram of the fundamental representation of an  $SU(N)$  gauge theory. The theory is expected to be conformal inside the shaded region, the conformal window. Below the lowest line  $\beta_1$  is negative and there is no infrared fixed point, resulting in a QCD-like running coupling. Above the conformal window  $\beta_0$  is positive and asymptotic freedom is lost, resulting in a QED-like theory. The walking behavior is expected to occur just below the conformal window.

this should be the case for any possible beyond SM fermions. As it turns out, walking behavior can be obtained with a modest number of new fermions if the fermions transform under given higher representations of the technicolor gauge group. A complete list of candidates for a walking or conformal  $SU(N)$  gauge theories with at least two flavors is given in [13]. Here we will concentrate on a couple of prime candidates.

**Two-index symmetric representation** For  $N$  colors and  $N_f$  flavors in the two-index symmetric representation  $R$ , the relevant group theory factors are

$$\begin{aligned} d(R) &= \frac{1}{2}N(N+1), & C_2(R) &= 2(N-1)(N+2), \\ d(G) &= N^2 - 1, & C_2(G) &= 2N^2. \end{aligned} \quad (2.30)$$

Substituting these to equations (2.25) and (2.28) yields the upper and lower bounds of the conformal window

$$N_f < \frac{11N}{2(N+2)}, \quad N_f^c = \left( \frac{N}{N+2} \right) \frac{83N^2 + 66N - 132}{20N^2 + 15N - 30}. \quad (2.31)$$

The resulting phase diagram is presented in the left panel of figure 2.5. As can be seen from the figure, the point  $(N_f, N) = (2, 2)$  lies just below the conformal window and is therefore proposed as a good candidate for a walking theory.

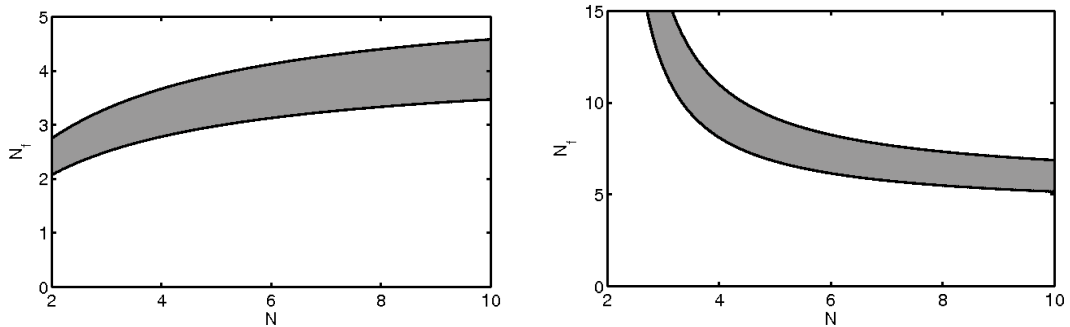


Figure 2.5: Phase diagrams of the two-index symmetric (left) and two-index antisymmetric (right) representations of an  $SU(N)$  gauge theory. In the two-index symmetric representation walking behavior is expected to occur with a modest number of techniquarks, therefore providing good candidates for a walking technicolor theory.

Here the number of new fermions remains small, since we only need to add one generation of techniquarks to the SM. This model, two flavors in the two-index symmetric representation of  $SU(2)$ , is called minimal walking technicolor (MWT) [21], [22], since in this model the walking behavior is achieved with the minimal possible amount of techniquarks<sup>1</sup>. We shall explore this model in detail later. Another possibility is the next to minimal walking technicolor (NMWT) [23], which incorporates two flavors in the two-index symmetric representation of  $SU(3)$ . The phenomenology of these models have been studied in e.g. [24] and [25]. For an ETC construction based on the MWT model see [26].

**Two-index antisymmetric representation** For  $N$  colors and  $N_f$  flavors in the two-index antisymmetric representation, the dimension and the quadratic Casimir operator are given by

$$d(R) = \frac{1}{2}N(N-1), \quad C_2(R) = 2(N-2)(N+1), \quad (2.32)$$

yielding the upper and lower bounds of the conformal window

$$N_f < \frac{11N}{2(N-2)}, \quad N_f^c = \left( \frac{N}{N-2} \right) \frac{83N^2 - 66N - 132}{20N^2 - 15N - 30}. \quad (2.33)$$

The phase diagram of two-index antisymmetric representation is presented in the right panel of figure 2.5. We conclude that in this case, a large number of technifermions is needed for walking or conformal behavior, and thus this representation does not offer good candidates for a walking technicolor theory.

<sup>1</sup>Recently an ultra minimal model with even less new matter content has been proposed, see [27]

**Adjoint representation** For fermions in the adjoint representation the equations (2.25) and (2.28) simplify to constants independent of the number of colors. The conformal window is between the constant limits  $2.075 < N_f < 2.75$ . For the case of  $SU(2)$  the adjoint representation is equal to the two-index symmetric representation presented above. Here we will not consider models of adjoint fermions in  $SU(N)$  for  $N$  larger than two.

## 2.2 A natural next generation

A very straightforward way to expand the SM is the addition of a fourth generation of quarks and leptons. The SM itself does not contain any explanation for the number of fermionic matter generations. Experimental evidence restricts the number of light SM-like neutrinos to three, but if one assumes a heavy neutrino, there is no apparent reason why there could not be a fourth generation.

The addition of a fourth fermion generation does not, in itself, solve the problems of naturalness, unification or dark matter, although it might help to explain the origin of baryonic matter via CP-violation [28]. Therefore simply adding the fourth generation seems quite ad hoc. In models of walking technicolor, however, the fourth generation may arise as a natural consequence of adding the correct amount of technifermions needed to achieve the walking behavior. In this case the fourth generation may be significantly different from the three known generations of the SM.

### 2.2.1 Anomaly cancellation

#### Witten anomaly

As was shown by Witten [29] in 1982, an  $SU(2)$  gauge theory with an odd number of fermion doublets is mathematically inconsistent, since the path integral of such theory vanishes. This result limits the possible extensions of the SM. In the SM there are three lepton doublets and nine quark doublets (three flavors in the fundamental representation of  $SU(3)$ ), and thus the total number of fermion doublets coupled to the weak  $SU(2)$  gauge group is twelve. Therefore the SM is free of the Witten topological anomaly. Accordingly, in any extension of the SM the number of new fermion doublets coupled to the weak  $SU(2)$  group must be even.

#### Gauge anomaly

The fundamental properties of a quantum field theory result from the gauge symmetries of the theory. With each gauge symmetry there exists an associated conserved current. Radiative corrections may, however, violate the conservation of that current through loop diagrams. This process is called anomalous breaking of the gauge symmetry, and must be absent for the gauge symmetry to be in

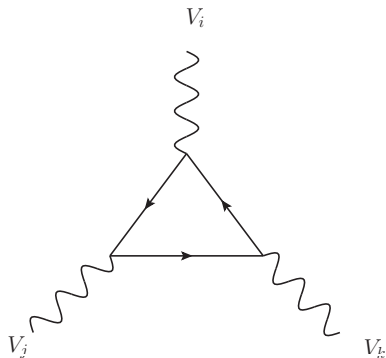


Figure 2.6: The triangle diagram responsible for generating the axial current anomaly. The fermion in the loop is a lepton or a quark, and the gauge bosons  $V_{i,j,k}$  represent any SM gauge bosons.

effect. In the SM, there is a gauge anomaly related to the conservation of the axial current  $j^{\mu 5a} = \bar{\psi} \gamma^\mu \gamma^5 t^a \psi$ , where  $t^a$  is a generator of an SM gauge group. The anomaly arises from the triangle diagram shown in figure 2.6.

The contribution of diagram 2.6, and the corresponding diagram with the fermion current running in the other direction in the loop, to  $\partial_\mu j^{\mu 5a}$  is proportional to the trace

$$\mathcal{A}_{i,j,k}^{abc} = \pm \text{Tr}(t_i^a \{t_j^b, t_k^c\}), \quad (2.34)$$

where  $\{t_j^b, t_k^c\}$  is the anticommutator of the generators  $t_j^b$  and  $t_k^c$  of the gauge groups  $j$  and  $k$ , and the sign is minus one for right handed fermions and plus one for left handed fermions in the loop. Thus, in order to conserve the axial current, the sum of the factors  $\mathcal{A}_{i,j,k}^{a,b,c}$  must vanish when summed over all gauge bosons in the external legs and all fermion species in the loop.

In the SM this cancellation takes place as follows: let us label the  $SU(N)$  gauge bosons of the SM with  $N$ , so that the trace factor related to the diagram with two external  $U(1)$  gauge bosons and one  $SU(2)$  gauge boson is  $\mathcal{A}_{1,1,2}^{abc}$  and so on. Now, since  $t_1 = Y$  is just a number and the generators  $t_2^a = \tau^a$  of  $SU(2)$  and  $t_3^a = t^a$  of  $SU(3)$  are traceless, all trace factors containing odd number of  $SU(2)$  or  $SU(3)$  generators vanish. The remaining factors are  $\mathcal{A}_{1,1,1}^{abc}$ ,  $\mathcal{A}_{1,2,2}^{abc}$ ,  $\mathcal{A}_{1,3,3}^{abc}$ , and

$\mathcal{A}_{1,G,G}^{abc}$ , where  $G$  refers to the graviton. These are given as follows:

$$\begin{aligned}
\mathcal{A}_{1,1,1}^{abc} &= 2 \sum_f \epsilon Y_f^3, \\
\mathcal{A}_{1,2,2}^{abc} &= \frac{1}{2} \delta_{ab} \sum_{f_L} Y_{f_L}, \\
\mathcal{A}_{1,3,3}^{abc} &= \frac{1}{2} \delta_{ab} \sum_q \epsilon Y_q, \\
\mathcal{A}_{1,G,G}^{abc} &= 2 \sum_f \epsilon Y_f,
\end{aligned} \tag{2.35}$$

where the sum index  $f_{(L)}$  refers to sum over all (left handed) fermions and  $q$  to sum over all quark species.  $Y$  is the hypercharge of the fermion and  $\epsilon$  is minus one for right handed and plus one for left handed fermions. Substituting the hypercharges of the SM fermions we indeed see that each of the sums in equation (2.35) indeed vanish. This should be noted as a nontrivial consistency check of the SM itself.

The cancellation of these anomalies restricts the class of possible extensions to the SM. First, to avoid the Witten anomaly, the total number of new weak doublets added to the SM must be even. Second, the hypercharge of the new fermions must be assigned correctly to cancel the axial current anomaly. One could in principle entertain the thought of very exotic hypercharge assignments, such as doubly or even fractionally charged fermions, but here we will be mainly interested in models where the hypercharge assignments resemble either the quarks or the leptons of the SM.

In [III] we present a list of anomaly free technicolor models, when the technicolor sector has been chosen to consist of two flavors in the two index symmetric representation of either  $SU(2)$  or  $SU(3)$ . The idea goes as follows: we choose a technicolor model that is expected to satisfy the constraints from existing experimental data on electroweak precision measurements and flavor physics. We restrict ourselves to consider only SM-like hypercharge for the techniquarks, namely  $Y(Q_L) = \pm\frac{1}{2}$  or  $Y(Q_L) = \pm\frac{1}{6}$ . We then add extra generations of SM like leptons or QCD quarks to cancel Witten and gauge anomalies. The idea of anomaly cancellation between BSM fields, designed for electroweak symmetry breaking, and extra SM-like matter fields, was first proposed in the context of adding one lepton doublet to a walking technicolor model in [21]. In [III] we expand this idea to cover different models for the technicolor sector and different settings of SM-like matter.

We fix the hypercharge of the right handed techniquarks as  $Y(U_R) = Y(Q_L) + T_3(U_L)$  and  $Y(D_R) = Y(Q_L) + T_3(D_L)$ . Denoting the number of new SM-like lepton generations with  $N_l$  and the number of new QCD quark generations with  $N_q$ ,

the anomaly cancellation leads to conditions

$$\begin{aligned}\mathcal{A}_{1,1,1}^{abc} &= -\frac{3}{4}(N_q - N_l) + d(R)(2Y^3(Q_L) - Y^3(U_R) - Y^3(D_R)) = 0, \\ \mathcal{A}_{1,2,2}^{abc} &= -\frac{1}{2}(N_l - N_q) + d(R)Y(Q_L) = 0,\end{aligned}\tag{2.36}$$

where  $d(R)$  is the dimension of the technicolor group representation. The anomalies  $\mathcal{A}_{1,G,G}^{abc}$  and  $\mathcal{A}_{1,TC,TC}^{abc}$  vanish trivially. Equations (2.36) can be written as a constraint for the difference of the number of new lepton and quark generations  $\Delta N = N_l - N_q$ , yielding

$$\Delta N = 2d(R)Y(Q_L).\tag{2.37}$$

Since we only consider  $Y(Q_L) \neq 0$ , as explained above, this implies a nonzero number of new lepton or quark generations.

### 2.2.2 Minimal walking technicolor

The minimal walking technicolor (MWT) model consists of two flavors of techniquarks in the adjoint representation of the technicolor gauge group  $SU(2)$ . In this model, the number of fermion doublets added to the SM is three, since the dimension of the adjoint representation is  $d(R) = 3$ . Therefore the model suffers from Witten anomaly, which must be cured by adding another odd number of fermion doublets. The relative number of new lepton and QCD quark generations is controlled by equation (2.37). Choosing  $Y(Q_L) = \pm\frac{1}{2}$  would thus lead to three new generations of leptons or QCD quarks. Such large amount of new fermionic matter is not desirable, and we will not consider these scenarios further. Instead, choosing  $Y(Q_L) = \pm\frac{1}{6}$  leads to only one additional lepton or quark generation. The interesting scenarios are then MWT with  $Y(Q_L) = \frac{1}{6}$  accompanied by fourth generation leptons, and  $Y(Q_L) = -\frac{1}{6}$  with fourth QCD quark generation. We will consider the phenomenology of these models in chapter 4.

### 2.2.3 Next to minimal walking technicolor

The next to minimal walking technicolor (NMWT) model includes two techniflavors in the two index symmetric representation of  $SU(3)$ . This representation is six-dimensional, so there is no Witten anomaly. However, equation (2.37) implies a nonzero  $\Delta N$  and therefore we must add new leptons or quarks to cancel the gauge anomaly.<sup>2</sup> Again, choosing  $Y(Q_L) = \pm\frac{1}{2}$  would result in six new generations of quarks and leptons, which seems totally unacceptable. Hence we are left with two possible scenarios:  $Y(Q_L) = \frac{1}{6}$  with two new lepton generations or  $Y(Q_L) = -\frac{1}{6}$  with two additional generations of QCD quarks. The phenomenology of these models will also be examined in chapter 4.

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<sup>2</sup>Obviously, one may release the restriction on the techniquark hypercharge and allow  $Y(Q_L) = 0$ . Then the NMWT model is anomaly free without any new SM-like matter fields.

## 2.3 Unnatural origin of fermion masses

Before going into phenomenological analysis, let us go back to the issue of fermion masses in technicolor. As was explained in section 2.1.2, dynamical symmetry breaking from the technicolor sector is only capable of producing masses for the electroweak gauge bosons, but leaves the fermions massless. This is usually taken care of by the ETC interaction, whose details remain unknown.

The downside of this approach is that any phenomenology resulting from the ETC sector remains uncalculable, since we do not have an effective description of the dynamics of this sector. The simplest attempt to build such an effective model is to assume that the Higgs-Yukawa sector of the SM describes these dynamics in the low energy regime. The role of the Higgs particle is then played by the technicolor composite. This approach gives a simple and calculable description of the low energy sector of an extended walking technicolor model, providing masses for the weak gauge bosons and fermions alike. However, this description lacks any information about the interplay between the technicolor sector and the sector responsible for fermion masses.

To gain an insight into this interplay, in [II] we study a model of bosonic technicolor. In this model, the technicolor sector (which we have chosen to be the next to minimal walking technicolor for simplicity) is accompanied by a SM Higgs-like scalar particle. The existence of an elementary scalar would obviously destroy almost completely the benefits of dynamical symmetry breaking. Thus we assume that this scalar is really the low energy effective description of the sector responsible for fermion masses, be that ETC or something else.

### 2.3.1 The model

The model we are considering consists of the SM accompanied by the technicolor sector, here NMWT. The SM Higgs is now interpreted as a low energy effective description of the sector responsible for fermion masses. The potential of this scalar does not necessarily need to have the symmetry breaking form with negative  $\mu^2$ , since the symmetry breaking occurs in the technicolor sector regardless of the scalar potential. The scalar has Yukawa couplings with both the SM fermions and techniquarks. The Lagrangian reads:

$$\mathcal{L}_{UTC} = \mathcal{L}_{SM} \Big|_{\text{Higgs}=0} + \mathcal{L}_{TC} + \mathcal{L}_{\text{Higgs}} + \mathcal{L}_{\text{Yukawa}}, \quad (2.38)$$

where the Higgs sector has been separated from the SM and  $\mathcal{L}_{\text{Yukawa}}$  contains all the Yukawa couplings of the model. Here we consider non SM-like hypercharge assignments for the techniquarks,  $Y(Q_L) = 0$ , since this configuration is anomaly free without the need for new leptons or QCD quarks. However, the analysis performed here is valid also in the case of extra generations. To construct the effective low energy Lagrangian, we write the technicolor sector as

$$M = \frac{1}{\sqrt{2}}(s\mathbb{I}_2 + 2i\pi_M), \quad \langle s \rangle = f, \quad (2.39)$$

where  $f$  is the technipion decay constant. The Higgs field is written in an analogous form:

$$H = \frac{1}{\sqrt{2}}(h\mathbb{I}_2 + 2i\pi_H), \quad \langle h \rangle = v. \quad (2.40)$$

Here we used a short hand notation  $\pi = \pi^i \tau_i$ , where  $\tau_i$  are the generators of  $SU(2)$ . The Higgs Lagrangian in (2.38) reads

$$\begin{aligned} \mathcal{L}_{\text{Higgs}} &= \frac{1}{2} \text{Tr} [DH^\dagger DH] - V_H, \\ V_H &= \frac{1}{2} m_H^2 \text{Tr} [H^\dagger H] + \frac{\lambda_H}{4!} \text{Tr}^2 [H^\dagger H], \end{aligned} \quad (2.41)$$

where all contracted Lorenz indices have been omitted. The Yukawa terms  $\mathcal{L}_{\text{Yukawa}}$  in (2.38) are written as

$$\mathcal{L}_{\text{Yukawa}} = - \sum_{i=q,l,Q} \bar{\Psi}_{L,i} H Y^i \Psi_{R,i}, \quad Y^i = y^i \mathbb{I}_2 + \delta y^i \sigma_3. \quad (2.42)$$

where the sum is over all SM fermions and techniquarks. The Yukawa coupling between techniquarks and the Higgs,  $-\bar{Q}_L H Y_Q Q_R$ , is of special importance, since it breaks the  $(SU(2)_L \times SU(2)_R)^2$  symmetry arising from the Higgs sector and the TC sector, down to  $SU(2)_L \times U(1)_R$ , which is then broken further by the electroweak gauging. Applying naive dimensional analysis [30], and omitting terms of order  $\mathcal{O}(Y_Q^2)$  and higher, the low energy effective Lagrangian of the technicolor sector and its coupling with the scalar may be written as

$$\begin{aligned} \mathcal{L}_{TC-\bar{Q}_L H Y_Q Q_R} &\rightarrow \frac{1}{2} \text{Tr} [DM^\dagger DM] + \frac{1}{2} (c_3/\alpha) \text{Tr} [DM^\dagger D H Y_Q] - V_M, \\ V_M &= \frac{1}{2} m_M^2 \text{Tr} [M^\dagger M] + \frac{\lambda_M}{4!} \text{Tr}^2 [M^\dagger M] \\ &\quad - \frac{1}{2} (\alpha c_1) f^2 \text{Tr} [M^\dagger H Y_Q] - \frac{1}{24} (\alpha c_2) \text{Tr} [M^\dagger M] \text{Tr} [M^\dagger H Y_Q] \\ &\quad - \frac{1}{24} (c_4/\alpha) \text{Tr} [H^\dagger H] \text{Tr} [M^\dagger H Y_Q] + \text{h.c.} \end{aligned} \quad (2.43)$$

The factors  $c_1 \dots c_4$  are dimensionless quantities of order one, and are taken to be real to preserve the CP symmetry.

The idea of bosonic technicolor is not new. It was originally presented by Simmons [31], Kagan and Samuel [32], and Carone and Georgi [33], [34]. Recently similar models have been studied in [35], [36], [37]. See also [38] and [39] for work on related topics.

Compared to the previously existing literature on the topic we have performed a more extensive analysis by including in the effective Lagrangian all dimension four operators with at most one mixing between the two scalar sectors. We have provided an extensive scan of the parameters of the model, not limiting to any special case for the mass parameter of the fundamental scalar. We have updated the comparison with measurements to up to date values of experimental data, and linked the dynamical sector with modern models of walking technicolor, namely the NMWT model.



### 2.3.2 Particle spectrum

The kinetic terms from equations (2.41) and (2.43) form a mixing Lagrangian of the form

$$\mathcal{L}_{KE} = \frac{1}{2} \text{Tr} [DM^\dagger DM] + \frac{1}{2} \text{Tr} [DH^\dagger DH] + \frac{c_3}{2\alpha} \text{Tr} [(DM^\dagger DH + DH^\dagger DM) Y_Q], \quad (2.44)$$

which is diagonalized by

$$\begin{pmatrix} M \\ H \end{pmatrix} = \frac{1}{\sqrt{2}} \begin{pmatrix} \frac{1}{\sqrt{1-(c_3/\alpha)y_Q}} & \frac{1}{\sqrt{1+(c_3/\alpha)y_Q}} \\ -\frac{1}{\sqrt{1-(c_3/\alpha)y_Q}} & \frac{1}{\sqrt{1+(c_3/\alpha)y_Q}} \end{pmatrix} \begin{pmatrix} M_- \\ M_+ \end{pmatrix}. \quad (2.45)$$

Here the index  $\pm$  is just a label and does not denote electric charge. The fields  $M_\pm$  are coupled to the electroweak gauge bosons via the usual covariant derivative. Three out of the six Goldstone bosons of the model are absorbed as the longitudinal components of the weak gauge bosons, and three remain in the physical particle spectrum, accompanied with two scalars. The fields  $M_\pm$  are expressed in terms of the physical pions as

$$M_\pm = \frac{1}{\sqrt{2}} \left( s_\pm + f_\pm \mp 2i \frac{f_\mp}{v_w} \pi \right), \quad (2.46)$$

where

$$f_\pm \equiv \frac{1}{\sqrt{2}} \langle \text{Tr} M_\pm \rangle = \frac{\sqrt{1 \pm (c_3/\alpha)y_Q}}{\sqrt{2}} (f \pm v). \quad (2.47)$$

The constants  $f_\pm$  have to satisfy the condition  $f_+^2 + f_-^2 = v_{weak}^2$  in order to generate the correct mass for the  $W$  boson. It should be noted that due to the kinetic mixing this condition does not hold directly for  $f$  and  $v$ , which may both be large if one is negative and the other positive.

Finally, the physical scalars  $s_p$  and  $h_p$  are given as linear combinations of  $s$  and  $h$  according to the mixing

$$\begin{pmatrix} s \\ h \end{pmatrix} = U_{\text{mix}} \begin{pmatrix} s_p \\ h_p \end{pmatrix}, \quad (2.48)$$

with the mixing matrix

$$U_{\text{mix}} = \frac{1}{2} \begin{pmatrix} \frac{1}{\sqrt{1-(c_3/\alpha)y_Q}} & \frac{1}{\sqrt{1+(c_3/\alpha)y_Q}} \\ -\frac{1}{\sqrt{1-(c_3/\alpha)y_Q}} & \frac{1}{\sqrt{1+(c_3/\alpha)y_Q}} \end{pmatrix} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} \cos \theta & \sin \theta \\ -\sin \theta & \cos \theta \end{pmatrix}. \quad (2.49)$$

The rotation angle is given by

$$\tan 2\theta = -\frac{y_Q}{\sqrt{\alpha^2 - y_Q^2 c_3^2}} \frac{c_4 v^2 + (2c_1 + c_2) f^2 \alpha^2 + c_3 (N_H + N_M)}{N_H - N_M}, \quad (2.50)$$

where

$$N_H = m_H^2 + \frac{\lambda_H v^2}{2} - y_Q f v \frac{c_4}{\alpha}, \quad N_M = m_M^2 + \frac{\lambda_M f^2}{2} - y_Q f v c_2 \alpha. \quad (2.51)$$

We will consider the phenomenology of this model in chapter 4

# Chapter 3

## Constraints

### 3.1 Detecting new physics

Assuming that there exists new physics beyond the SM, there are two ways of detecting evidence of it in collider experiments. The new particles may be observed via direct detection, i.e. by producing these particles in high energy collisions and detecting either the particles or their decay products. This method would enable a detailed study of the properties of the new particles, such as mass, electric charge, spin and couplings to the SM particles. However, this kind of detection is only possible if the production rate of these particles in collider experiments is large enough, and/or the detection signal deviates enough from the SM background. If this is not the case, the new particles could possibly be detected indirectly, via their effects on known processes of the SM particles. These kind of effects would typically arise as quantum corrections to tree level SM processes.

Correspondingly, the lack of evidence of new physics, i.e. the good agreement between experiments and SM predictions, may be used to constrain the supposed models of new physics. Since there is no direct evidence of detecting particles beyond the SM, any model that expands the SM must include only such particles that may evade all existing direct search experiments. This implies that either the masses of the new particles must be large enough so that the center of mass (CMS) energy in current experiments is not enough to produce these particles, or the couplings of these particles to the SM particles must be so weak, that the expected production rate falls below the detectable limit. Another set of constraints arises from the indirect effects of new physics. Some SM processes have been measured with very high precision, so that even the quantum corrections to these processes are constrained by experiment. These constraints may be especially important when one considers models of technicolor.

## 3.2 Oblique corrections

One of the most precise sets of measurements ever performed in the field of particle physics is related to scattering of light leptons and muon decay. These electroweak precision measurements have been carried out at the  $Z$ -resonance energy in LEP and at low energies in various laboratories. The results of the electroweak precision measurements generally agree quite well with the SM predictions, placing stringent constraints on new physics. These results are especially useful for constraining models of new physics that fulfill the following conditions:

- The new physics does not expand the electroweak gauge sector of the SM.
- The new physics couple to the electroweak gauge bosons more strongly than to the light SM fermions.
- The intrinsic scale of the new physics is large compared to the masses of the electroweak gauge bosons

If all of the above holds, the effects of new physics may be expressed in terms of the oblique corrections, the Peskin-Takeuchi parameters  $S$ ,  $T$  and  $U$ . [40], [41]

In the electroweak precision measurements the observed particles are light SM fermions, typically electrons or muons. This means that these measurements test processes that can generally be described by the schematic diagram presented in figure 3.1. Here the external fermion lines are the light SM fermions that are measured by the experiment. They interact via an electroweak gauge boson. If the assumptions stated above hold, then the electroweak gauge sector is that of the SM and hence this boson is one of the SM electroweak gauge bosons. Furthermore, if new physics couple mainly to the electroweak gauge bosons, then the fermion-gauge boson vertices remain unchanged and all the corrections enter as radiative corrections to the gauge boson propagator. In the figure the gray blob represents all radiative corrections to the gauge boson propagator. The loop corrections may be calculated in the SM, and any deviations from those values then indicate new physics. Correspondingly, the size of the deviation may be used to constrain the new physics.

The last assumption that the intrinsic scale of the new physics is large compared to the electroweak scale is needed for calculation of the radiative corrections induced by the new physics. New physics introduce corrections to the gauge boson vacuum polarization:

$$\Pi_{ab}(q^2) = \Pi_{ab}^{SM}(q^2) + \delta\Pi_{ab}(q^2). \quad (3.1)$$

Here  $\Pi_{ab}^{SM}$  is the SM contribution, including the radiative corrections due to SM particles, and  $\delta\Pi_{ab}$  represents the corrections induced by the new physics. The indices  $a$  and  $b$  refer to the electroweak gauge bosons  $a, b = \gamma, W^\pm, Z$ . Since we

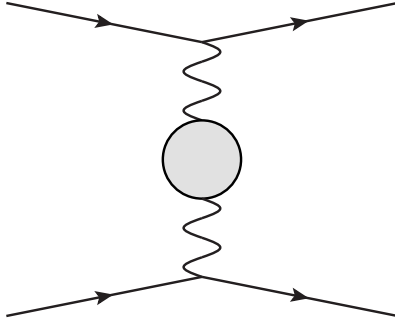


Figure 3.1: Schematic drawing of an electroweak precision measurement. The external lines are light fermions and the wavy line represents an electroweak gauge boson. The blob represents radiative corrections to the gauge boson propagator.

assume that the intrinsic scale of the new physics is large, we may expand in orders of  $q^2$  and simply work in linear order:

$$\delta\Pi_{ab}(q^2) \approx A_{ab} + B_{ab}q^2. \quad (3.2)$$

Now there are in total eight independent quantities  $A_{ab}$  and  $B_{ab}$ , with  $ab = \gamma\gamma, Z\gamma, ZZ$  or  $WW$ , that describe the effects of new physics at low energies.  $A_{\gamma\gamma} = \delta\Pi_{\gamma\gamma}(0)$  and  $A_{Z\gamma} = \delta\Pi_{Z\gamma}(0)$  are zero by gauge invariance and three other may be fixed by the renormalization of the three SM input parameters, e.g.  $\alpha$ ,  $M_Z$  and  $G_F$ . Thus all the effects of new physics are given by three linear combinations of  $A_{ab}$  and  $B_{ab}$ . Traditionally these are chosen to be the Peskin-Takeuchi parameters  $S$ ,  $T$  and  $U$ , defined by

$$\begin{aligned} \frac{\alpha S}{4s_w^2 c_w^2} &= \frac{\delta\Pi_{ZZ}(M_Z^2) - \delta\Pi_{ZZ}(0)}{M_Z^2}, \\ \alpha T &= \frac{\delta\Pi_{WW}(0)}{M_W^2} - \frac{\delta\Pi_{ZZ}(0)}{M_Z^2}, \\ \frac{\alpha U}{4s_w^2} &= \frac{\delta\Pi_{WW}(M_W^2) - \delta\Pi_{WW}(0)}{M_W^2} - c_w^2 \frac{\delta\Pi_{ZZ}(M_Z^2) - \delta\Pi_{ZZ}(0)}{M_Z^2}. \end{aligned} \quad (3.3)$$

Here  $s_w$  and  $c_w$  are, respectively, sine and cosine of the Weinberg angle.

The data collected from various experiments may then be evaluated to form a universal fit for the oblique corrections. This has been done by the PDG<sup>1</sup> [5], and the result is shown in figure 3.2. Since the SM Higgs has not been found, its mass has to be taken as an input parameter to the fit. Figure 3.2 shows the  $1\sigma$  constraints for  $S$  and  $T$  assuming Higgs mass of 117 GeV and setting  $U$  to

<sup>1</sup>Similar fits are available also from other groups, such as the LEP electroweak working group (LEPEWWG) [42] and Gfitter [43]. There are some differences between the different groups in what data is taken into account in the fit, but the procedure is similar and the obtained results are quantitatively compatible with each other.

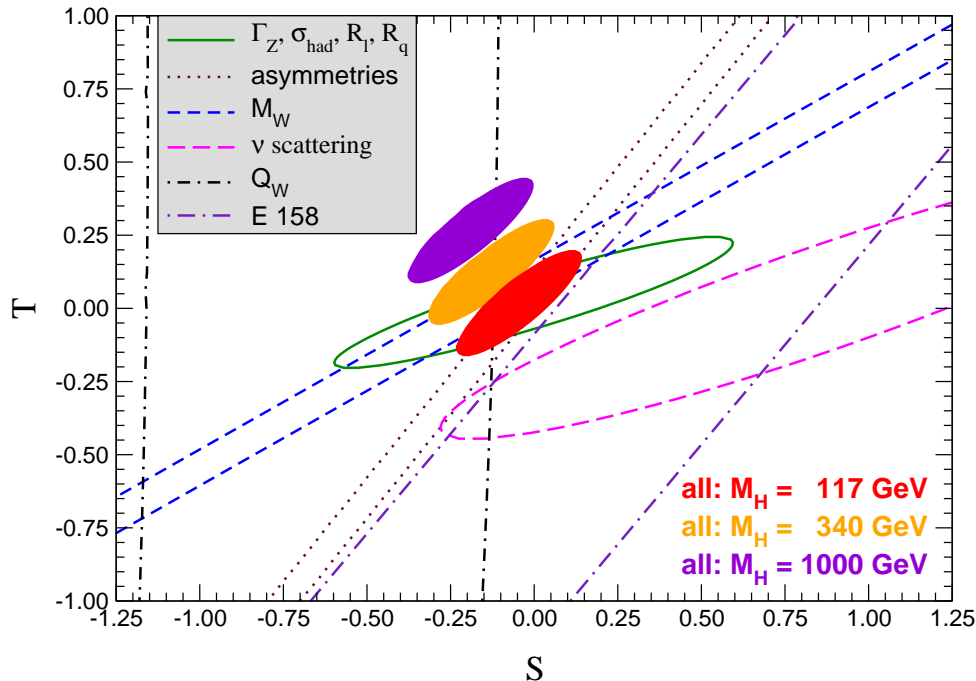


Figure 3.2: The  $1\sigma$  constraints for the electroweak precision parameters  $S$  and  $T$  obtained from various measurements, assuming  $U = 0$  and  $M_H = 117$  GeV, and the 90% CL contours allowed by all data, for  $M_H = 117$  GeV (bottom right),  $M_H = 340$  GeV and  $M_H = 1000$  GeV (top left). Figure from [5].

zero. The coloured ellipses are the 90% confidence limit contours allowed by all data for Higgs masses of 117 GeV, 340 GeV and 1000 GeV, with Higgs mass rising towards upper left corner.

In technicolor models the first two assumptions hold quite well: there are no new electroweak gauge bosons, although there may be technimesons with the same quantum numbers as the  $W$  and  $Z$ . Also the techniquarks do not couple directly to the SM fermions, but are gauged under the electroweak gauge group and hence couple primarily to the electroweak gauge bosons. The last assumption about the heaviness of the technicolor scale is on a weaker ground. To induce correct masses for the electroweak gauge bosons the technicolor scale  $\Lambda_{TC}$  must be of the order of the electroweak scale. However, as it turns out,  $S$ ,  $T$  and  $U$  are still usually applicable to constrain technicolor theories. For the model studied in [II] we have checked this by computing the derivative  $d/dq^2 \delta\Pi_{ZZ}(q^2)$  numerically in the range  $0 < q^2 < M_Z^2$  and verified that the results agree with

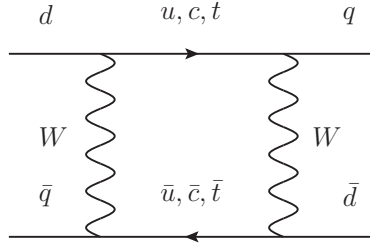


Figure 3.3: The SM diagram contributing to the  $\Delta q = 2$  FCNC processes, for  $q = s, b$ .

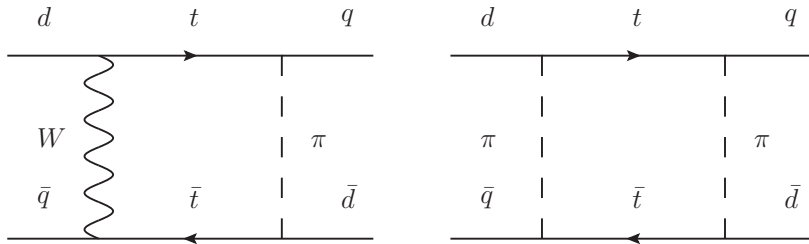


Figure 3.4: The diagrams contributing to the  $\Delta q = 2$  FCNC processes, for  $q = s, b$ , in the bosonic technicolor model of [II]. The top quark loop contribution is dominant over up and charm, due to the large Yukawa coupling and hence large couplings to the physical pions.

the approximation obtained by the definition of equation (3.3). It is also possible to discard the assumption of the heavy intrinsic scale. This leads to addition of three other precision parameters, named  $V$ ,  $X$  and  $W$  in [44]. Here we have chosen not to calculate the corrections arising from these new parameters, though we have checked that these values remain small in the area allowed by  $S$ ,  $T$  and  $U$ .

### 3.3 Flavor changing neutral currents

As was briefly explained in section 2.1.3, the flavor changing neutral current processes are strongly suppressed in the SM. Any new physics that induce sizeable contributions to these processes would therefore be easily detected in FCNC experiments. The most stringent constraints arise from processes where  $\Delta s = 2$  or  $\Delta b = 2$ . In the SM these processes are forbidden in tree level and only arise from loop diagrams, such as the one presented in figure 3.3.

In technicolor theories, there are two sources of FCNCs. First of all, the ETC interactions inevitably contribute to the FCNC processes, as explained in

section 2.1.3. These contributions are suppressed by the walking dynamics, and the actual numerical values of these contributions remain uncalculable, except for the naive estimates presented in section 2.1.3, as long as the details of the ETC sector are unknown. Another source of FCNCs is in the technicolor sector itself. As the techniquarks are confined at low energies, they form a zoo of composite resonances, just as happens in QCD. Some of the vector mesons will have the quantum numbers of the  $W$  boson, and hence mix with it. These vector resonances will contribute to the diagram of figure 3.3. This effect may be used to constrain the technicolor models, especially the masses of the vector resonances, as presented in [45].

In [II] we presented a calculable model for the origin of fermion masses, as opposed to the usual picture of an unknown ETC sector. In this case, the FCNC contributions become perturbatively calculable as well. The leading contribution comes from diagrams presented in figure 3.4. Here the pions are the physical pions that remain in the mass spectrum of the model, as explained in section 2.3. We have calculated the values of these contributions and used them to constrain the parameters of the model.



# Chapter 4

## Results

### 4.1 Fourth generation fermions

As explained in section 2.2, the existence of a fourth generation of SM-like quarks or leptons can be viewed as a natural consequence of anomaly cancellations in walking technicolor models. Here we will study the effects of these new fermions on particle phenomenology.

#### 4.1.1 MWT and fourth generation of leptons

In some sense this is the most minimal set up. The matter content added to the SM has exactly the same electroweak quantum numbers as a SM generation of quarks and leptons. There are three techniquark doublets and one lepton doublet. Here we will be interested in the phenomenology arising from the leptons.

#### The Lagrangian

The question of neutrino masses in the SM still remains unanswered. It is not clear whether the SM-neutrinos are Dirac- or Majorana particles. Here we will take no prejudice in this matter. The technicolor sector is presented at low energy by the chiral effective theory. Out of the many particles present in this effective model, the composite Higgs  $H$  is the only one that couples directly to SM matter fields. For the lepton masses we therefore consider the following Lagrangian, including operators up to dimension five:

$$\begin{aligned} \mathcal{L}_{\text{Mass}} = & (y\bar{L}_L H E_R + h.c.) + C_D \bar{L}_L \tilde{H} N_R + \frac{C_L}{\Lambda} (\bar{L}^c \tilde{H})(\tilde{H}^T L) \\ & + \frac{C_R}{\Lambda} (H^\dagger H) \bar{N}_R^c N_R + h.c., \end{aligned} \tag{4.1}$$

where  $\tilde{H} = i\tau^2 H^*$  and  $\Lambda$  is a cut off scale of the order of one TeV, below which the effective theory is valid.  $y$  is a hypercharge parameter,  $y = \frac{1}{3}$  corresponding

to the SM-like hypercharge assignment we are interested in. This Lagrangian results in the usual Dirac mass term for the charged lepton  $E$ , and neutrino mass term of the form

$$-\frac{1}{2}\bar{n}_L^c M n_L + h.c., \quad M = \begin{pmatrix} M_L & m_D \\ m_D & M_R \end{pmatrix}, \quad (4.2)$$

where  $n_L = (N_L, N_R^c)^T$ ,  $m_D = C_D v / \sqrt{2}$  and  $M_{L,R} = C_{L,R} v^2 / (2\Lambda)$ . This reduces to Dirac mass if one removes the dimension five operators, and to pure left handed Majorana mass if one removes the right handed field  $N_R$ . Here we will keep all of the terms, resulting in the most general mass structure for the neutrino. The mass eigenstates are two Majorana neutrinos,  $\chi_1$  and  $\chi_2$ , associated with the eigenvalues

$$\lambda_{1,2} = \frac{1}{2} \left( (M_L + M_R) \pm \sqrt{(M_L - M_R)^2 + 4m_D^2} \right). \quad (4.3)$$

Since these may be positive or negative, we define the masses  $M_{1,2} = \rho_{1,2} \lambda_{1,2}$ , where  $\rho_{1,2} = \pm 1$ , so that  $M_{1,2} > 0$ . This phase may be absorbed in the diagonalizing matrix

$$\mathcal{U} = \begin{pmatrix} \sqrt{\rho_1} \cos \theta & \sqrt{\rho_2} \sin \theta \\ -\sqrt{\rho_1} \sin \theta & \sqrt{\rho_2} \cos \theta \end{pmatrix}, \quad (4.4)$$

where the mixing angle  $\theta$  is given by  $\tan(2\theta) = 2m_D / (M_R - M_L)$ . The mass eigenstates are given as

$$\chi = \begin{pmatrix} \chi_1 \\ \chi_2 \end{pmatrix} = \mathcal{U}^\dagger n_L + \mathcal{U}^T n_L^c, \quad (4.5)$$

The relevant interaction terms are the electroweak gauge interactions and the effective interactions between the composite Higgs and the neutrino, which in the mass eigenbasis read

$$\begin{aligned} W_\mu^+ \bar{N}_L \gamma^\mu E_L &= \frac{\cos \theta}{\sqrt{\rho_1}} \bar{\chi}_{1L} W_\mu^+ \gamma^\mu E_L + \frac{\sin \theta}{\sqrt{\rho_2}} \bar{\chi}_{2L} W_\mu^+ \gamma^\mu E_L, \\ Z_\mu \bar{N}_L \gamma^\mu N_L &= \cos^2 \theta Z_\mu \bar{\chi}_{1L} \gamma^\mu \chi_{1L} + \sin^2 \theta Z_\mu \bar{\chi}_{2L} \gamma^\mu \chi_{2L} \\ &\quad + \frac{\sin(2\theta)}{2\sqrt{\rho_1}^* \sqrt{\rho_2}} Z_\mu \bar{\chi}_2 \gamma^\mu (\alpha - \beta \gamma^5) \chi_1, \\ \mathcal{L}_{Higgs} &= C_{22} h \bar{\chi}_2 \chi_2 + C_{11} h \bar{\chi}_1 \chi_1 + C_{12} h \bar{\chi}_1 (\beta + \alpha \gamma^5) \chi_2 + \mathcal{O}(h^2), \end{aligned} \quad (4.6)$$

where we have defined the following constants:

$$\begin{aligned}
\alpha &= \frac{1}{2}(1 - (\sqrt{\rho_1^*} \sqrt{\rho_2})^2), \\
\beta &= \frac{1}{2}(1 + (\sqrt{\rho_1^*} \sqrt{\rho_2})^2), \\
C_{11} &= \frac{M_1}{v} \left( 1 - \frac{1}{4} \sin^2(2\theta) \left( 1 - (\sqrt{\rho_1^*} \sqrt{\rho_2})^2 \frac{M_2}{M_1} \right) \right), \\
C_{22} &= \frac{M_2}{v} \left( 1 - \frac{1}{4} \sin^2(2\theta) \left( 1 - (\sqrt{\rho_1} \sqrt{\rho_2^*})^2 \frac{M_1}{M_2} \right) \right), \\
C_{12} &= -\frac{M_2}{4v} \sqrt{\rho_1} \sqrt{\rho_2^*} \sin(4\theta) \left( 1 - (\sqrt{\rho_1^*} \sqrt{\rho_2})^2 \frac{M_1}{M_2} \right).
\end{aligned} \tag{4.7}$$

We have neglected any interaction terms of order  $\mathcal{O}(h^2)$ , since we are only interested in the Higgs decays into neutrinos.

### Parameter space

The parameters  $M_L$ ,  $M_R$  and  $m_D$  are simply coupling constants in our formulation, and hence may be positive or negative. The parameter space is divided into three domains, corresponding to  $\rho_1 = \rho_2 = \pm 1$  and  $\rho_1 = -\rho_2 = 1$ . In each domain, the physical masses  $M_1$  and  $M_2$  assume all positive values and  $0 \leq \sin \theta \leq 0.5$ . These domains are presented in figure 4.1, showing an  $m_D = \text{constant}$  -plane in the  $(M_L, M_R, m_D)$ -parameter space. Here the hyperbolas correspond to surfaces  $m_D^2 = M_L M_R$  and the line in the middle corresponds to the plane  $M_R = -M_L$ . The corresponding values of the  $\rho$ -parameters are shown in the figure. The parameter space is symmetric with respect to the plane  $M_R = -M_L$  with replacement  $M_1 \leftrightarrow M_2$ . We may therefore restrict to the upper half, corresponding to  $M_R > -M_L$ .

Typical special cases considered in the literature are Dirac neutrino ( $M_L = M_R = 0$ ), pure left Majorana neutrino ( $M_R = M_D = 0$ ) and type I seesaw mass matrix ( $M_L = 0$ ). Here we explore the whole parameter space. It should be noted, that all of the special cases above correspond to setting either  $M_L$  or  $M_R$  to zero, driving the model into the domain  $\rho_1 = -\rho_2 = 1$ . The domain  $\rho_1 = \rho_2$  is only accessible if both  $M_L$  and  $M_R$  are nonzero. Our analysis is therefore fully general in the sense that we are not restricted to any of the domains of figure 4.1.

### Oblique corrections

The lepton loop diagrams contributing to  $S$  and  $T$  may be calculated in terms of the physical mass parameters using equation (4.6). The resulting expressions for  $S$  and  $T$  are presented in [I]. For the technicolor sector we use the naive perturbative estimate  $S \approx 1/(2\pi)$ ,  $T \approx 0$ . We have chosen  $M_R > -M_L$ , and hence  $M_1 > M_2$ . Therefore  $\rho_1$  is positive and the sign of  $\rho_2$  depends on the ratio of  $m_D^2$  and  $M_R M_L$ , so that  $\rho_2$  is negative for  $m_D^2 > M_R M_L$ . As it turns out,

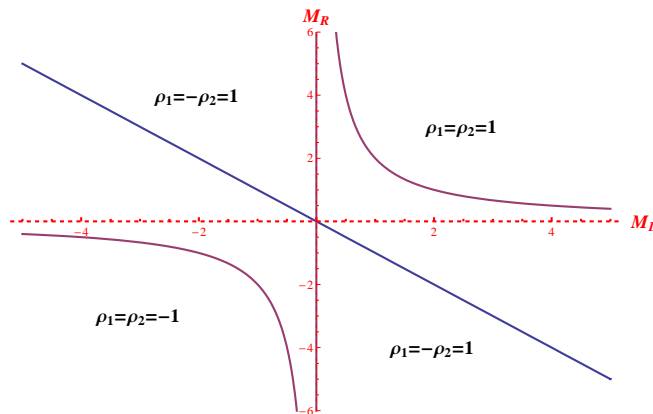


Figure 4.1: Schematic figure of the parameter space corresponding to a fixed value of  $m_D$ . The hyperbolas and the straight diagonal line correspond to sections of surfaces  $m_D^2 = M_L M_R$  and  $M_R = -M_L$ , respectively. The  $m_D$ -axis is perpendicular to the  $(M_L, M_R)$ -plane.

all our results are practically independent of the sign of  $\rho_2$ , and hence we will only consider the case  $\rho_2 = -1$ . All of the results are applicable also in the case  $\rho_2 = 1$ , with only slight modifications.

As the new physics, i.e. the new leptons, might be comparably light, it is questionable whether  $S$  and  $T$  are a good enough indication of the total effects of new physics to electroweak precision measurements. This is equal to asking, whether it is sufficient to approximate the effects of new physics to only linear order in  $q^2$ . We have checked this by comparing our results for the  $S$  parameter using the finite difference formula of equation (3.3), and the corresponding numerically calculated derivative. The differences in the total value of  $S$  are at few percent level at worst. The expression of  $T$  is dependent on a cut-off scale  $\mu$ , due to the fact that the dimension five operator responsible for the mass of the left handed Majorana state is not renormalizable. The dependence is of the form of  $\sim M_L \log(\mu)$ . We fix the scale by the mass of the heavier neutrino eigenstate  $\mu \propto M_1$ , and estimate the uncertainty resulting from the choice of the constant of proportionality. We find that this uncertainty is roughly on the level of  $\mathcal{O}(10\% \dots 30\%)$  when  $\mu$  is varied between  $1.5M_1$  and  $2M_1$ . Keeping this in mind, we fix  $\mu = 1.5M_1$  in what follows.

In total there are four physical parameters that enter the calculation for  $S$  and  $T$ : the three masses of the leptons and the mixing angle  $\theta$ . We vary these parameters and calculate the resulting values for  $S$  and  $T$ . It turns out that  $S$  is practically independent of  $\theta$ , and in general less restrictive than  $T$ . The area of the parameter space allowed by  $T$  is shown in figure 4.2. In this area  $S$  is between 0.1 and 0.2. In the left panel we have chosen  $M_2 = 0.5M_Z$ , and in the right panel  $M_2 = M_Z$ . The two sets of curves presented in each panel correspond to  $\sin \theta = 0.1$  and  $\sin \theta = 0.5$ . The  $x$  and  $y$  axis are the relative mass splittings

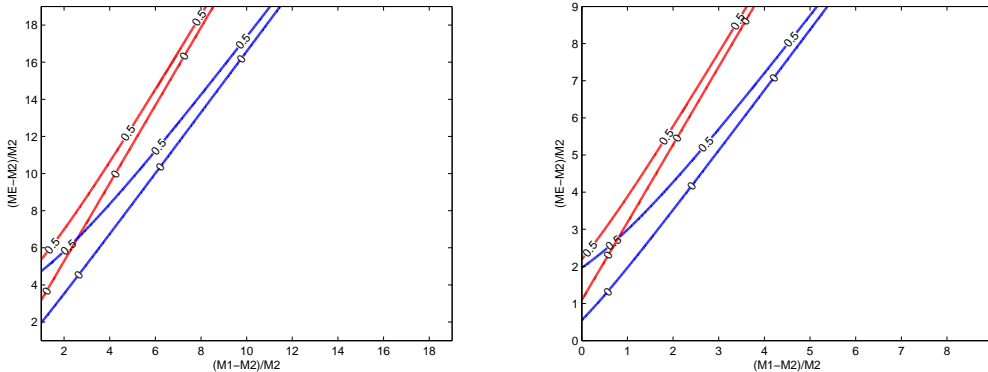


Figure 4.2: Constant  $T$ -contours in the  $((M_1 - M_2), (M_E - M_2))$ -plane for the choice of masses  $M_2 = 0.5M_Z$  (left) and  $M_2 = M_Z$  (right). Within each panel, the two sets of curves correspond to  $\sin \theta = 0.1$  (left set) and  $\sin \theta = 0.5$  (right set).

$(M_1 - M_2)/M_2$  and  $(M_E - M_2)/M_2$ . From the figures we conclude that the mass pattern allowed by the electroweak precision data is roughly  $M_E \sim 2M_1 - M_2$ . The general features of this analysis hold also for larger values of  $M_2$ , but in the following we will be mainly interested in the phenomenology arising from  $M_2 \sim 0.5M_Z \dots M_Z$ , since this mass range should be easily accessible in the LHC.

We have also scanned the  $(M_L, M_R, m_D)$ -space and searched for areas that produce acceptable values for  $S$  and  $T$ . Especially we are interested in seeing whether a certain hierarchial pattern of these parameters is favored by the electroweak precision analysis, possibly suggesting a Dirac-mass or a seesaw-type mass for the fourth generation neutrino. As it turns out, no such clue towards a single paradigm is to be found. All different hierarchies between the mass parameters of the effective Lagrangian we considered, contained areas that produce allowed values for  $S$  and  $T$ .

## Production of new leptons

Production of the fourth generation leptons should give a visible signal in the LHC experiment. Here we focus on the production of the new Majorana neutrinos, since these are expected to be lighter than the charged lepton. These may be produced either through the neutral current, i.e.  $Z^* \rightarrow \chi\chi$ , or the charged current, i.e.  $W^* \rightarrow l^\pm\chi$ , where  $l$  is a charged SM lepton. The neutral current process may be enhanced by vector boson fusion mechanism, via  $VV \rightarrow H \rightarrow \chi\chi$ , depending on the mass of the composite Higgs boson and the mass of the neutrino. The production cross section of a neutrino pair in 14 TeV  $pp$  collision as a function of the neutrino mass is shown in figure 4.3. The solid black line is the

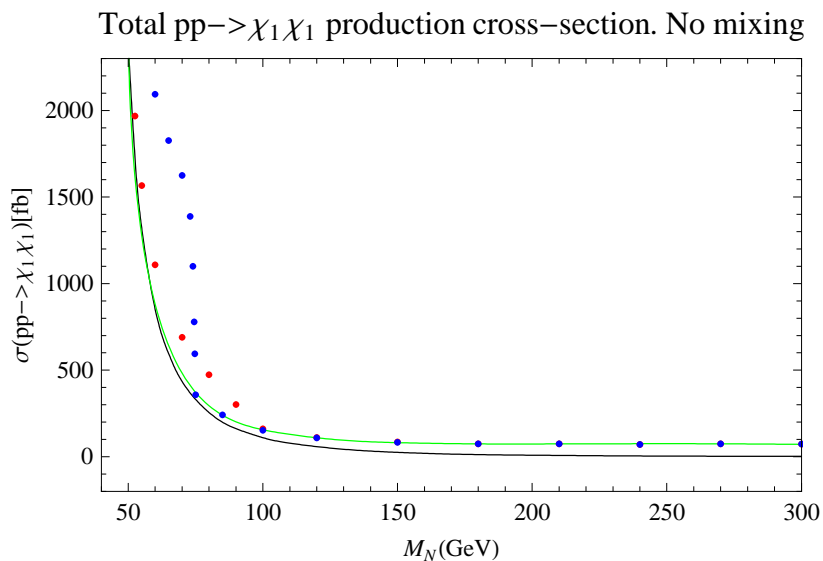


Figure 4.3: The production cross section of a  $\chi_1 \chi_1$ -pair in 14 TeV  $pp$  collision for  $\sin \theta = 0$ . The solid black line is the off-shell  $Z$ -boson channel, and the solid green is the total cross section including the vector boson fusion channel for Higgs mass of 100 GeV. The blue and red dotted lines are the corresponding total cross sections for Higgs masses of 150 GeV and 200 GeV, respectively.

off-shell  $Z$ -boson channel, and the solid green is the total cross section including the vector boson fusion channel for Higgs mass of 100 GeV. The blue and red dotted lines are the corresponding total cross sections for Higgs masses of 150 GeV and 200 GeV, respectively.

Assuming that the neutrino mixes with the SM leptons, it should decay via  $\chi \rightarrow lW$ . Therefore the interesting final states following the pair production of the neutrino are  $2l + 4j$  or  $3l + 2j + \cancel{E}$ , where  $j$  stands for a jet and  $\cancel{E}$  for missing energy. In [1] we have tabulated some basic search strategies for the two and three lepton final states with estimates for the signal to background ratio. We find the three lepton final state to be the most promising channel to look for the new neutrino.

## Higgs decay

The existence of a fourth generation of leptons could, depending on the relevant masses, have a significant effect on the decay rates and branching ratios of the composite Higgs boson. In the most interesting scenario the neutrinos are light so that the composite Higgs may decay into pair of neutrinos. If the Higgs mass and the neutrino masses happen to be in just the right range, the neutrino channel could even be the dominant decay branch of the Higgs boson.

In general, the Higgs couples directly to the mass of a particle, and thus tends to decay into the heaviest particles that are kinematically allowed. Therefore

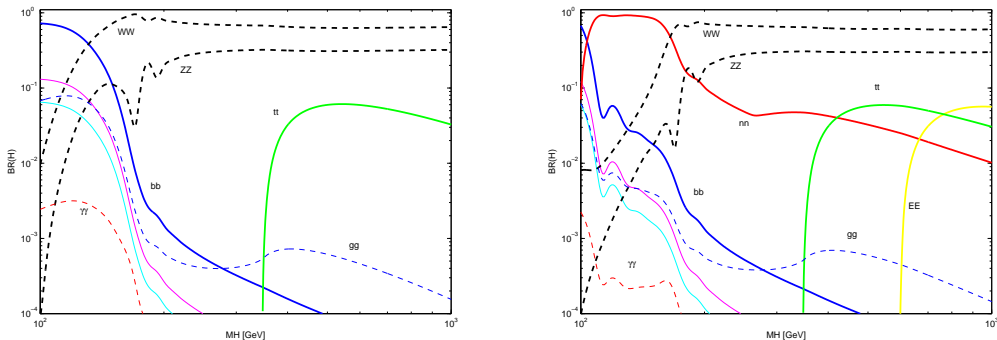


Figure 4.4: Left panel: The decay branching ratios of the SM Higgs boson. Right panel: The decay branching ratios of a MWT composite Higgs particle, assuming  $M_2 = 50$  GeV,  $M_1 = 130$  GeV,  $M_E = 300$  GeV and  $\sin \theta = 0.45$ .

the most interesting effects arise, if the Higgs is not heavy enough to decay into two  $W$  bosons, and if the neutrino mass is light enough to allow for the decay  $H \rightarrow \chi\chi$ . This effect is demonstrated in figure 4.4. The left panel of the figure shows the familiar decay widths of the SM Higgs boson, as a function of the Higgs mass. In the right panel we show the relative decay widths of the composite Higgs, assuming a fourth generation with neutrino masses of  $M_2 = 50$  GeV,  $M_1 = 130$  GeV. The mass of the charged fourth generation lepton is chosen to be  $M_E = 300$  GeV, and the mixing angle is  $\sin \theta = 0.45$ . This mass pattern is allowed by the electroweak precision data, yielding  $(S, T) = (0.17, 0.6)$ . Obviously this pattern is chosen to produce maximally visible effects in the Higgs decays, but even if the masses are not chosen as conveniently as here, one may still see notable effects.

#### 4.1.2 MWT and fourth generation of QCD quarks

Another possibility is to replace the fourth lepton generation with a fourth generation of QCD quarks. This requires choosing  $Y(Q_L) = -\frac{1}{6}$  for the hypercharge of the techni-quarks, instead of  $Y(Q_L) = \frac{1}{6}$ .

##### Oblique corrections

Since the new QCD quarks are Dirac particles, the contributions to  $S$ ,  $T$  and  $U$  may be calculated analytically. The required formulae are presented e.g. in [46]. For the techni-quarks, we again use the naive perturbative estimate  $S \approx 1/(2\pi)$ , but here we include a reduction that is expected to result from the walking behavior of the technicolor coupling constant [47]. We estimate this reduction to be of the order of 30%.

We vary the masses of the fourth generation quarks independently of each other, i.e. we do not require the new up-type quark to be heavier than the

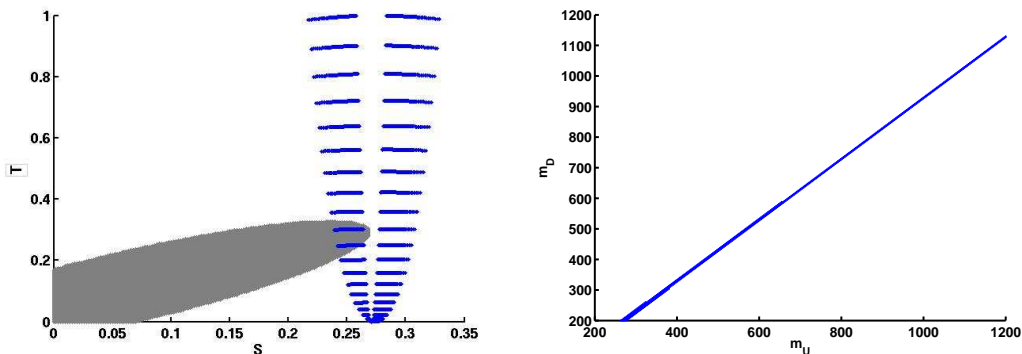


Figure 4.5: Left panel: The  $(S, T)$ -spectrum of a fourth generation of QCD quarks, and the allowed  $3\sigma$ -region. Right panel: Masses (in GeV) of the fourth generation quarks, allowed by the electroweak observables.

down-type quark, and calculate the resulting values of  $S$ ,  $T$  and  $U$ . The  $(S, T)$ -spectrum of the model is shown in the left panel of figure 4.5. The gray ellipsis is the  $3\sigma$ -allowed region obtained from the LEPEWWG global fit [42]. The right panel shows the points of the  $(m_U, m_D)$ -space that result in  $S$  and  $T$  inside the ellipse. The LEPEWWG fit assumes  $U = 0$ , and here we have required  $U < 0.05$ , so that no large deviations from the fit at  $U = 0$  should be expected. We note that the mass scale of the quarks is not restricted by the electroweak observables, but the mass difference is limited by the  $T$  parameter to the range of  $m_U - m_D \sim 50 - 75$  GeV.

### Higgs production

The existence of a fourth generation of QCD quarks would enhance the coupling of the composite Higgs to gluons. This effective coupling is generated by a quark loop with two external gluon legs and one Higgs leg. Since the Higgs coupling to quarks is directly proportional to the mass of the quark, the contribution of light quark loops is negligible. Thus, in the SM, the Higgs gluon coupling is dominantly generated by the top loop. Adding new heavy quarks would thus enhance this coupling dramatically. The heavier the quarks in the loop are, the larger is the coupling to Higgs, but, correspondingly, the larger is the off-shellness of the quarks. These effects approximately cancel and thus any new heavy quark gives approximately the same contribution to the effective coupling as the top. For a new quark generation one would thus expect an enhancement of the order of three to the Higgs gluon coupling, yielding an enhancement of the order of nine to the Higgs production cross section in gluon fusion channel, as well as to the Higgs partial decay width to the two gluon channel. Since gluon fusion is the dominant production channel for the Higgs boson in the LHC, this effect would dramatically enhance the Higgs production in the LHC, and correspondingly



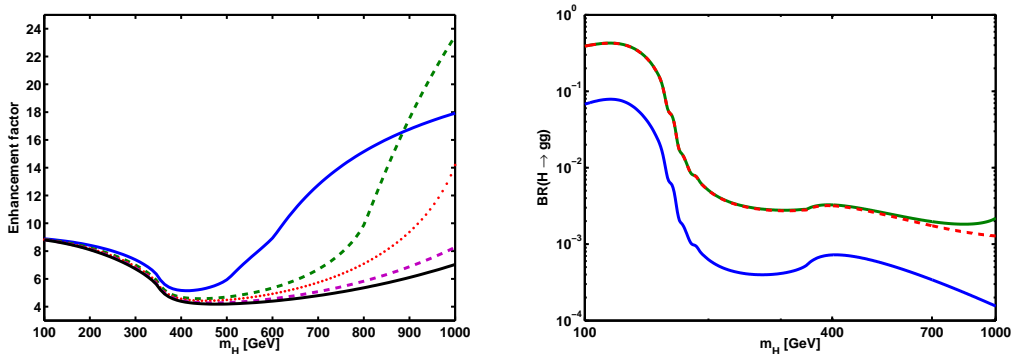


Figure 4.6: Left panel: The enhancement factor  $\sigma_4(gg \rightarrow H)/\sigma_{SM}(gg \rightarrow H)$  as a function of the Higgs mass for quark masses  $(m_U, m_D) = (250, 300)$  GeV (blue solid line),  $(400, 400)$  GeV (green dashed line),  $(500, 500)$  GeV (red dotted line),  $(800, 850)$  GeV (purple dashed line) and  $(\infty, \infty)$  (black solid line). Right panel: The Higgs decay branching fraction  $\Gamma(H \rightarrow gg)/\Gamma(H \rightarrow X)$  with fourth generation quark masses  $(500, 500)$  GeV (green solid line) and  $(800, 850)$  GeV (red dashed line), and in the SM (blue solid line).

lower the required integrated luminosity for detection of the Higgs.

The actual value of the enhancement factor depends on the masses of the new quarks and on the Higgs mass. The enhancement factor is plotted as a function of the Higgs mass in the left panel of figure 4.6, for various choices for the quark masses. The right panel of the figure shows the Higgs decay branching fraction to two gluons in the SM, and in presence of fourth generation quarks. A more detailed analysis for the effects of a fourth generation of QCD quarks on Higgs boson searches in the LHC is presented in [48].

## CP violation

A fourth QCD quark generation would change the physics of CP violation in the quark sector. In the case of three generations, there is only one CP-violating phase in the CKM matrix. This phase has to be fitted to explain all CP violation in the quark sector of the SM. However, if there is a fourth generation, the resulting four by four CKM matrix has in total three CP violating phases. This leaves a lot more freedom to the physics of CP violation. This in turn may be adequate to explain the origin of baryonic matter, i.e. the matter-antimatter asymmetry of the universe, through the electroweak baryogenesis. The additional CP violation would also explain the recently observed anomalies in the B-meson decays, as has been shown in [49] and [50], assuming quark masses in the range of 400-600 GeV.

### 4.1.3 NMWT and two new lepton generations

If the technicolor gauge group is  $SU(3)$  instead of  $SU(2)$ , and we choose  $Y(Q_L) = \frac{1}{6}$  for the hypercharge of the techniquarks, then we need to add two new lepton generations to the SM. The techniquarks transform under two-index symmetric, i.e. sextet representation of the gauge group  $SU(3)$  and thus, from the electroweak viewpoint, the particle content of the model looks like two new generations of quarks and leptons.

#### Oblique corrections

Here we restrict to the case of Dirac neutrinos, for simplicity. If one wishes to include the possibility of Majorana mass for the fourth and fifth generation neutrinos, one may simply follow the procedure presented in [I] for the fourth generation neutrino. The most important difference between Majorana and Dirac neutrinos in terms of the oblique corrections is, that for Dirac particles  $T$  is positive definite, but Majorana particles may produce negative contributions to  $T$ . As the precision data seem to suggest a small positive value for  $T$ , this feature is generally not that important. But in case there are other beyond SM fields besides the new leptons, that produce large positive contributions to  $T$ , one may wish for a negative contribution from the leptonic sector to cancel this effect.

Restricting to the Dirac case, there are total of four mass parameters that enter the calculation of the oblique corrections from the leptonic sector. For the TC sector we use the naive estimate  $S \approx 1/\pi$ , and apply a reduction of 30% due to walking dynamics. In terms of the fermion masses, the fourth and fifth generations may or may not be hierarchial, i.e. the fifth generation leptons may both be heavier than both of the fourth generation leptons, or not. Another question is, whether the charged lepton is always heavier than the corresponding neutrino. We examine whether there is a clue towards any of these scenarios in electroweak precision data.

The  $(S, T)$ -spectrum of the model is presented in the left panel of figure 4.7. The green points correspond to the hierarchial case, where the fifth generation is heavier than the fourth. The blue crosses are points of the parameter space, where at least one of the fourth generation leptons is heavier than at least one of the fifth generation leptons. We conclude that both the hierarchial and non-hierarchial scenario produce values of  $S$  and  $T$  inside the experimentally allowed region. The right panel of the figure shows the points allowed by  $S$  and  $T$ , and with  $U < 0.05$ , in the plane defined by the mass differences between the charged lepton and the neutrino in each generation. This plot does not show the absolute scale of the masses, but we have checked that practically any values between  $\sim 100$  GeV to a couple of TeV are allowed. As is seen from the plot, the maximal mass difference between the charged lepton and the neutrino of a single generation is around 120 GeV, and can be either positive or negative, implying that either the charged lepton or the neutrino may be the heavier

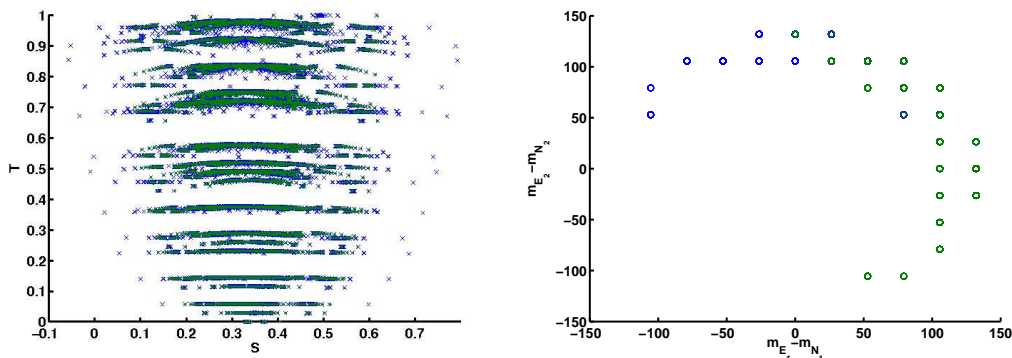


Figure 4.7: Left panel: The  $(S, T)$ -spectrum of two new SM-like lepton families. The green points correspond to the hierarchial case, where the fifth generation is heavier than the fourth, and the blue crosses correspond to the non-hierarchial case. Right panel: the mass differences (in GeV) of the charged lepton and the neutrino within each family, allowed by  $S$ ,  $T$  and  $U$ .

lepton of a given generation. However, the maximal allowed mass difference of a given generation is affected by the mass difference of the other. If there is a large mass splitting in one of the new lepton generations, the other one must have nearly degenerate masses. Moreover, both of the mass differences are not allowed to be negative at the same time. This implies that at least one of the charged leptons must be heavier than the corresponding neutrino. If stabilized by additional symmetry, such neutrino could provide a dark matter candidate as discussed in [51].

### Collider signals

The collider signatures are expected to resemble those described in section 4.1.1. Depending on the masses, the fifth generation may or may not contribute substantially. The new neutrinos may have a significant effect on decays of the composite Higgs, as explained in section 4.1.1. If one of the charged leptons is lighter than the corresponding neutrino, the only decay channel available for the charged lepton is mixing with the lighter generations. Depending on the strength of the mixing, this could lead to a comparably long lifetime for the charged lepton, and interesting collider signals.

### 4.1.4 Unification

The fourth generation of SM-like fermions, together with the technicolor sector, may also play a role in achieving the unification of the SM gauge couplings. As is presented in [III], in some of the scenarios considered here unification is achieved to some degree, if we allow for the existence of additional Weyl fermions,

transforming under the adjoint representation of  $SU(3)$  color and  $SU(2)_L$ .

However, one must bear in mind that here we only consider the unification of the three gauge couplings of the SM. In the context of walking technicolor theory, there are two additional gauge couplings: the technicolor coupling  $\alpha_{TC}$  and the coupling constant of the ETC interaction  $\alpha_{ETC}$ . To form a complete grand unified theory (GUT) in the context of technicolor, one should aim at the unification of all the coupling constants of the model. Therefore too much emphasis should not be placed on this observation. One may, however, state that unification of the SM couplings is not exclusively a property of supersymmetric theories, but is achieved quite naturally also in some scenarios of technicolor theory.

Besides the cases considered here, one may relax the requirement of SM-like hypercharge assignments for the techniquarks, and possibly even consider additional leptons with exotic charges such as a charged neutrino and a doubly charged lepton. We have outlined such a scenario in [III], and obviously there are many more possibilities. The cases presented here are in a way the most minimal scenarios, and therefore should serve as a starting point for searches of this type of new physics. We want to point out, that the story of the possible fourth matter generation may be much richer than the typically considered sequential one.

## 4.2 Bosonic technicolor

In section 2.3 we presented a model framework [II] for the origin of fermion masses in technicolor, where a SM Higgs-like scalar boson is added to the usual technicolor sector to produce the Yukawa terms responsible for fermion masses. Here we will study the phenomenology and constraints of such a model.

### 4.2.1 Oblique corrections and FCNCs

As explained in section 2.3, the low energy spectrum of the model consists of three massive pions and two Higgs-like scalars. We calculate the oblique corrections resulting from these particles, as well as the contributions to flavor changing neutral current processes. The Feynman diagrams relevant for FCNCs are presented in figure 3.4, and those relevant for the calculation of the oblique corrections are shown in figures 4.8 and 4.9

We then scan the parameter space of the model by varying the parameters of the underlying Lagrangian (2.38), independently of each other. From these we calculate the masses of the physical particles and the mixing angle  $\theta$ , which in turn enter the calculation of the oblique corrections and the FCNCs. An additional restriction comes from direct Higgs boson search experiments, which limits the low mass range of the scalars of the model. The limits obtained for SM Higgs boson may be alleviated, depending on the mixing angle  $\theta$ , by

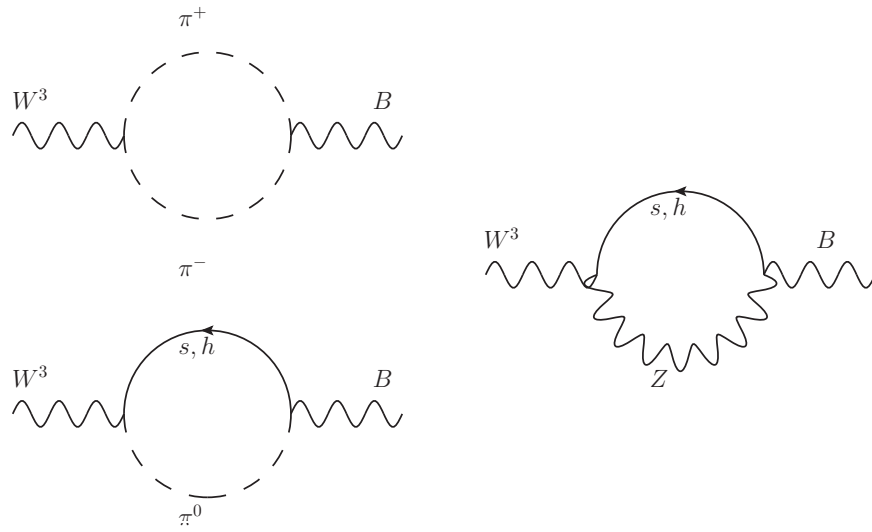


Figure 4.8: The diagrams contributing to  $S$ .

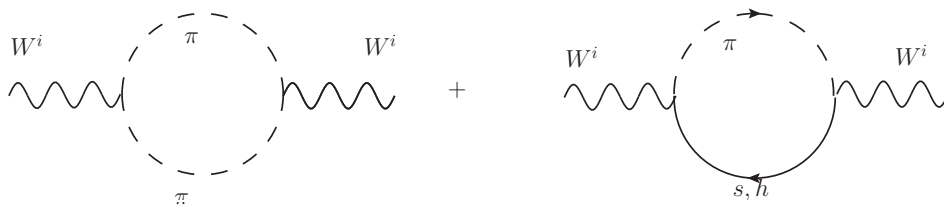


Figure 4.9: The diagrams contributing  $T$ .

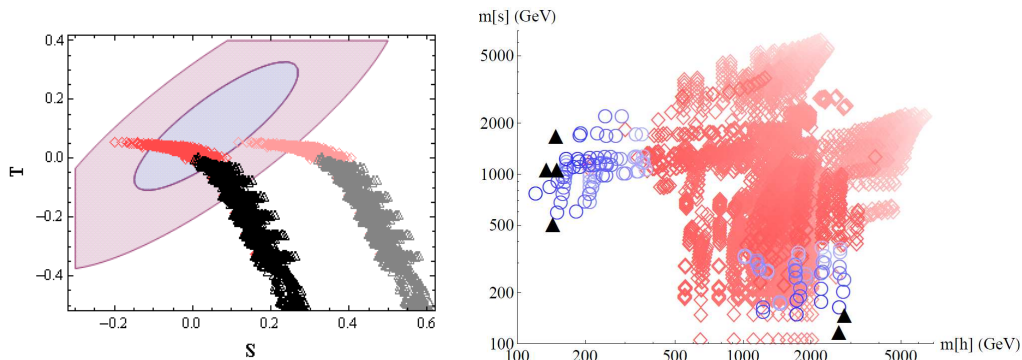


Figure 4.10: Left panel: The  $(S, T)$ -spectrum of the model, and the 90% confidence limit. The light red diamonds are ruled out by direct search limits. All points pass the FCNC tests. Right panel: The masses (in GeV) of the scalars of the model. The black triangles correspond to points inside the 90% confidence limit of the left panel, and the blue circles correspond to points inside the larger ellipse. The red diamonds are ruled out by  $S$  and  $T$ .

the weakened coupling of the light scalar and the Z boson. For details of the calculations to obtain these limits see [II].

## 4.2.2 Results

The  $(S, T)$ -spectrum of the model is shown in the left panel of figure 4.10. The inner ellipse is the 90% confidence limit contour. All of these points pass the FCNC constraints, but the light red diamonds are ruled out by direct search experiments. The leftmost set of points are achieved by the perturbative calculation performed with the physical particle spectrum of the low energy effective theory. The rightmost set is achieved by adding a naive estimate for the nonperturbative contribution of the technicolor sector. Since the contribution of the technipions, that are assumed to be the lightest particles of the TC sector, is already included in the calculation of the leftmost points, the rightmost points include some amount of double counting and therefore tend to exaggerate the value of  $S$ . On the other hand, the leftmost points completely ignore the effect of higher resonances, and the actual values may lie between these two data sets. In the following analysis we concentrate on the leftmost values, however.

The right panel of figure 4.10 shows the corresponding parameter space points in  $(m_h, m_s)$ -plane. Here the black triangles correspond to the points inside the 90% confidence limit contour, the blue circles correspond to triangles in the left panel that are inside the larger ellipse and the red diamonds correspond to triangles even farther out. All the points in the right panel pass the direct search and FCNC limits.

From these figures we conclude that most of the parameter space of the model is ruled out by experiment. The points of the parameter space that produce the

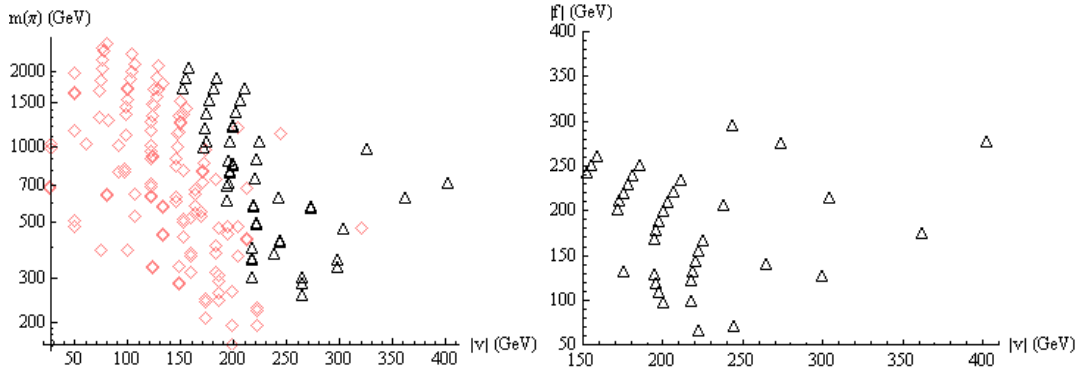


Figure 4.11: Left panel: The FCNC constraints on the pion mass as a function of the scalar vacuum expectation value  $v$ . The black triangles pass the FCNC tests, and the red diamonds are ruled out. Right panel: The allowed values of  $f$  and  $v$ , after taking all constraints into account.

most acceptable values for  $S$  and  $T$  correspond to a very light scalar, hence ruled out by direct search experiments. Correspondingly, the points that produce large values for both scalars are ruled out by  $S$  and  $T$ . However, there are corners of the parameter space that survive all experimental tests, and hence the model is viable, though heavily restricted. A clear prediction of the model is the existence of one light and one heavy Higgs-like scalar.

The left panel of figure 4.11 shows the points in the  $(v, m_\pi)$ -plane that are consistent with the 90% confidence limit of  $S$  and  $T$ , and pass the direct search limits. The red diamonds are ruled out by the FCNC limits, and the black triangles are allowed by all data. We conclude that the pion mass may have practically any value from couple of hundred GeV to a few TeV. The right panel of the figure shows the points in the  $(v, f)$ -plane that are allowed by all data. The relative size of  $v$  and  $f$  reflects the relative strength of the symmetry breaking in the fundamental and composite sector. As is seen in the figure, a very wide area of this plane is covered by the allowed points, implying that the origin of the electroweak symmetry breaking in this model may originate either mostly from the composite sector, or mostly from the fundamental scalar. Or it might be an even combination of both. This balance is then reflected on the mixing angle and on how the physical pions are composed of the technipions and the Goldstone bosons of the fundamental scalar sector.





# Chapter 5

## Conclusions

### 5.1 Technicolor and fourth generation of fermionic matter

Walking technicolor is a simple and natural way to address the intrinsic problems of the SM Higgs sector. The basic idea of technicolor theory is to incorporate the mechanism of dynamical symmetry breaking, a process already observed in nature in QCD and in the phenomena of superconductivity, to act as the origin of electroweak symmetry breaking and gauge boson masses. A simple copy of QCD is, however, not capable of producing the observed masses of the weak gauge bosons while evading the limitations from electroweak precision tests and flavor physics. This problem is cured by walking, i.e. nearly conformal dynamics of the technicolor coupling constant, which is achieved by putting the technifermions in a non fundamental representation of the technicolor gauge group.

To avoid the global and gauge anomalies associated with the technicolor sector, a non sequential fourth generation of SM-like fermionic matter may naturally arise. We have studied the scenarios where one or two new generations of SM-like leptons appear as a consequence of cancelling the anomalies of a technicolor sector, but there are no new quarks charged under QCD color, and complementary to that, also the scenario where a new QCD generation arises, without a new lepton generation. We have found each of these possibilities viable in light of all existing electroweak and flavor precision data.

These models can be considered natural from many viewpoints. The naturalness and fine tuning problems of the SM Higgs are trivially avoided as a result of the dynamical origin of electroweak symmetry breaking. Also the existence of the new, non sequential fermion generations is a natural consequence of anomaly cancellation. The new QCD quarks, if they exist, may help to explain some recent observations of deviations from the SM predictions on CP-violating physics in the B meson experiments, and they may even play a role in explaining the matter-antimatter asymmetry, the origin of all baryonic matter in the universe. On the other hand, the fourth generation neutrino could be a viable WIMP

candidate, hence helping to solve the long lasting puzzle of dark matter. All these effects arise naturally, without invoking any additional global symmetries or adding new fields ad hoc.

The existence of a fourth generation of matter, be it leptons or quarks, would be detectable in the LHC experiment. We have outlined some basic search strategies, and more detailed studies in the context of a sequential fourth SM generation have been carried out by other groups. The physics of the Higgs particle is also sensitive to the fourth generation. A fourth generation of quarks would dramatically enhance the production rate of the Higgs through gluon fusion, whereas a fourth lepton generation could significantly alter the decay channels of the Higgs. From our point of view, the most interesting would be the detection of a non sequential generation of fermionic matter. As there are currently no other schemes that suggest that we should only find leptons but not quarks, or vice versa, this kind of observation would strongly point towards the idea of dynamical electroweak symmetry breaking and technicolor. As the collider signals of the technicolor sector itself could be more difficult to distinguish from other model building paradigms, the existence of a non sequential fourth generation could be the single most striking evidence towards this direction.

## 5.2 The origin of fermion masses

Technicolor models do not provide an explanation of fermion masses. In the SM, the Higgs boson acts as a source for both the gauge boson and fermion masses. Technicolor models generate the correct gauge boson masses, but leave the SM fermions massless. Typically one assumes that fermion masses are generated by an ETC gauge symmetry, broken at a very high scale, resulting in a low energy effective theory that resembles the Yukawa sector of the SM.

We have explored a simple model framework where the technicolor sector is accompanied by a fundamental scalar boson, much like the SM Higgs. This scalar sector is taken to represent the low energy effective model of an ultraviolet complete ETC theory. This framework allows well defined perturbative calculations of the particle masses and their effects on electroweak and flavor precision experiments. We find the model to be allowed, although heavily restricted, by all existing data. The model predicts the existence of one light and one heavy Higgs-like scalar. On top of that, there are massive pions and other resonances, which may or may not be light enough to be observed in the early years of the LHC experiment.

## 5.3 Outlook

In this work we have considered a dynamical explanation for the origin of electroweak symmetry breaking and some simple nontrivial implications this would

have on flavor physics. Our simple framework for the origin of fermion masses is yet void of any explanation for the hierarchical nature of the SM generations: Why are there exactly three generations of fermionic matter and why are their masses arranged as they are? Interestingly, we find that the mechanism of dynamical electroweak symmetry breaking might naturally induce new generations of fermionic matter, as a result of anomaly cancellation. If the fermion masses are generated by effective Yukawa couplings to the scalar Higgs sector, representing the yet unknown ETC sector at low energies, then fermion masses of the order of the electroweak symmetry breaking scale  $v_{weak}$  appear most natural, since this implies Yukawa couplings of order one. It is the lighter SM fermions that need a richer explanation than just simple Yukawa couplings. The full gauge theory of electroweak symmetry breaking, including ETC to generate fermion masses, could also include a dynamical mechanism for generating the hierarchy of the effective Yukawa couplings, resulting in fermion mass hierarchy.

As the LHC generates more data we will begin to see deeper into the mechanism behind electroweak symmetry breaking. If this mechanism indeed turns out to be technicolor, then the next task will logically be to understand the origin of fermion masses. Hopefully this understanding will also give us an answer to the long lasting puzzle of fermion mass hierarchy.



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