U N I V E R S I T Y   O F   J Y V Ä S K Y L Ä

School of Business and Economics

THE BAYESIAN ESTIMATION OF PRIVATE INVESTMENT IN FINLAND

Economics

Master's Thesis

December 2009

Author: Samuli Pietiläinen

Supervisor: Professor Kari Heimonen

Supervisor: Lecturer Jani Luoto

UNIVERSITY OF JYVÄSKYLÄ SCHOOL OF BUSINESS AND ECONOMICS

Author

Samuli Pietiläinen

Title

The Bayesian Estimation of Private Investment in Finland

Major	**Description**
Economics	Master's Thesis
Time	**Pages**
December 2009	45+16

Abstract

This paper estimates an investment equation for private investment using Bayesian estimation techniques.. In the paper we derive the optimal capital accumulation behavior in the model economy from the households' optimization problem of utility. The equation is derived as in Smets and Wouters (2003). The model contains costly adjustment of investment and random shocks to adjustment cost function. The driving variable of investment is Tobin Q variable.

The empirical proxy for Tobin Q in this paper is the ratio of OMX Helsinki Cap Index to the price index of the physical capital. The investment series is the seasonally adjusted private investment in quarterly national accounts.

The AR(1) modeled investment shocks are found to be less persistent in Finland than in the euro area. The estimated median of persistence parameter ρ for Finland is 0.485. Also the shocks to investment adjustment cost function are found to vary less in Finland as in the euro area. The estimated standard deviation of the shocks is 0.065. The adjustment cost parameter κ is roughly the same for both data sets. The results are robust to loosening the strict prior of discount factor, β=0.99. The paper also provides discussion about adjustment cost parameter κ and we investigate the behaviour of the posterior chain of κ with different prior distributions for the parameter.

Keywords

Private investment, DSGE model, Bayesian inference, investment adjustment costs

Storing location

University of Jyväskylä / School of Business and Economics

JYVÄSKYLÄN YLIOPISTO TALOUSTIETEIDEN TIEDEKUNTA

Tekijä

Samuli Pietiläinen

Työn nimi

The Bayesian Estimation of Private Investment in Finland

Oppiaine	**Työn laji**
Kansantaloustiede	Pro gradu-tutkielma
Aika	**Sivumäärä**
Joulukuu 2009	45+16

Tiivistelmä

Tässä pro gradussa estimoidaan yhtälö yksityisille investoinneille bayesilaisella menetelmällä. Tässä työssä optimaalinen pääoman akkumulointi mallikansantaloudessa johdetaan kotitalouksien hyödyn optimointi-ongelmasta. Investointiyhtälö johdetaan kuten Smets'n ja Wouterin (2003) artikkelissa. Malli sisältää investointien sopeutuskustannukset ja satunnaisia shokkeja sopeutuskustannus-funktioon. Investointien selittäväksi muuttuja on Tobin Q -muuttuja.

Empiirinen vastine teoreettiselle Tobin Q muuttujalle on OMX Helsinki Cap indexin arvo suhteutettuna fyysisen pääoman hintaindeksillä. Työssä käytetty investointisarja on kausitasoitettu yksityisten investointien sarja kansantalouden neljännestilinpidossa.

Investointishokit ovat AR(1)-prosessi. Shokit osoittautuvat vähemmän pysyviksi Suomessa kuin euroalueella. Estimoitu AR(1)-kerroin investointishokeille on 0.485. Investointishokit myös vaihtelevat vähemmän Suomessa kuin euroalueella, sillä estimoitu shokkien keskihajonta on 0.065. Investointien sopeutuskustannus on likipitäen samankokoinen Suomessa ja euroalueella. Tulokset ovat robusteja kiinnitetyn diskonttausparametrin $\beta=0.99$ löysäämiselle antamalla β:lle eri priorijakaumia. Tässä työssä myös keskustellaan sopeutuskustannusparametrista κ ja tutkitaan κ:n posterioiriketjujen käyttäytymistä kun parametrille κ annetaan eri priorijakaumia.

Asiasanat

investoinnit, DSGE-malli, Bayesiläinen estimointi, investointien sopeutuskustannus

Säilytyspaikka

Jyväskylän yliopisto / Taloustieteiden tiedekunta

ACKNOWLEDGEMENTS

The author would like to thank University of Jyväskylä for the opportunities provided during the years in the university. Now it is easy to see that the first step towards this Masters' thesis was taken already in late 2005 when I became one of the students to take part in the student exchange program between my home university and University of Toronto in spring 2007. The courses I took in Toronto gave my studies their own unique contents.

The author would like to thank also the School of Business and Economics and its staff members. Especially the help from the faculty when I was looking for an internship is greatly appreciated. Then of course, the author would like to thank the School for the financial aid during the internship in Ministry of Finance in fall 2008.

This thesis has had several supervisors. First of all, I would like to thank Professor Kari Heimonen for the supervision and the help in making this paper close to something that is called Masters's thesis. I'd like to thank Professor Heimonen for the support and help when things got difficult. This leads us to thank also Lecturer Jani Luoto for his help and support with the econometric methods used in the paper.

The author would like to thank the Economics department of Ministry of Finance for the possibility to join the department as a university intern for three months. I would like to thank the members of the staff for their support during the internship. Especially I would like to thank Mika Kuismanen for the challenging task and the challenging subject for thesis. Though it has not been the easiest thing to do, it has given me a lot and it sure has taught me a lot. Mika, thanks for teaching new things, the useful comments and the support. Also thanks to Mikko Sariola and Meri Obstbaum as well as Sami Ylä-Outinen in the Economics department.

Last but not least, I need to mention my family: mum, dad, sister and brother; I guess I just cannot thank you enough for just being there. Thanks also to my friends in and outside the university; I think there were times that you guys believed in me and the work more than I did. Your support is definitely acknowledged and hey! We're all happy to hear that soon we can change at least one subject over the lunch hour or gym exercise!

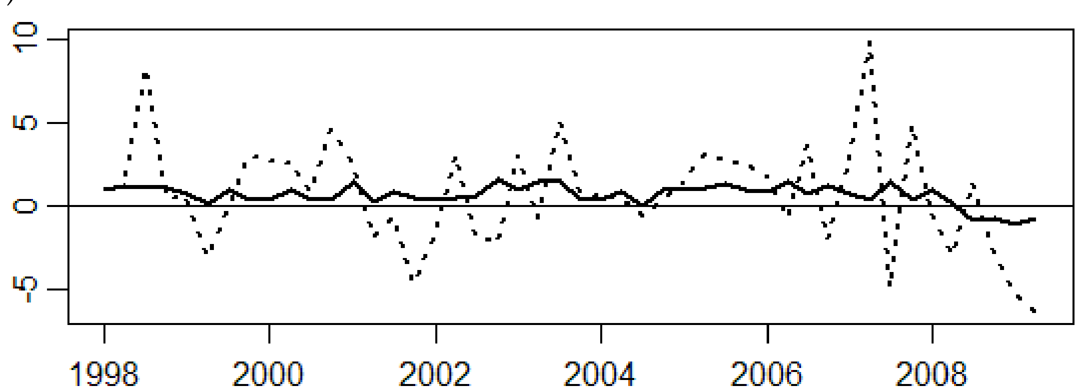
CONTENTS

1 INTRODUCTION

Understanding and even anticipating the growth rates of the gross domestic product (GDP) helps the government to adjust the fiscal policy according to the current phase of the business cycle. The knowledge about the current business cycle is also essential to the private sector. In addition for example the semiannual publication of Economic Outlook by Bank of Finland among other economic forecasts attracts wide interest in the media and in the public. This paper develops a model for private investment that is assumed to be one source of economic fluctuation. In the national accounts the private investment is volatile quantity compared to for example the private consumption. The growth rates of the private investment from period to period in the quarterly national accounts vary considerably more than those of private consumption. In the Figure 1, the growth rate of private consumption remains small and positive for the most of the time period whereas the growth rate of private investment varies from -5% to nearly 10% and hardly remains stabile. The changes in the private investment have a great impact on the aggregate domestic demand and thus to the domestic production.

In the contemporary macroeconomic research the dynamic and stochastic general equilibrium (DSGE) models have become very popular. Several central banks and research institutions have built their own models to simulate and forecast the economy. This paper searches an answer to question whether modern macroeconomic model and investment equation therein is possible to estimate using the Finnish data. The estimation of the model is done using the Bayesian estimation techniques that provide good way of identifying a model that contains strong structural base and assumptions about structural parameters. To our knowledge an exercise like this have not yet been carried out using the Finnish data.

Figure 1: The growth rates of private investment and private consumption (solid line)



The model of this paper is an investment model that follows closely the work of Smets and Wouters (2003) and Christiano, Eichenbaum and Evans (2005). The model is based on behaviour of utility maximizing households in a model economy. The neoclassical model is made "more real" by adding frictions such as investment adjustment costs that occur when the amount of investment in changed. As in Smets and Wouters (2003) the investment adjustment cost function is subject to random shocks. Our model is a closed economy model as in Smets and Wouters (2003) and the results are naturally compared to those of Smets and Wouters (2003). The recent work of Adolfson, Laséen, Lindé and Villani (2007) using Swedish data is a good reference to compare the results between Finnish and Swedish economy although the model in Adolfson et al. (2007) is an open economy model. The both papers Smets and Wouters (2003) and Adolfson et al. (2007) use a fixed discount factor β as done in the base case our paper but in addition we conduct tests of robustness of the discount factor parameter β by letting the data choose the β freely with different prior distributions. In this paper the role of the investment adjustment cost parameter κ is discussed more than in a standard DSGE model paper and a test of robustness of the parameter κ is conducted.

In this paper we derive a model that is referred in the literature an Euler equation model. From another point of view, the model can also be seen as an application of Tobin Q theory presented in Tobin (1969) because the driving variable of our theoretical investment equation is *marginal Q*. The existing literature is somewhat inconclusive whether Tobin Q theory is actually a good model for investment. The theory suggests that the variable Q includes all the information that is essential for making the decision about investment in physical capital (Hayashi 1982). However, for example Vilmunen (2002) argues clearly that the Tobin Q or the Euler equation models fail in empirical studies repeatedly. On the contrary our model seems to work well because the predictive series for investment simulated using model fits the actual investment series rather well.

As already being said the Euler equation model of investment as in Smets and Wouters (2003) is based on optimal behaviour of the households. The households own the capital stock and they decide the utilization rate of capital and the capital accumulation. The households maximize their utility that depends on consumption and supply of labour. The optimization problem has two constraints: the capital stock constraint and the budget constraint. The theoretical Tobin Q variable in this paper is the ratio of the Lagrange multipliers of the constraints. The empirical proxy for the theoretical Q is constructed as in Takala (1995) where the variable q is a ratio of stock price index to the price index of physical capital.

We estimate our investment model using Bayesian techniques that have become a popular way of estimating DSGE models. The advantages of the Bayesian method is that it allows to restrict the parameters to plausible areas of real axis thus the results are economically reasonable. In this econometric exercise the investment equation is linear model where the coefficients depend on structural parameters. The structural parameters are given the prior distributions as in Smets and Wouters (2003).

The main finding in this paper is that the $AR(1)$ modelled investment shocks are less persistent in Finland as in the euro area. The median of $AR(1)$-coefficient for Finland is 0.485 whereas the parameter value for the euro area is 0.913 (Smets and Wouters 2003). The standard error of the shocks to investment adjustment cost function is also smaller for Finland than for the euro area. The investment adjustment cost parameter κ is found to be relatively equal for both Finland and the euro area. The posterior median of the parameter $\kappa = 6.617$.The estimation is done with fixed discount factor $\beta = 0.99$ and the results from the estimation are checked for robustness using different prior distributions for the parameter β. We find against our own prior expectations that posterior of β is not (to large extent) dependent on the prior distribution of β.

The paper is organized as follows. The next section reviews the literature of the neoclassical investment and Tobin Q theories. The section ends with derivation of the theoretical model of this paper. The section 3 describes the data and manipulation of the theoretical model into form that we are able to estimate. Before the estimation there is a brief discussion of the Bayesian method. The results are presented and discussed in the end of section 3. The section 4 discusses the some weaknesses of the estimation and concludes the work.

2 THEORETICAL FOUNDATIONS

2.1 The review of relevant literature

In the neoclassical capital theory, optimal capital accumulation depends on the opportunity cost of capital K. A firm can own the capital or rent it from the market, but either way, cost of it is the market determined real interest rate. As presented in Romer (2006) if real price of renting capital is r_K and firm's profit function is given by $\pi(K, \mathbf{X}) - r_K K$, where $\mathbf{X}$ is vector of other inputs to production and some other variables such as price of produced good that are taken as given. If we assume partial derivatives respect to capital $\pi_K > 0$ and $\pi_{KK} < 0$ first order condition for profit maximization is

$$\pi_K(K^*, \mathbf{X}) = r_K \tag{1}$$

and it implicitly implies that firm adjusts its capital to level K^* where marginal product of capital equals rental price r_K. K^* is referred in literature as the desired level of capital.

In this section the development of neoclassical investment literature is presented. First we take a look at Jorgenson (1963) that is the benchmark model in literature. Then we investigate what happens if price of installed capital is allowed to differ from purchasing price (Tobin 1969). The theory is taken further adding capital adjustment costs as presented in Hayashi (1982). Abel and Blanchard (1983) expand the optimization problem to contain both firms and households, that maximize their utility. The firms face capital adjustment costs when investing and the households can adjust the capital utilization rate. The investment with the adjustment costs in DSGE model is presented as in Christiano Evans Eichenbaum (2001). In section 2.2 the model used in this paper is derived.

2.1.1 The neoclassical baseline model

In the neoclassical investment literature, Jorgenson (1963) is the reference to start with. Jorgenson (1963) studies the optimal behaviour of a representative firm that in competitive market chooses a production such that it maximizes the utility of the firm over time. The maximization of utility is achieved by maximizing the net worth of the firm over infinite planning horizon. The net worth is defined as an integral

$$\int_0^\infty e^{-rt} \left[R_t - D_t \right] dt \tag{2}$$

where R_t is revenue before taxes, D_t direct corporate taxes and r is discount interest rate. The revenue R_t is defined

$$R_t = pQ_t - sL_t - qI_t \tag{3}$$

where p is price of production Q_t, s is price of variable input, for example labor L_t, and q is the price of investment goods I_t. The direct taxes D_t are

$$D_t = u\left[pQ_t - sL_t - (v\delta q + wrq - x\dot{q})K_t\right]$$

where u is corporate tax rate, v and w are shares of tax credit of replacement cost and interest of capital, respectively and x is proportion of capital losses chargeable against income. The dot above a variable as in $\dot{q}$ denotes time derivative of variable in question. Assume that the production function $Q = F(K, L)$ has Gobb-Douglas form and the equation of the motion of capital is

$$\dot{K} = I - \delta K, \tag{4}$$

where δ is the depreciation of physical capital. The first order conditions for the marginal productivity of labour and capital respectively are

$$\frac{\partial Q}{\partial L} = \frac{s}{p} \tag{5}$$

$$\frac{\partial Q}{\partial K} = \frac{q\left[\frac{1-uv}{1-u}\delta + \frac{1-uw}{1-u}r - \frac{1-ux}{1-u}\frac{\dot{q}}{q}\right]}{p} \equiv \frac{c}{p} \tag{6}$$

In Jorgenson (1963) the last term in numerator in (6) is assumed to be zero, since all capital gains are assumed transitory. The user cost of capital c is then a reduced version of numerator in (6), the shadow price of capital:

$$c \simeq q\left[\frac{1-uv}{1-u}\delta + \frac{1-uw}{1-u}r\right]. \tag{7}$$

The shadow price of the capital (7) has two sources of variation in the short or medium term: The interest rate r and the price of investment goods q. The change in other variables in (7) is slower than the change in r or q and thus u, v, w and δ can be assumed constant in the short run. Under the first order conditions (5) and (6) the behavior of the firm is an iterative process in a way that i) at a given capital stock K_t, labour is set at level L_t by (5) and production is therefore $Q_t = F(K_t, L_t)$. Then ii) using (6) the firm sets desired level of capital K_{t+1} for the period $t+1$. iii) The Firm uses then capital K_{t+1} to set the labour L to new level according to (5) and so on. One can calculate the elasticity of output with respect to capital, γ, that is used in Cobb-Douglas production function:

$$\gamma = \frac{\partial Q}{\partial K}\frac{K}{Q}$$

from which one can derive using (6) and (7) the desired level of capital K^* :

$$K^* = \gamma \, \frac{p \; Q}{c} \; . \tag{8}$$

According to Jorgenson (1963), equation (8) relates changes in r to changes in desired level of capital K^*. The iterative process described above is the way how capital evolves in the economy and the investment is the change in capital from period to period. At each period t new investment projects I_t^N are undertaken but only certain share, ω_t is completed at the same period. Thus, there is a distribution of lags that determine when project I_t^N is fully accounted into capital stock. The lag structure is thus a sort of friction of investment, but not in the way the frictions are later taken into account in this paper. According to Jorgenson (1963) the investment turns fully into physical capital eventually and in that sense the investment is frictionless. The total of the investment in new projects I_t^E is the sum of new projects starts at time t and the past starts that are completed at time t:

$$I_t^E = \sum_{\tau=0}^{\infty} \omega_\tau \, I_{t-\tau}^N = \omega(L) \, I_t^N \; ,$$

where $\omega(L)$ is a power series in the lag operator. It is assumed that in each period I_t^N is chosen such that it closes the gap between desired capital stock K^* and the sum of actual capital K_t and completions of the past investment projects at time t. This leads to form

$$I_t^E = \omega(L)(K_t^* - K_{t-1}^*) \; .$$

The total investment I_t at period t is the sum of investment in new projects and the replacement investment. The latter is defined as a constant share of capital stock in same period, thus

$$I_t = \omega(L)(K_t^* - K_{t-1}^*) + \delta K_t \; , \tag{9}$$

where δ is the depreciation rate.

In conclusion the basic neoclassical view is that the investment is dependent on the present and past changes in desired capital K_t^* that is determined *ceteris paribus* by user cost of capital as (8) defines (Jorgenson 1963). The driving variables of the user cost of capital is interest rate r and the price of the investment goods q.

There are difficulties with baseline model. Romer (2006) demonstrates them considering a discrete change in interest rate. The interest rate r is a component of user cost of capital (7) and causes a discreet change in c. The change in c causes also a discrete change in desired capital K^*. The motion of capital is $\dot{K} = I - \delta K$ by assumption and discrete change in desired capital ($\dot{K}$ suddenly infinite) would require infinite rate of investment. Such infinite investment is not possible in any economy, because the economies are limited by their ability to produce. The second problem is that neoclassical investment model by Jorgenson (1963) is not forward looking. The investment is undertaken when firms believe that more capacity is needed in the future or proportion of inputs need to change. (Romer 2006)

2.1.2 Tobin Q theory

In the seminal paper Tobin (1969) presents a model that describes the behavior of the balance sheet of the economy. The main assumption in Tobin (1969) is that the economic agents make the spending decisions and the portfolio decisions independently. This means that for example households make decisions how to distribute their wealth among different assets or debts (the portfolio decisions) regardless of consumption (the spending decision). Each sector of the economy such as the households, the firms or the public sector, choose their portfolio according to given interest rate of each asset. In this framework one possible asset is physical capital. In Tobin (1969) variable q is defined differently as in Jorgenson (1963), where q is defined as price of investment goods. In Tobin (1969) the variable q is the market price or valuation of the installed capital and it may differ from the price that capital is purchased. The variable q enters the model as a coefficient of the capital stock K.

As an example in a simple one sector economy with capital and money as possible assets, the real wealth W is defined as

$$W = qK + \frac{M}{p}$$

where M is amount money in the economy and p the price of the consumer and investment goods. The variable q appears also in so-called rate of return equation that is defined $r_K\, q = R$ and manipulating this, one easily gets equation for q :

$$q = \frac{R}{r_K} \tag{10}$$

In equation (10), R is regarded as a continuous stream of return generated by non depreciating capital K and r_K is rate of return of capital. Since the economy is assumed to be perfectly competitive r_K is also the cost of renting the capital K. Tobin (1969) argues that there is obvious direct connection between the variable q and the rate of investment. When $q = 1$ the market valuation of capital K and its replacement cost are equal and there is no temptation to make capital investment. When $q > 1$ the market valuation is higher than the replacement cost of capital which induce households allocate greater share of wealth to capital causing positive investment. If $q < 1$ the direction of the allocation is the opposite. The change in the portfolio as assumed to be frictionless, and it has nothing to do with other decisions made in the economy. Tobin (1969).

The definition of variable q in (10) has lead to two interpretations of q: average q and marginal q. *The average q* is a simple application of q theory: q is a ratio of the market valuation of the existing capital to the replacement cost of that capital. *The marginal q* on the other hand is a ratio of change in the market valuation to change in the replacement cost when one unit of capital is added (Hayashi 1982, 214). The average q is fairly easily observed by econometrician and there are studies where researchers try to identify relation between the average q and the investment in physical capital (Hayashi 1982, 218). On the contrast according to Hayashi (1982) the marginal q should be used to investigate the drivers of the investment. Hayashi (1982) develops model with adjustment costs (more below) and also presents conditions under which the average and the marginal q are equal.

2.1.3 The capital adjustment costs

The Tobin Q theory and The Jorgensonian neoclassical model both assume that all the resources used in the investment in physical capital transform into new physical capital in the economy. It is fairly easy to argue that the models above are against the everyday life in the economy and the investment is associated some extra costs other that the cost of new capital goods. The installing of the investment goods is costly, because additional goods and services are used in the installing process but these goods and services are not accounted in the physical capital. Some of the investment goods might be simply destroyed while they are being installed. Already planning an investment is costly as Lucas (1967) suggests. The denominator of all the types of the capital adjustment costs is that the cost arises when new investment project is undertaken and there will be changes in the stock of capital.

Lucas (1967) considers some modifications to Jorgensonian neoclassical investment (Jorgenson 1963) by modelling the firms with two divisions, one for production and one for planning. Both the divisions use the same inputs, physical capital and variable input. The production function F has third argument $I(t)$ to reflect the fact that planning division cuts the final production quantity because all inputs are not used for production. Thus the profit function of the firm π_t is

$$\pi_t = p_t F[K_t, N_t, I_t] - w_t N_t - v_t I_t, \tag{11}$$

where N_t is variable input, I_t investment and w_t and v_t their prices, respectively. The use of inputs of the planning division is rising in investment, and both the divisions are supposed to operate under the decreasing returns to scale. This implies that the greater investments are the larger sacrifices must be made in form of lost output. Put formally, partial derivative of F with respect to investment is negative, $F_I < 0$. Lucas (1967)

The Tobin Q theory suggests that investment is a function of q as presented above and the capital is adjusted freely but Jorgenson (1963) has been criticized by Hayashi (1982) because the role of production in Tobin Q theory is unclear. Tobin (1969) assumes the spending and the capital accumulation decisions to be made separately and this according to Hayashi (1982) is inconsistent with the perfectly competitive environment where the firms operate. In Jorgenson (1963) the rate of investment is not determined by theory, it adjusts according to a certain mechanism based on the differences in the desired and the current level of capital. The link between the capital adjustment costs and Tobin Q theory is Lucas & Prescott (1971) which show that the investment theory with the adjustment costs and Tobin Q theory are equivalent. Hayashi (1982) thus incorporates the adjustment costs and Tobin Q theory in neoclassical setting and presents what is called "very general model of firm's present value maximization".

The Hayashi (1982) model starts with the assumption that a firm maximizes its present value of future after-tax net receipts:

$$V(0) = \int_0^\infty R_t \exp\left(-\int_0^\infty r_s\, ds\right) dt, \tag{12}$$

where r_s is nominal discount rate. The net receipts R_t in (12) are:

$$R_t = [1 - u_t]\,\pi_t \,+\, u_t \int_0^\infty D_{x,t-x} p_t^I\,(t-x)\,dx \,-\, [1 - k_t]\,p_t^I\,I_t, \tag{13}$$

where u_t is the corporate tax rate and π_t is the gross profits before tax. $D_{x,t-x}$ is the depreciation allowance per dollar of investment for tax purposes on an asset of age x according to tax code that was in effect at time $t - x$, p_t^I is the price of investment goods, I_t investment and k_t the rate of investment tax credit. Hayashi (1982) uses the neoclassical profit function π_t:

$$\pi_t = p_t\,F[K_t, N_t, t] - w_t\,N_t, \tag{14}$$

where p_t is price of final good and N_t is variable input with price w_t. The firm maximizes (12) subject to capital accumulation equation, that is a modified version of (4):

$$\dot{K} = \psi(I, K; t) - \delta K, \tag{15}$$

where ψ is increasing and concave *installation function* in I. The difference between equations (4) and (15) is that in latter all of the investment I does not turn into capital, only $\psi \times 100$ per cent does. The Hamiltonian for the firm's optimization problem (12) using (13) and (14) subject to (15) can be written as

$$\begin{aligned} H(I_{t,}, N_t, t, \lambda_t) \;=\; & \int_0^\infty \left[(1-u)\pi - (1-k-z_t)p_t^I I\right] \exp\left(-\int_0^\infty r\, ds\right) \\ & + \int_0^\infty \left\{u(t)\left[\int_{-\infty}^0 D(t-v,v)p_t^I(v)I(v)dv\right] \exp\left(-\int_0^\infty r\, ds\right)\right\} dt \\ & + \lambda_t[\psi(I,K;t) - \delta K] \end{aligned} \tag{16}$$

where λ_t is the shadow price of the constraint (15). z_t is defined:

$$z_t = \int_0^\infty u(t+x)D(x,t)\exp\left(-\int_0^\infty r(t+s)\,ds\right)dx. \tag{17}$$

Verbally, first term in (16) is firm's after tax present value that takes into account investment tax deductions and second term is present value of current and future tax deductions due to investments made in the past. The second term in (16) is independent of result of the optimization problem.

The first order conditions are:

$$\begin{array}{lcl} \frac{\partial H}{\partial N_t} = p_t F_N - w_t = 0 & \Leftrightarrow & F_N = \frac{w_t}{p_t} \\ \frac{\partial H}{\partial I_t} = -(1-k-z_t)\; p_t^I + \lambda\,\psi_I = 0 & \Leftrightarrow & (1-k-z_t)\,p_t^I = \lambda\psi_I \\ \dot{\lambda} = -\frac{\partial H}{\partial K_t} & \Leftrightarrow & \dot{\lambda} = (r+\delta-\psi_K)\lambda - (1-u)\pi_K \end{array} \tag{18}$$

and the transversality condition is that in the limit, the product of shadow price and capital stock - in other words - market valuation equals zero:

$$\lim_{t\to\infty} \lambda_t K_t \exp\left(-\int_0^t r\,ds\right) = 0.$$

The first order conditions (18) bring insight to the investment behavior. After rearranging the terms and adding λ in both sides, one can write the second condition $\frac{\partial H}{\partial I_t}$ in following form:

$$(1-k)\,p_t^I + (1-\psi_I)\lambda = \lambda + z_t\,p_t^I, \tag{19}$$

that states essential the nature of optimum: the marginal cost of investing one unit must equal the marginal benefit of doing so. On the left hand side of (19) the first term is the purchasing price of the new investment goods. The firm need not to pay full market price p_t^I since the investment tax credits (rate k) reduce the actual purchasing price. The second term on the left hand side is the adjustment costs associated with investment in terms of market value. One can think that ψ_I represents the ratio of new physical capital in capital stock from investment project X to size of that project. If there are no adjustment costs $\psi_I = 1$ so there are no market value foregone due to investment costs. The marginal benefit of investing one unit is increased market value λ and present value of investment tax credit $z_t\,p_t^I$.

Above we discussed about Tobin Q theory, of which Hayashi (1982) interprets in a way that *average* q is ratio of V to $p_I K$, verbally, ratio of discounted net present value of firm (12) to purchasing price of the existing capital stock. The *marginal* q is instead defined:

$$q = \frac{\lambda}{p_t^I}. \tag{20}$$

The link between the marginal q and the adjustment costs can be seen if one uses (20) to write $\frac{\partial H}{\partial I}$ in (18) following form:

$$\frac{q}{1-k-z} = \tilde{q} = \frac{1}{\psi_I}.$$

Under the optimal investment decisions this identity must hold and thus investment is a function of $\tilde{q}$ (Hayashi calls it *modified* q) and K :

$$I = F(\tilde{q}, K; t).$$

When a firm knows q, K, k and z, firm's optimal investment depends only on the adjustment costs ψ_I. The variable q contains all necessary information of the production function of the firm, the demand for products and the expectations about the future. To get an idea how q evolves in time, one can write (20) as $\lambda = q\, p^I$ and derive it with respect to time to obtain another definition for $\dot{\lambda}$. Using this definition and one in (18) and (20) one gets definition for the time derivative of q :

$$\dot{q} = \left(r + \delta - \frac{\dot{p}^I}{p^I} - \psi_K\right) q - (1-u)\frac{\pi_K}{p^I}.$$

However, the marginal q is still difficult to observe. Hayashi (1982) shows in which circumstances the average q and the marginal q are essentially the same and uses that connection to derive the average q. Those conditions are: The firm is prace-taker in the final goods market and both production and investment installation functions are linearly homogenous. Hayashi (1982) also tests empirically the theory, but finds no solid support for the theory.

The conditions for equality of marginal q and average q is widely referred in literature, see for example Vilmunen (2002). In our work the equality plays no role because Hayashi (1982) presents a model of the net value optimizing firms but below we will see that investment in our model is a choice made by the utility maximizing households.

2.1.4 Towards general equilibrium model

So far the studies that have been mentioned, have assumed that the firms own the capital they use for production or they rent it from "somebody". The models have been so called partial equilibrium models where the focus is only in few markets of the economy. As a contrast to this view, Abel and Blanchard (1983) have studied investment in an model economy that contains both firms and households and thus the number of the markets of the economy that need to be cleared rises. The households maximize their utility from consumption subject to given budget constraint. The decision variables of the households include consumption, supply of labour (fixed in Abel and Blanchard 1983 for simplicity) and saving. The saving is allowed since Abel and Blanchard (1983) add bond market to the model.

In Abel and Blanchard (1983) also the firms optimize their behaviour. A firm maximize the present value of its cash flow using given production and profit functions. The investment and the capital accumulation is one decision variable of the firm. In previous models that have been discussed firms have used retained earnings to finance their investment projects. In Abel and Blanchard (1983) the firms are able to issue bonds and they can borrow cash to finance investment projects. The bond market connects the households and the firms. The firms own the capital, and they pay wages, interest rate on bonds and dividends to households. The optimization problem of the firms is constrained by capital accumulation. As being already said the objective of a firm is to maximize the present value of its cash flow

$$f(k_t) - i_t \left[1 + h\left(\frac{i_t}{k_t}\right)\right] - w_t - r_t B_t \tag{21}$$

subject to the motion in capital

$$\dot{k} = i_t + \delta k_t.$$

In (21) f is the production function, i_t investment, $h(\cdot)$ adjustment cost function and w_t wage bill, r_t is interest rate on the outstanding bonds B_t. The first order condition in this case is similar to above in (18): in optimum the marginal cost of investing equals the shadow value of the installed capital. In Abel and Blanchard (1983) households' utility is increasing in consumption that is defined as the production less the investment[1]. The households' maximization problem is constrained by wealth that is sum of wages, dividends from firms and interest on bond holdings less consumption and new saving e.g. purchases of bonds. The optimum for households is that the marginal utility of consumption equals the shadow price of the wealth constraint. Abel & Blanchard (1983) show that the variable q enters both optimization problems and thus has impact on both path of consumption and path of investment.

The way Abel & Blanchard (1983) model the economy with the households and the firms has several features that can be found in the recent literature of the economic modelling. We now turn to the recent general equilibrium models.

[1] In this model production is either consumed or used in investment, thus that is the goods market equilibrium.

2.1.5 The investment adjustment costs in DSGE model

In recent years the work of Christiano, Eichenbaum and Evans (2005) and Smets and Wouters (2003) have been frequently referred. Both the papers develop complete macroeconomic model where the theory has microeconomic foundations and the model contains several nominal rigidities for example in prices and investment. These two models are very much alike and the reason is that Smets and Wouters (2003) follow closely a NBER working paper by Christiano, Eichenbaum and Evans (2001). The model that is presented in Christiano et al. (2001) contains the utility maximizing households and the net present value maximizing firms as in Abel and Blanchard (1983), and the model economy is thus neoclassical by nature. In Christiano et al. (2001) the modelers try to make the model economy closer the real world by adding rigidities such as stickiness in prices or wages and costs arising from adjusting the capital stock. In the following we present the Christiano et al. (2001) model relevant to this paper. In Christiano et al. (2001) the capital accumulation is the decision variable of the households and the households also own the capital stock and decide how much capital services to supply to the firms.

In Christiano et al. (2001) there are two types of firms: Final good producers and intermediate goods producers. The latter type firm $j \in (0,1)$ -a monopolist- uses labour and capital services as inputs to production of intermediate good j. The inputs are rented from the perfectly competitive factor markets. These rents of the use of the factors are paid to the households. The intermediate good producers use certain production function to produce goods to the final good producers. The intermediate producers are able to reoptimize the price of their product $\tilde{P}_t$ with a constant probability $1-\xi_p < 1$.[2] The intermediate producers' problem is to maximize the discounted present value of profits choosing path of $\tilde{P}_t$ such that following sum (22) is maximized:

$$E_{t-1}\left[\sum_{l=0}^{\infty}(\beta\xi_p)^l v_{t+l}(\tilde{P}_t X_{tl} - s_{t+l}P_{t+l})Y_{j,t+l}\right], \tag{22}$$

where

$$X_{tl} = \begin{cases} \pi_t \times \pi_{t+1} \times \cdots \times \pi_{t+l} & \text{if } l \geq 1 \\ 1 & \text{if } l = 0 \end{cases} \text{, where } \pi_t = \frac{P_t}{P_{t-1}}.$$

E_{t-1} is the expectation operator and it is assumed that the firm is using all the relevant information at time $t-1$. $P_t X_{tl}$ is the revenue per one unit of intermediate good and $s_{t+l}P_{t+l}$ is the cost of production per one unit. $Y_{j,t+l}$ is the amount of production of the firm j at time $t+l$. β is the discount factor, v_t is value of a dollar to the household, that is, the Lagrange multiplier of asset constraint equation in optimization problem (exogenous for the firm) and ξ_p is probability that a firm *cannot* set the price at desired level.

[2]More about setting the price, see: Calvo 1983.

The model is constructed in a way that it is the household that makes most of the decisions in the economy. They decide among others:

- consumption
- capital accumulation
- the supply of capital services
- purchases of securities
- wage (if possible).

The maximization problem therefore is complex, there are both the asset and the budget constraints that contain together several variables. The stock of physical capital $\bar{k}_t$ evolves in familiar way:

$$\bar{k}_{t+1} = (1-\delta)\bar{k}_t + F(i_t, i_{t-1}), \tag{23}$$

where function $F(\cdot)$ describes the technology that changes investment i_t to new capital. Note that transformation from investment to capital takes one period of time. Also note that function $F(\cdot)$ has as argument investment in previous period $t-1$. The Function $F(\cdot)$ contains the investment adjustment cost function $S(\cdot)$ that is discussed more below. In Christiano et al. (2001), the investment is not always necessary. The households may also change the rate of capital utilization, u_t. Above, intermediate good producers use capital services, k_t, as an input to production process and there is a simple way to connect all these three variables:

$$k_t = u_t \, \bar{k}_t.$$

The fact that households own the capital stock ($\bar{k}_t$) and decide how much to use it (u_t), can be seen in asset constraint of the households in which households earn $R_t^k \, u_t \, \bar{k}_t$ from supplying capital services, but there is also cost of adjusting the utilization rate $a(u_t)\bar{k}_t$. R_t^k is rate of return from capital and the cost function of adjusting the capital utilization rate $a(u_t)$ is convex in u_t.

The first order conditions in Christiano et al. (2001) with respect to capital and investment respectively are:

$$E_{t-1}\psi_t = \beta E_{t-1}\psi_{t+1} \left[\frac{u_{t+1} r_{t+1}^k - a(u_{t+1}) + P_{k',t+1}(1-\delta)}{P_{k',t}} \right] \tag{24}$$

$$E_{t-1}\psi_t = E_{t-1}\left[\psi_t \, P_{k',t} \, F_{1,t} + \beta \, \psi_{t+1} \, P_{k',t+1} F_{2,t+1}\right]. \tag{25}$$

In (24) and (25) ψ_t is defined $\psi_t = v_t P_t$ where P_t is the price of consumption good and v_t as above. ψ_t is thus a measure of change in one unit of household assets in consumption good units. ψ_t equals thus marginal utility of consumption. $P_{k',t}$ is the shadow value of unit of $\bar{k}_{t+1}$in consumption units. In Christiano et al. (2001), the final goods market clearing is done in a way that all production is either consumed, invested or used in capital utilization adjustment. The equation (24) states that the cost of giving up one unit of consumption, $E_{t-1}\psi_t$, must equal the expected return of adding it to the capital stock. In the same way, (25) states that cost of giving up one unit of consumption must equal the expected gains using it to investment. In the equation (25) $F_{j,t}$ equals the partial derivative of the installation function $F(i_t, i_{t-1})$ with respect to j^{th} argument. On the right hand side of (25) the term with $F_{1,t}$ is value of $\bar{k}_{t+1}$ in consumption goods after adding investment one unit. The term in (25) with $F_{2,t+1}$ catches the effect of change in i_t on installed capital in period $t+1$ in utility terms.

There is assumption about the functional form of $F(i_t, i_{t-1})$ is

$$F(i_t, i_{t-1}) = (1 - S\left(\frac{i_t}{i_{t-1}}\right))i_t,$$

and the adjustment cost function $S(\cdot)$ is assumed to have following properties in the steady state: $S(1) = S'(1) = 0$ and $S''(1) \equiv \kappa > 0$. Note that now argument of the adjustment cost function is ratio of investment in two successive periods. It is argued in Christiano et al. (2001) that function $S(\cdot)$ that uses the change in investment, $\frac{i_t}{i_{t-1}}$, rather than the level of investment, $\frac{i_t}{k_t}$, gives better empirical fit. To get empirical formulation of investment, (25) is log-linearized about steady state:

$$\hat{P}_{k',t} = \kappa E_{t-1}\left\{\hat{\imath}_t - \hat{\imath}_{t-1} - \beta\left[\hat{\imath}_{t+1} - \hat{\imath}_t\right]\right\}, \tag{26}$$

where circumflex above a variable denotes deviation from its steady state value. One can solve i_t from (26) to get the equation for investment.

2.2 The model of investment of this paper

In this paper, the work of Smets & Wouters (2003) is followed in setting up the model in which the behaviour of investment is studied. The Smets & Wouters (2003) model is very much like Christiano et al. (2001). It brings together the neoclassical profit maximizing firms, the utility maximizing households, the investment that is costly to adjust and the idea of Tobin Q theory. The derivation of the model and linearization of the mode are done in this section. The Smets & Wouters (2003) model is a comprehensive macroeconometric model for the euro area, but in this paper only the investment section is of interest, so we concentrate on that and leave other features aside.

Assume that there is continuum of households $z \in [0,1]$. The Households' instantaneous utility depends on level of consumption C and labour supply L. The households own the physical capital and supply capital services as in Christiano et al. (2001) to the firms. The households also supply differentiated labour services to the firms and act as wage setters in the monopolistically competitive markets. The factor market for the physical capital is competitive but subject to real frictions in the form of costly changes of capital utilization rate and costly changes in capital accumulation. The households decide each period how much to consume, accumulate capital and how much to supply capital services. Thus the market equilibrium in final goods market is that all production at time t is consumed, invested or spend on adjusting capital utilization rate. The household optimization problem is exposed to random shocks, ϵ, that follow first order autoregressive progress with i.i.d. zero mean normal error term. That is $\epsilon_t = \rho\, \epsilon_{t-1} + \eta_t$, where $\eta_t \backsim N(0,1)$ and $|\rho| < 1$. The shocks are associated with preferences, the supply of labour or capital adjustment costs.

In the following the discussion gets rather mathematical. The more detailed presentation of the model and derivation of the model is presented in the Appendix A. The utility function of a household z is familiar constant relative risk aversion form utility function:

$$U_{t,z}(C_{t,z}) = \frac{1}{1-\delta_c}\epsilon_t^C\,(C_{t,z})^{1-\delta_c} - \frac{1}{1+\delta_L}(L_{t,z})^{1+\delta_L}, \tag{27}$$

where $C_{t,z}$ and $L_{t,z}$ are the consumption and the labour supply of the household z at time t,respectively. The variable ϵ_t^C is preference shock of consumption, δ_c is the risk aversion term and δ_L is the inverse of the labour supply elasticity. Assume that households are homogenous and drop z. Thus the preferences of the household are presented as a maximization problem:

$$\max \sum_{j=0}^{\infty} \beta^j \epsilon_{t+j}^C \left[\frac{1}{1-\delta_c}(C_{t+j})^{1-\delta_c} - \frac{\epsilon_{t+j}^L}{1+\delta_L} {L_{t+j}}^{1+\delta_L} \right], \tag{28}$$

where the supply of labour is subject to supply shock ϵ^L and β^j is the discount factor.

The households maximize their utility subject to budget and capital accumulation constraints. The income of the household, INC_t, in nominal terms is

$$INC_t = W_t(1-\tau_t^A)L_t + R_t^K(1-\tau_t^K)u_t K_{t-1} + DIV_t + OTT_t,$$

where

$$\begin{array}{rcl} W_t & = & \text{nominal wage rate} \\ \tau_t^A & = & \text{tax rate on labour income} \\ \tau_t^K & = & \text{tax rate on capital income} \\ u_t & = & \text{utilization rate of capital} \\ R_t^K & = & \text{nominal rental rate on capital} \\ K_t & = & \text{physical capital stock} \\ P_t & = & \text{unit price of final good} \\ DIV_t & = & \text{dividends from firms} \\ OTT_t & = & \text{transfers from monetary authority.} \end{array}$$

The budget constraint contains wealth of households at time t. The household's budget constraint reads:

$$\begin{array}{c} \left[\frac{E_t B_{t+1}^F}{R_t^F(1-\Gamma)} - E_t B_t^F\right] + \\ + \left[\frac{B_{t+1}^H}{R_t^H} - B_t^H\right] \end{array} = \begin{array}{c} INC_t + \\ -P_t\left[(1+\tau_t^C)C_t + (1+\tau_t^I)I_t + \Psi(u_t)(1+\tau_t^I)K_{t-1}\right] \end{array}, \tag{29}$$

where

$$\begin{array}{rcl} E_t & = & \text{exchange rate} \\ B_t^F & = & \text{foreign bonds} \\ B_t^H & = & \text{domestic bonds} \\ R_t^F & = & \text{discount factor on foreign bonds} \\ R_t^H & = & \text{discount factor on domestic bonds} \\ \Gamma & = & \text{risk premium} \\ \tau_t^C & = & \text{consumption value added tax} \\ \tau_t^I & = & \text{tax on investment} \\ \Psi(u_t) & = & \text{cost function of varying capital utilization rate} \\ P_t^C & = & \text{price of consumption} \\ I_t & = & \text{investment in physical capital} \end{array}$$

The budget constraint (29) states that the households at time t spend income INC_t on consumption, investment and on the adjustment costs of capital utilization. Excess income is used to buy domestic or foreign bonds. Also financing consumption from bond market is allowed. There is a risk premium Γ associated to the foreign bonds.

The physical capital accumulation is the second constraint to the household maximization problem. The capital is accumulated according to

$$K_t = (1-\delta)K_{t-1} + \left[1 - \Phi\left(\epsilon_t^I, I_t, I_{t-1}\right)\right] I_t, \tag{30}$$

where

$$\begin{array}{rcl} \delta & = & \text{the depreciation rate} \\ \Phi(\cdot) & = & \text{investment adjustment cost function} \\ \epsilon_t^I & = & \text{shock to investment adjustment function.} \end{array}$$

Assume that the investment adjustment cost function $\Phi(\cdot)$ takes as arguments the investment I_t in two successive periods.

The Lagrangian of the optimization problem in real terms is

$$\begin{aligned} L &= \sum_{j=0}^{\infty} \beta^j \left\{ \epsilon^C_{t+j} \left[\frac{1}{1-\sigma_c} (C_{t+j})^{1-\sigma_c} - \frac{\epsilon_{L,t+j}}{1+\sigma_L} {L_{t+j}}^{1+\sigma_L} \right] \right. \\ & -\Lambda_{t+j} \left[\begin{array}{c} \frac{E_t B^F_{t+1}}{r^F_t (1-\Gamma(\cdot))} - \frac{E_t B^F_t}{P_t} + \frac{B^H_{t+1}}{r^H_t} - \frac{B^H_t}{P_t} + \\ -INC_t + (1+\tau^C_t) C_t + (1+\tau^I_t) I_t + \Psi(u_t)(1+\tau^I_t) K_{t-1} \end{array} \right] \\ & \left. -\Xi_{t+j} \left[K_t - (1-\delta) K_{t-1} - \left[1 - \Phi \left(\epsilon^I_t \frac{I_t}{I_{t-1}} \right) \right] I_t \right] \right\}. \end{aligned} \tag{31}$$

The first order conditions of the optimization problem for K_t, $I_t$and u_t are, respectively:

$$\Xi_t = \beta(\Lambda_{t+1}) \left[r^K_{t+1} (1-\tau^K_t) u_{t+1} - \Psi(u_{t+1})(1+\tau^I_{t+1}) \right] + \beta \Xi_{t+1} (1-\delta) \tag{32}$$

$$\begin{aligned} \Lambda_t (1+\tau^I_t) &= \Xi_t \left[1 - \Phi \left(\epsilon^I_t \frac{I_t}{I_{t-1}} \right) - \Phi'(\epsilon^I_t \frac{I_t}{I_{t-1}})(\epsilon^I_t \frac{I_t}{I_{t-1}}) \right] \\ & \quad + \beta \left(\Xi_{t+1} \right) \Phi' \left(\epsilon^I_{t+1} \frac{I_{t+1}}{I_t} \right) \left[\epsilon^I_{t+1} \left(\frac{I_{t+1}}{I_t} \right)^2 \right] \end{aligned} \tag{33}$$

$$r^K_t (1-\tau^K_t) = \Psi'(u_t)(1+\tau^I_t) \tag{34}$$

The first order condition of the capital utilization rate is the easiest to interpret. The function Ψ is increasing in u_t, so the capital utilization rate u_t is raised up to the level at which the change of costs rising the utilization rate one unit equals the real return on capital. The Lagrangian multipliers Λ_t and Ξ_t contain the most relevant information for optimization problem. Ξ_t is the Lagrangian multiplier that indicates the present value of additional unit of capital for the household. Λ_t can be interpreted as marginal cost of using one unit of final good on capital in stead of consumption. In other words, Λ_t is the cost of acquiring one unit of capital. Now one can define Tobin Q as the ratio of the two multipliers. This variable Q_t is by definition *marginal q* because in the nominator there is the change in market value caused by adding one unit of capital and in the denominator there the cost of additional unit of capital:

$$Q_t = \frac{\Xi_t}{\Lambda_t}. \tag{35}$$

Use definition (35), and the equation (32) to get

$$
\begin{aligned}
Q_t &= \frac{\beta(\Lambda_{t+1})}{\Lambda_t}\left[r^K_{t+1}(1-\tau^K_t)u_{t+1} - \Psi(u_{t+1})(1+\tau^I_{t+1})\right] + \beta\frac{\Lambda_{t+1}}{\Lambda_t}\frac{\Xi_{t+1}}{\Lambda_{t+1}}(1-\delta) \\
&= \frac{\Lambda_{t+1}}{\Lambda_t}\beta[Q_{t+1}(1-\delta) + r^K_{t+1}(1-\tau^K_t)u_{t+1} - \Psi(u_{t+1})(1+\tau^I_{t+1})].
\end{aligned}
$$

In the same way use the definition (35) to equation (33) to get

$$
(1+\tau^I_t) = \begin{array}{c} Q_t\left[1-\Phi\left(\epsilon^I_t\frac{I_t}{I_{t-1}}\right) - \Phi'(\epsilon^I_t\frac{I_t}{I_{t-1}})(\epsilon^I_t\frac{I_t}{I_{t-1}})\right]+ \\ +\beta\frac{\Lambda_{t+1}}{\Lambda_t}Q_{t+1}\Phi'\left(\epsilon^I_{t+1}\frac{I_{t+1}}{I_t}\right)\left[\epsilon^I_{t+1}\left(\frac{I_{t+1}}{I_t}\right)^2\right] \end{array}
$$

One can show that term that contains investment tax τ^I_t has no impact on result of the optimization it is just a scale factor. To get analysis more tractable, assume from now on that $\tau^K_t = \tau^I_t = 0$ and thus conditions of optimum for capital, investment and capital utilization are

$$
Q_t = \frac{\Lambda_{t+1}}{\Lambda_t}\beta[Q_{t+1}(1-\delta) + r^K_{t+1}u_{t+1} - \Psi(u_{t+1})] \tag{36}
$$

$$
\begin{array}{c} Q_t\left[1-\Phi\left(\epsilon^I_t\frac{I_t}{I_{t-1}}\right) - \Phi'(\epsilon^I_t\frac{I_t}{I_{t-1}})(\epsilon^I_t\frac{I_t}{I_{t-1}})\right]+ \\ +\beta\frac{\Lambda_{t+1}}{\Lambda_t}Q_{t+1}\Phi'\left(\epsilon^I_{t+1}\frac{I_{t+1}}{I_t}\right)\left[\epsilon^I_{t+1}\left(\frac{I_{t+1}}{I_t}\right)^2\right] \end{array} = 1 \tag{37}
$$

$$
r^K_t = \Psi'(u_t).
$$

The steady state of the model describes the equilibrium of the model. The steady state is derived using the household optimum conditions of capital, investment and capital utilization above. The bar above the variable denotes the steady state value of the variable.

$$
\begin{aligned}
\bar{Q} &= 1. \\
r^K &= \frac{1-\beta(1-\delta)}{\beta} \\
\bar{I} &= \delta
\end{aligned}
$$

In the steady state, the variable $\bar{Q}$ equals one, and as defined in the Tobin Q theory, there is no incentive to invest more or less. As it is assumed there are no adjustment costs in the steady state, because the investment is at constant level. The capital K_t is unchanged in the steady state, because only the replacement investment is done, that is $\bar{I} = \delta$.

The log-linearized version of the model is

$$i_t = \frac{1}{1+\beta} i_{t-1} + \frac{\beta}{1+\beta} E_t[i_{t+1}] + \frac{1}{1+\beta} \frac{1}{\kappa} q_t + \frac{\beta E_t[\epsilon_{t+1}] - \epsilon_t}{1+\beta}. \tag{38}$$

The equation (38) is the investment function of our model. At time t investment is a function of lagged investment i_{t-1} and expected investment i_{t+1}. The investment depends also on the variable q_t that contains information about the installed capital and its valuation. The investment adjustment costs have an impact on the investment through the second derivative of the adjustment cost function Φ. The investment function (38) also contains the random shock term, originally modelled to cause randomness in the investment adjustment cost function. Since the investment i_t in (38) depends on the second derivative of the adjustment cost function, κ, it is clear that the adjustment cost function Φ must be a function of minimum second order. The model is forward looking because it contains the lead of the investment variable i_{t+1} and the variable q_t contains market valuation that contains all the information available at time t about the future.

In the next section the investment equation (38) needs to be manipulated into a form that we are able to estimate. Before that we describe the data and treatment of the time series.

3 DATA, ESTIMATION METHOD AND RESULTS

In this chapter the data used in estimation is described and the manipulation of the theoretical model into the form used in estimation is presented. In addition, a brief introduction to Bayesian inference is presented and the estimation of the model is done. The results are discussed and compared to selected studies in the field. To see how the estimated model performs, the model is simulated. At the end of the chapter tests of robustness for variables β and κ is done and the results of those tests are presented and discussed.

3.1 Data

There are two variables in the theoretical model, investment and Tobin Q with lead and lagged values. The base of the investment series is the time series of private investment in the quarterly national accounts by Statistics Finland. The series is seasonally adjusted using TRAMO/SEATS[3] method by Statistics Finland. The investment series is in real terms and available from the first quarter of 1990 to the second quarter of 2009 in the Statistics Finland web service, StatFin. The investment series was read in the online database on 20th Sep 2009.

To get the proxy for variable Tobin Q_t, empirical work by Takala (1995) is followed. Assume that the capital in real terms that works in the companies at time t is K_t. Then we need to assume that the equity market is functioning well and the prices in the stock market reflect "the right market value" of the capital K. Assume that the stock market price index is s_t. At the time t the market valuation of the capital K_t in nominal terms, is product of K_t and s_t. The replacement cost of the physical capital in nominal terms is $p^K \times K$, where p^K is the price index of physical capital K. The proxy for Tobin Q is the ratio of the two:

$$Tobin\ Q_t = \frac{s_t \times K_t}{p_t^K \times K_t} = \frac{s_t}{p_t^K}.$$

The deflator of capital, p_t^K, is calculated as a ratio of capital in nominal terms to capital in real terms at time t. The capital series used in this paper are the total of fixed assets series (in real and nominal terms) of the Finnish economy from the tables of the gross stock of fixed capital. The capital account time series provided by Statistics Finland are in annual frequency from year 1975 to 2008 and the capital

[3]The abbreviation TRAMO stands for "Time Series Regression with ARIMA Noise, Missing Observations and Outliers" and the abbreviation SEATS stands for "Signal Extraction in ARIMA Time Series".

series was read in StatFin on 20th Sep 2009. The quarterly time series is calculated linearly interpolating the annual series. The interpolation is presented in more detail in the Appendix B.

The choice of the stock price index or more generally speaking the proxy of the market valuation price index of the capital K has a major influence on the model estimation and model performance. For example Barro (1990) shows how stock price index dominates the Tobin Q proxy. As the stock price index s_t we use the OMX Helsinki Cap index. It is a weight limited index so the large companies or one single large company do not dominate the price index. To use the OMX Helsinki Cap index we must assume the index proxies more generally the market value of the installed capital in the Finnish companies. One source of problem using the Finnish data is that the index available do not necessarily cover the whole private sector of the Finnish economy. The private investment series includes all the investment done in Finland in the private sector, but only a small fraction of the Finnish companies are included in the stock index. Also in Helsinki Stock Exchange the mobile phone manufacturer Nokia is a giant compared to other traded companies, but weight limit should alleviate this problem. From the original daily frequency data three-month averages are calculated to get the quarterly series. The OMX Helsinki Cap index is available from 1991Q1 to present. The source of the stock index is OMX Nordic Exchange and DataStream. The investment and the Tobin Q series used in estimations are presented in Appendix C

The statistical inference is done around the steady state of the model. The steady state commonly in the literature and in our case is the quadratic trend of the data sample. Thus for example the investment series used in estimation is the logarithm of the seasonally adjusted investment series less the quadratic trend series. The Figure (4) plots the log-deviation series of the investment and Tobin Q proxy. If full data sample available $(1991Q1 - 2008Q4)$ is used, the fitted quadratic investment series is convex but the fitted quadratic Tobin Q series is concave. The reason for this is that in the beginning of the series investment is at high level and for the first four year investment decline. The values at the beginning of the series are high enough to make the quadratic trend convex. To make both quadratic series concave the subsample starting from the first quarter of 1995 is used and thus number of observations is 56. Another argument for using the subsample is that the Finnish economy suffered from a very severe recession in the early 1990s. The changes in the economy were dramatic during the recession and the economical recovery started in 1994 (Pekkarinen & Sutela 2002). The Figure (2) plots the logarithm of the investment series with the quadratic trend and the Figure (3) plots the logarithm of the Tobin Q proxy with the quadratic trend.

Both the log-deviation series are tested for stationarity using augmented Dickey-Fuller test and Philips-Perron tests but results find no proof of stationarity. Harrison,

Figure 2: The logarithm of the investment series and the fitted quadratic trend

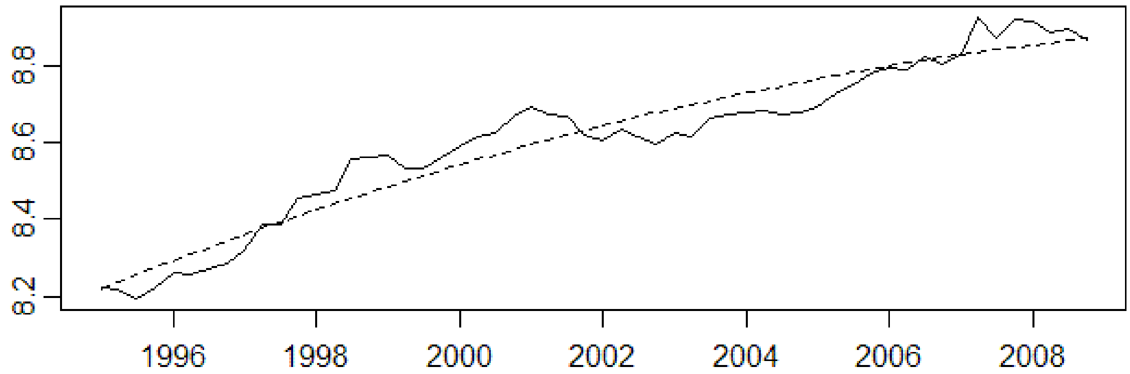
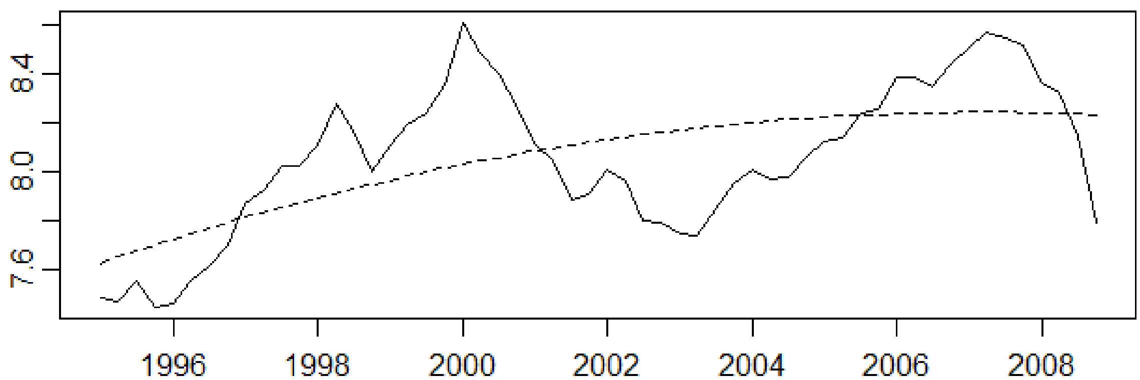
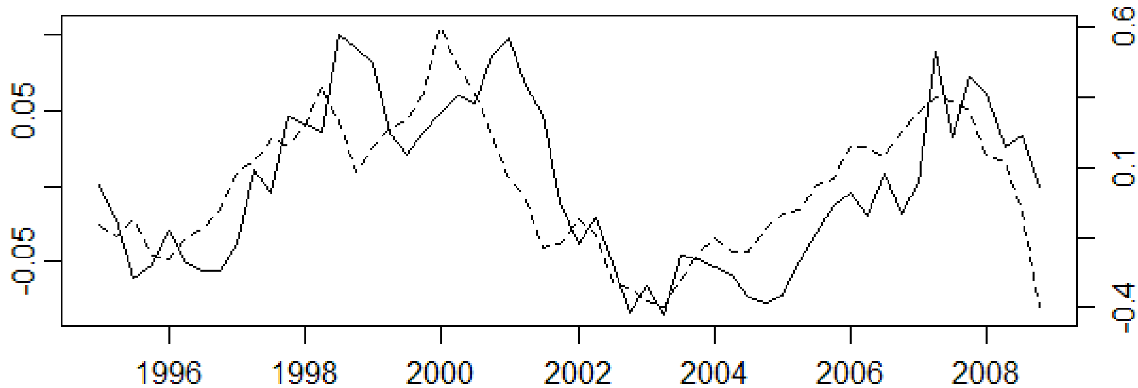


Figure 3: The logarithm of Tobin Q proxy series and the fitted quadratic trend



R.,Nikolov K., Quinn, M., Ramsay G., Scott, A. and Thomas, R. (2005) report the same problem concerning some of the variables of the Bank of England Quarterly Model. Harrison et al. (2005) suggest that reason for that the tests fail to reject the null hypothesis of the non-stationarity is the length of the data. Harrison et al. uses the sample of 100 - 105 observations depending on the test used. The sample in this paper is 56 observations which is a very small sample. An eye ball test suggests the stationarity of the series, see the Figure (4). The stationarity can also be argued because we examine the series around the steady state. The steady state is a fitted quadratic trend to the sample and thus the true values of the series vary around the steady state. Some of the true observations are above the trend and some are below the trend. The fitted quadratic trend should catch the non-stationary processes and thus the series of the deviations from the trend should be stationary.

Figure 4: The log-deviation series of investment (left, solid), and Tobin Q proxy



3.2 The manipulation of the estimation equation

Investment equation derived in previous section is

$$i_t = \frac{1}{1+\beta} i_{t-1} + \frac{\beta}{1+\beta} E_t[i_{t+1}] + \frac{\varphi}{1+\beta} q_t + \frac{\beta}{1+\beta}(E_t[\epsilon_{t+1}] - \epsilon_t), \tag{39}$$

where $\varphi = \frac{1}{\kappa}$.We need to manipulate the equation (39) because it contains variables at time $t+1$ that are unobservable to econometrician at time t. The detailed manipulation of the (39) is presented in the Appendix D. To begin the manipulation remember that the investment adjustment shock term ϵ_t in (39) is assumed to follow a stationary $AR(1)$ process $\epsilon_t = \rho\,\epsilon_{t-1} + \eta_t$, where $\eta_t \sim N(0,1)$. Thus the conditional expectation $E_t[\epsilon_{t+1}] = \rho\,\epsilon_t$. Using these properties one can solve (39) for ϵ_t and plug the error term back to investment equation. After some algebra one gets a simplified form

$$i_t = \gamma_1\, i_{t-1} - \gamma_2 i_{t-2} + \gamma_3 E_t[i_{t+1}] + \gamma_4\; q_t - \gamma_5\; q_{t-1} + \gamma_6 \eta_t, \tag{40}$$

where

$$\begin{aligned} \gamma_1 &= \tfrac{1+\rho+\rho\beta}{1+\beta+\beta\rho} & \gamma_4 &= \tfrac{\varphi}{1+\beta+\beta\rho} \\ \gamma_2 &= \tfrac{\rho}{1+\beta+\beta\rho} & \gamma_5 &= \tfrac{\rho\,\varphi}{1+\beta+\beta\rho} \\ \gamma_3 &= \tfrac{\beta}{1+\beta+\beta\rho} & \gamma_6 &= \tfrac{\beta\,(\rho-1)}{1+\beta+\beta\rho} \end{aligned} \tag{41}$$

One more manipulation is needed to do the estimation as in Lindé (2005). Consider the term $\gamma_3 E_t[i_{t+1}]$ with expectation in equation (40). Under the rational expectations

$E_t[i_{t+1}]$ equals $i_{t+1} + \xi_{t+1}$, where ξ_{t+1} is an independently and identically distributed (i.i.d.) zero mean error with finite variance σ^2_{ξ}. Thus one can write

$$\gamma_3 E_t[i_{t+1}] = \gamma_3 i_{t+1} + \gamma_3 \xi_{t+1}. \tag{42}$$

Replace then the expectation term in (40) with expression in line (42), solve the equation (40) for i_{t+1} and lag one period to get

$$i_t = \frac{1}{\gamma_3} i_{t-1} - \frac{\gamma_1}{\gamma_3}\, i_{t-2} + \frac{\gamma_2}{\gamma_3} i_{t-3} - \frac{\gamma_4}{\gamma_3}\; q_{t-1} + \frac{\gamma_5}{\gamma_3}\; q_{t-2} + v_t,$$

where $v_t = -\frac{\gamma_6}{\gamma_3}\eta_{t-1} - \xi_t$ is an i.i.d. zero mean error term with finite variance σ^2_v. This error term v_t contains now both the random shocks to investment adjustment cost function and the rational expectations error. Using the mapping (41) for γ_1, ..., γ_5 equation to be estimated can be written as

$$i_t = \frac{1+\beta+\beta\rho}{\beta}\, i_{t-1} - \frac{1+\rho+\rho\beta}{\beta}\, i_{t-2} + \frac{\rho}{\beta}\, i_{t-3} - \frac{\varphi}{\beta}\, q_{t-1} + \frac{\rho\,\varphi}{\beta}\, q_{t-2} - v_t, \tag{43}$$

Henceforth, the following mapping is used:

$$\begin{aligned} \theta_1 &= \tfrac{1+\beta+\beta\rho}{\beta} & \theta_4 &= \tfrac{\varphi}{\beta} \\ \theta_2 &= \tfrac{1+\rho+\rho\beta}{\beta} & \theta_5 &= \tfrac{\rho\,\varphi}{\beta} \\ \theta_3 &= \tfrac{\rho}{\beta} \end{aligned} \tag{44}$$

and thus (43) is written

$$i_t = \theta_1\, i_{t-1} - \theta_2\, i_{t-2} + \theta_3\, i_{t-3} - \theta_4\, q_{t-1} + \theta_5\, q_{t-2} - v_t. \tag{45}$$

3.3 Bayesian statistics

3.3.1 General discussion about Bayesian statistics

The Bayesian statistics draws conclusions about the parameter θ (which can also be a vector of parameters) in terms of probability statements. The idea in Bayesian inference is to combine the known information ("the data") and some prior believes about θ. The data and the prior believes both have some probability distributions and together they produce the posterior distribution of the parameter of interest.

The following general discussion follows the presentation of the Bayesian basics as in Koop (2003). It is assumed that the researcher knows precisely the distribution of the data or proposes a distribution for the data conditional on the parameter θ. The distribution of the data $\mathbf{y}$ could be for example normal and thus one could write $\mathbf{y} \mid \mu, \sigma^2 \sim N(\mu, \sigma^2)$ and so the parameter vector $\theta = (\mu, \sigma^2)$. One could also write the model as a probabilistic distribution $p(\mathbf{y} \mid \theta)$ which is known in Bayesian statistics as *the likelihood of the data*. The researcher has also some information or some belief about the parameter θ before collecting the data. This information could arise for instance from previous research done in the field. With the prior information the researcher also can restrict the estimation results. For example in economics there are parameters or variables that are restricted to certain interval and the prior can be used to restrict the parameter values to economically plausible values. Important is that the prior information is not included in the data. The prior information is taken into the inference in the form of probabilistic distribution $p(\theta)$ that is called *the prior distribution*.

Together the likelihood and the prior distribution form a joint probability distribution $p(y, \theta) = p(y \mid \theta)\, p(\theta)$. Since the researcher is interested in parameter θ, the joint distribution $p(y, \theta)$ that contains the parameter of interest and the data needs to be conditioned on the known data to reveal the conditional distribution of the parameter θ. To get this conditional distribution of θ, we use the Bayes rule. The derivation of the rule is based on the theory of the joint probability distributions. Notice that the joint probability distribution $p(y, \theta)$ can be written also as $p(\theta, y)$ and the both distributions are essentially the same. By definition the following two equations hold: $p(y, \theta) = p(y \mid \theta)\, p(\theta)$ and $p(\theta, y) = p(\theta \mid y)\, p(y)$. Rearrange the equations to get

$$p(\theta \mid y)\, p(y) = p(y, \theta) = p(y \mid \theta)\, p(\theta)$$

From this form, one can solve the probability distribution of the θ conditional on y. The following is known as the Bayes rule.

$$p(\theta \mid y) = \frac{p(\theta, y)}{p(y)} = \frac{p(\theta)\, p(y \mid \theta)}{p(y)} = \frac{p(\theta)\, p(y \mid \theta)}{\int p(\theta)\, p(y \mid \theta)\, d\theta}, \tag{46}$$

where $p(y)$ is *the marginal likelihood of* y. In case of continuous distributions $p(y)$ is an integral over all possible values of θ. The distribution $p(\theta|y)$ is *the posterior distribution* of θ.

3.3.2 An example of Bayesian statistics

As a text book example we will examine a simple one parameter model with a normal prior and normally distributed data. The presentation follows closely an example presented in the lecture notes by Penttinen (2009). Assume observations $\mathbf{y} = (y_1, \ldots, y_n)$ that are normally distributed with known variance v, thus $y_i|\mu \sim$

$N(\mu, v)$ and observations are independent conditional on μ. The variance v could be for example the known measurement error. The prior information about the unknown parameter μ, the expectation of the observations $\mathbf{y}$, is normally distributed, $\mu \sim N(m, w)$, where (hyper-)parameters m and w are known. The prior distribution of μ is

$$p(\mu) = \frac{1}{\sqrt{2\pi w}} \exp\left\{-\frac{1}{2\,w}(\mu - m)^2\right\} \propto \exp\left\{-\frac{1}{2\,w}(\mu - m)^2\right\},$$

where $\propto$ means that the left hand side is proportional to the right hand side. The term $\frac{1}{\sqrt{2\pi w}}$ is a scale factor and has no impact on the conclusions based on the posterior distribution of μ conditional of y. The likelihood based on the observations is

$$p(\mathbf{y} \,|\, \mu) = \prod_{i=1}^{n} p(y_i|\mu) \propto \exp\left\{-\frac{1}{2\,v}\sum (y_i - \mu)^2\right\}.$$

With this prior distribution and this likelihood one can show that the posterior distribution

$$p(\mu \,|\, \mathbf{y}) \propto \exp\left\{-\frac{1}{2\,w}(\mu - m)^2\right\} \times \exp\left\{-\frac{1}{2\,v}\sum_{i=1}^{n}(y_i - \mu)^2\right\} = \exp(-\frac{1}{2}Q),$$

where $Q = \left(\frac{1}{w} + \frac{n}{v}\right)(\mu - m_1)^2 + C$, $m_1 = \left(\frac{m}{w} + \frac{n\overline{y}}{v}\right) / \left(\frac{1}{w} + \frac{n}{v}\right)$ and C is a constant. The posterior distribution thus is a normal distribution, $\mu \,|\, \mathbf{y} \sim N(m_1,\, v_1)$ that is

$$\begin{aligned} p(\mu \,|\, \mathbf{y}) &\propto \exp\left\{-\frac{1}{2v_1}(\mu - m_1)^2\right\}, \\ m_1 &= \left(\frac{1}{w} + \frac{n}{v}\right)^{-1}\left(\frac{m}{w} + \frac{n\overline{y}}{v}\right), \\ v_1 &= \left(\frac{1}{w} + \frac{n}{v}\right)^{-1}. \end{aligned}$$

In conclusion, the prior parameters are m and v but the data $\mathbf{y}$ *updates* the parameters to m_1 and v_1. In a numerical example one can use the classic data[4] collected by Henry Cavendish who tried to determine the density of the earth. Assume *a priori* that the mean of the density is 5 and its variance is 0.5, thus $p(\mu) \sim N(5, 0.5)$. In addition assume the measurement error or variance v is 0.04. The descriptive statistics of the data show that the sample mean is 5.485 and there are 23 observations. Using the equations for m_1 and v_1 above, one can calculate the posterior parameter values 5.483 and 0.00173, respectively.

This was a nice and simple example how the data brings information to the prior belief. One of the criticism towards Bayesian inference is that it relies on the subjective

[4]The data presented in Penttinen (2009) is used here. The original study is most likely to be Cavendish (1789).

believes about prior information. An uninformative prior distribution can always be chosen but then one might ask why to use prior information and Bayesian inference in the first place if the prior information contains no information. In the numerical example above the prior of μ is quite uninformative with rather large variance v but the posterior is tightly around the posterior mean m_1 and the posterior variance v_1 is small. In this example the uninformative prior does its job as a prior distribution but it is easy to notice that the posterior mean does not fall far from the sample mean of the data. In the empirical part of this paper we will see how informative priors work in Bayesian inference.

If the choice of the prior distributions is somewhat arbitrary the same can be said about the choice of the likelihood model. There are no written rules about choosing the model and the researcher is free to use his own expertise. The vast choice of models carries also one strength of the Bayesian methods. The different models can be effortlessly tested and the most suitable model can be found. With "right" prior distributions the goodness of the results can be increased. Intuitively, Bayesian inference is tempting: The researcher can combine the results of the previous studies and his own view about the phenomenon under study.

3.3.3 Posterior simulation

The example above was based on choosing a conjugate prior for the data and thus the calculation of the posterior was easy. Sometimes the integral in the denominator of the posterior (46) is difficult and / or time consuming to calculate algebraically. In some cases the integral might not have a closed form solution at all. In these cases the posterior is simulated. There are few different methods of posterior simulation such as Metropolis algorithm, Metropolis-Hastings algorithm, Gibbs sampling or slice sampling. In this paper the simulation is done using Metropolis algorithm with symmetric and (in our case also) normal proposal distribution. The idea is following. Assume that i values of the posterior is already simulated. Thus the last value in the chain of simulated values is θ_i and the proposal for the next value $\theta\prime$ is drawn from the candidate generating density, which is not the same as posterior density. The density of the candidate draw $\theta\prime$ depends on the previous value θ_i. Then calculate acceptance probability $\alpha = \min\left\{1, \frac{p(\theta\prime|\mathbf{y})}{p(\theta^i|\mathbf{y})}\right\}$ using the posterior $p(\theta|\mathbf{y})$. The candidate $\theta\prime$ is always accepted, if $\frac{p(\theta\prime|\mathbf{y})}{p(\theta^i|\mathbf{y})} \geq 1$. If $\alpha = \frac{p(\theta\prime|\mathbf{y})}{p(\theta^i|\mathbf{y})} < 1$ the candidate $\theta\prime$ is chosen with probability α. In other words if $\alpha < 1$ accepting the $\theta\prime$ is like flipping a coin with the probability of α of the plausible result. In case the $\theta\prime$ is accepted set $\theta^{i+1} = \theta\prime$. Otherwise set $\theta^{i+1} = \theta^i$ and draw new candidate $\theta\prime\prime$.

The use of the proposal distribution and ratio of the successive values of the simulated posterior chain cancels the difficult integral. The simulated chain is called Markov Chain Monte Carlo, or just MCMC. The simulated chain is expected to stabilize after so called burn-in time. The convergence of the simulation is examined as proposed by Geweke (1992). The test is simple test of comparing the means of two different parts of the simulated chain. The widely used test in literature is the one, where after burn-in period of the simulated chain the mean of the first 10% of the chain is compared to the last 50% of the chain. The null hypothesis is that the difference of the means is zero.

The DSGE models and the investment equation in the case of this paper both contain a lot structure because the basis of the modern DSGE models is in the microeconomy. There are both firms and households that maximize their utility and thus number of structural parameters in the gets easily high. Luckily, the methods of Bayesian statistics allows the econometrician estimate the model as the theory suggests. The prior distributions can be chosen in a way that the posterior densities of the structural parameters of the model fall into plausible area of the real axis.

3.4 Estimation

3.4.1 The base case

The model (43) is estimated using the Bayesian method. The likelihood of the linear model (45) is formed on the assumption that the error term v_t is a zero mean i.i.d. variable with some finite variance σ^2 (the subscript v is dropped since henceforth we refer to the standard error of the error term v only with a Greek σ). The coefficients $\theta_1 ... \theta_5$ are linked to the structural parameters β, ρ and κ using mapping (44) and we need the prior distributions only for the structural parameters. In the literature the priors used by Smets & Wouters (2003) have become popular so those priors are a good starting point. In general, the prior distributions should not contain the same information than the data set contains. That means, we cannot use for example sample means and sample variances as prior distribution parameters. The prior distributions in Smets & Wouters (2003) are based on empirical findings using the U.S data, thus in our case they serve well as prior distributions.

The prior distribution of σ is an inverted gamma distribution which is plausible distribution for standard deviation since the gamma distribution has only positive values. Thus the inverted gamma has also only positive values but the emphasis is in the values less than 1. *A priori* we assume $\sigma \sim Igamma(2.0025, 1/0.10025)$ and thus $E(\sigma) = 0.1$ and $st.error(\sigma) = 2$. The mean is small but the standard error of two gives the distribution wide range of possible values. Above we have assumed that ρ is a coefficient in stabile autoregressive process of the investment shock. The $AR(1)$

process is stabile when $0 < \rho < 1$. The plausible prior distribution of ρ is the beta distribution which has values only between zero and one. Smets and Wouters (2003) use the beta distribution with mean 0.85 and standard error of 0.1 mainly to distinguish the $AR(1)$ modelled shocks from some other shocks. On one hand, in this exercise we do not have same kind of complete DSGE model around the investment equation and thus the prior distribution could be parametrized in some other way. On the other hand one could assume *a priori* that the $AR(1)$ process of investment shocks to be the same here as in more comprehensive models. The investment adjustment cost parameter κ is assumed *a piori* to follow normal distribution, $\kappa \sim N(4, 1.5^2)$ as in Smets and Wouters (2003). Finally as in Smets & Wouters (2003) in our base case the variable β is fixed at 0.99 to indicate 4% annual discount factor. The β is fixed because it has such a strong link to theory and previous research: the value of discount factor β simply can not be *whatever*. When a larger model is to be estimated the discount parameter β is fixed in order to decrease the number of parameters to estimate and thus to prevent possible problems during the model simulation. The assumption of fixed β is loosened later when we conduct some tests of robustness with different prior distributions of β.

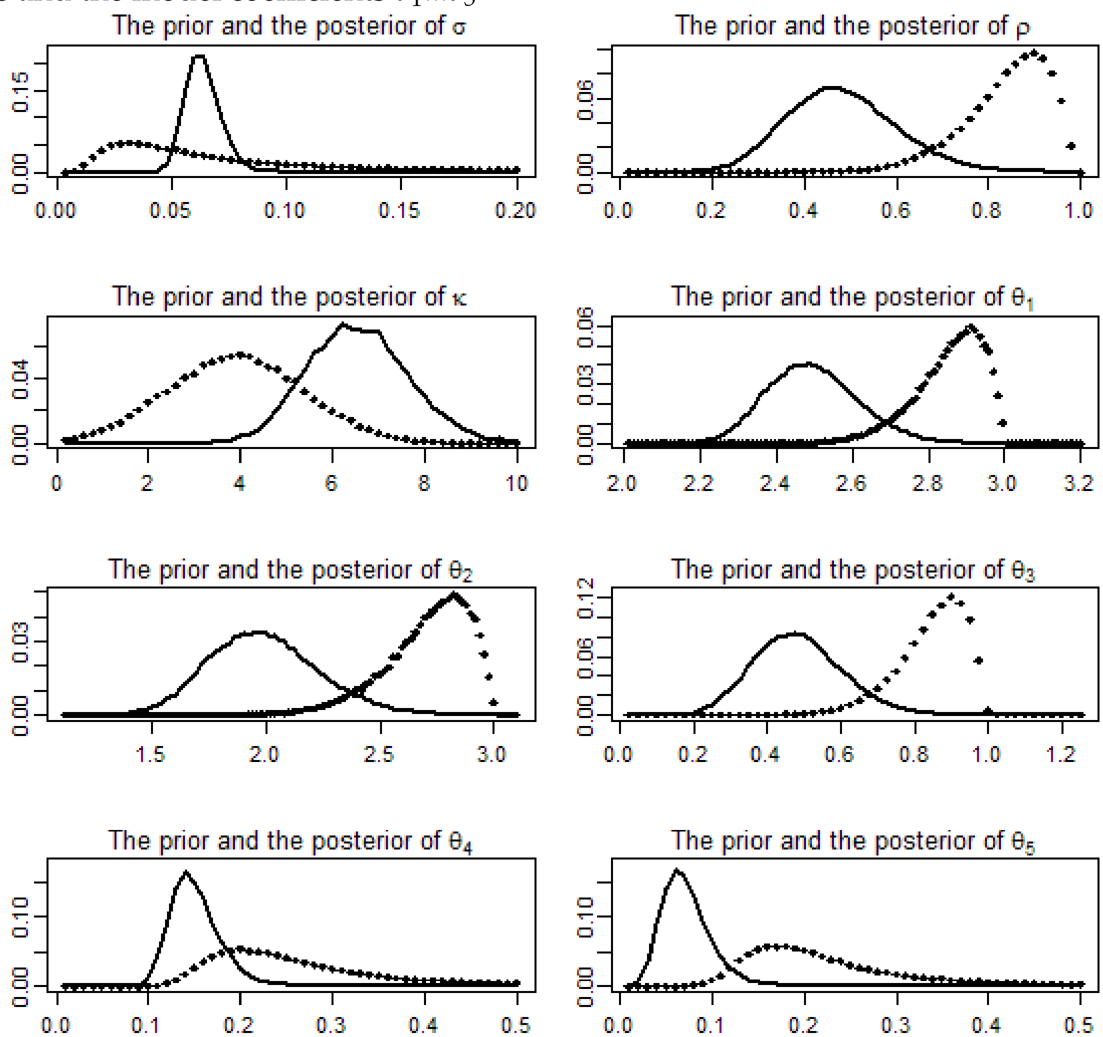
The Table 1 summarizes the base case of this paper. The mean and the standard error of the prior distributions are on the left panel of the Table 1. On the right panel of the Table 1 there are the posterior mean and standard error and 5%, 50% and 95% quantiles for the structural parameters σ, ρ and κ.

Table 1: The prior and posterior distributions.

	Prior distributions			Posterior				
		mean	st.error	mean	st.error	5%	50%	95%
σ	inv. gamma	0.1	2	0.066	0.0075	0.054	0.065	0.079
ρ	beta	0.85	0.10	0.492	0.118	0.309	0.485	0.698
κ	normal	4	1,5	6.653	1.107	4.92	6.617	8.536
θ_1				2.502	0.118	2.319	2.495	2.708
θ_2				1.999	0.237	1.631	1.986	2.497
θ_3				0.497	0.119	0.312	0.490	0.747
θ_4				0.156	0.0274	0.118	0.153	0.205
θ_5				0.077	0.0271	0.041	0.074	0.127

The iteration[5] is done in a way that the mapping (44) holds in every iteration. The Table 1 contains also the posterior statistics for parameters $\theta_1 ... \theta_5$. The prior distributions are plotted with the posterior distributions in the Figure (5) where the prior distributions are plotted with dotted line and the posterior distributions are plotted with solid line. The plots of the prior distributions in the Figure (5) are based on 80 000 draws from the prior distributions discussed above and presented in the Table 1.

Figure 5: The prior and posterior (solid line) distributions of the structural parametres and the model coefficients $\theta_1 ... \theta_5$



[5]Technical note:

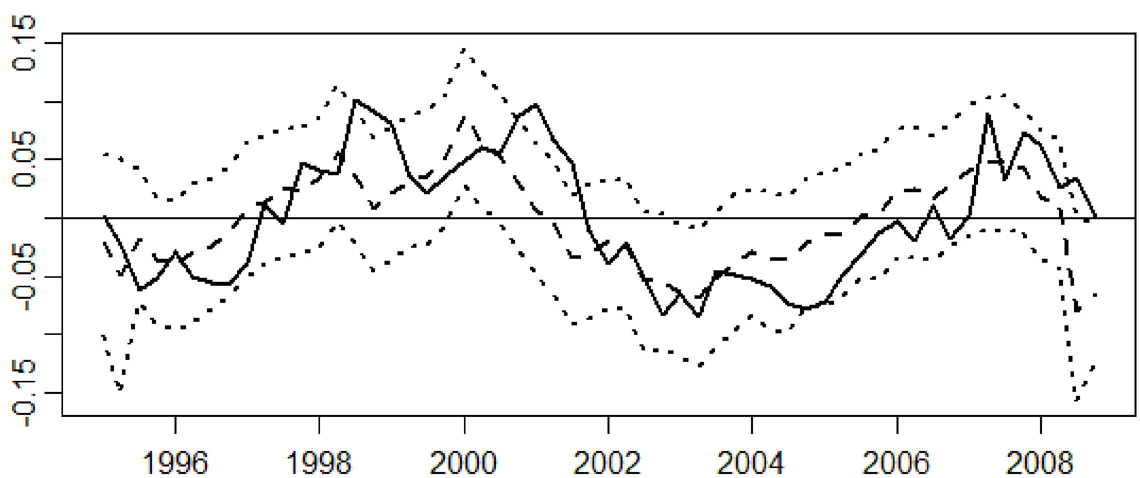
The estimation is done using WinBUGS software. The length of the simulated posterior chain is 100 000 iterations and the burn-in time is the first 20% of the chain. The posterior is simulated using Metropolis algorithm and the proposal disrtibution is symmetric normal distribution chosen by the program. More information about WinBugs, visit http://www.mrc-bsu.cam.ac.uk/bugs/welcome.shtml

The priors of the model coefficients $\theta_1 ... \theta_5$ are calculated draw by draw using the mapping (44). The posterior plots are based on the posterior chains that are stored during the estimation. The length of the posterior chains is 80 000 iterations. Looking at the plots of the priors and posteriors in the Figure (5) tells that although we have couple of very informative priors such as the prior of ρ or the priors of the model coefficients $\theta_1 ... \theta_3$ the data brings new information to our prior believes about the parameters. The new information is seen in the location of the posterior plot compared to the prior plot in the Figure (5).

3.4.2 The simulation of the posterior predictive belt and convergence

The Figure (6) shows the 90% posterior predictive belt of the model. The Figure (6) plots the actual investment series with the simulated investment series using the estimation results and 90% probability interval for the simulated series. The simulation is done using the structural parameters κ and ρ and the simulated standard deviation of the investment shock σ. The the procedure of the simulation is discussed more detail in the Appendix E.

Figure 6: The 90% posterior predictive belt of investment series 1995-2008.



The simulation itself and the posterior belt fits reasonably well to the data. There are few periods of time when the simulation does not fit well and the 90% band is violated. These periods are before and after year 2000 and at the end of the simulation, in year 2008. These the periods mentioned are exceptional times in the world economy. There was so called "dot-com" bubble in the stock markets that peaked on

10 March 2000[6]. The reader can easily identify the bubble also in the OMX Helsinki Cap index, see graph (3). The variable q_t is assumed exogenous in the simulation and it drives the simulated variables so a bubble in the stock market has an impact on the simulation. In addition, the model is log-linear which means that it works well, when the changes in the variables are small. In case of a bubble the model cannot do well. The same discussion applies to the simulation performance at the end of the simulation, in year 2008. The financial crisis that started in 2007 caused fast decline in market valuations of the share prices especially in year 2008. The falling Tobin Q variable results in rapidly falling predicted investment. The actual investment however do not react as fast as the predicted series suggests it should. One reason for this is that investment projects are long and once started they must be carried out as planned. If a particular investment project has started before it was clear that there will be a recession the managers might not be able to halt the project although it might be reasonable.

The convergence of the simulated posterior chains is tested using test proposed by Geweke (1992). The test compares means of first 10% of the series to the last 50% of the series. The burn-in time (the first 20 000 iterations) is excluded from the iterated chains. The Z-score reported in the Table 2 is the difference of the two means divided with the standard error of the sample. The results show that the iterated chains converge since none of the p-values is small enough and thus the null hypothesis of the equal means is not rejected. The results of the convergence tests are presented in the Table 2

Table 2: The Convergence

	Z-Score	p-value
θ_1	1.238	0.216
θ_2	1.237	0.216
θ_3	1.237	0.216
θ_4	-1.102	0.271
θ_5	-0.272	0.978
ρ	1.237	0.216
σ	0.166	0.868
κ	1.243	0.214

3.4.3 Discussion about the results

The parameter posteriors simulated here differ from those of Smets and Wouters (2003) which used the euro area data. The Smets and Wouters (2003) report following parameter medians: the standard error of investment shock $\sigma_{SW} = 0.105$, the

[6] http://en.wikipedia.org/wiki/Dot-com_bubble

median of investment shock persistence $\rho_{SW} = 0.913$ and the investment adjustment parameter $\kappa_{SW} = 6.920$. The 90% Bayes intervals for parameters σ_{SW}, ρ_{SW} and κ_{SW} are $(0.060, 0.196)$, $(0.864, 0.946)$ and $(5.148, 8.898)$ respectively. See the Table 3 for the selected statistics of Smets and Wouters (2003).

Table 3: Selected statistics in Smets and Wouters (2003)

	Prior distributions			Posterior			
		mean	st.error	st.error.	5%	50%	95%
σ_{SW}	inv gamma	0.2	2	0.030	0.060	0.105	0.196
ρ_{SW}	beta	0.85	0.10	0.022	0.864	0.913	0.946
κ_{SW}	normal	4	1.5	1.026	5.148	6.920	8.898

The estimated standard error of investment shock σ for Finland is lower than σ_{SW} but σ is included in the 90% Bayes interval of σ_{SW}. The smaller standard deviation of the investment shock indicates that variables in the model explain the behaviour of the depended variable i_t better when the Finnish data is used. From another point of view the smaller σ implies that the shocks to investment adjustment cost function are generally smaller in Finland than in the euro area. The adjustment cost parameter κ is not significantly greater or smaller than κ_{SW}, and the the 90% Bayes interval for κ_{SW} and κ are overlapping. The role of adjustment costs is roughly the same in Finland as in the euro area which would mean that the technology transferring the investment into new physical capital is roughly the same in the euro area and in Finland.

The greatest difference in results between Finnish and the euro area is in the persistence of investment shock. The estimated ρ is notably smaller than ρ_{SW}. The 90% Bayes intervals for ρ_{SW} and ρ do not overlap. The parameter ρ is coefficient in the $AR(1)$-process of investment adjustment cost shocks, so smaller ρ indicates that the impact of the shocks is weakened faster. In fact, the investment shocks in the euro area are persistent in the way that it takes 8 periods to half a shock which size is normalized to 1 in the first period. In contrast, the value of ρ implies that a shock size of 1 in the first period is alleviated to less than half as fast as in the following period. One reason for the difference is the data set. The aggregate euro area data gives different results than the single country data. The shocks remain in the euro area longer as they diffuse from initial country to other member states.

Adolfson et al. (2007) estimates a small open economy DSGE model using Swedish data. The model follows the work of Christiano et al. (2005) and Smets and Wouters (2003) so the foundations of the model are alike the foundations of the model in this paper. There are however some differences as well, that might make the comparison difficult. The prior distributions are close to priors chosen here, only the prior for κ differs by its expectation. Another difference concerning the priors of parameters is that Adolfson et al. (2007) fixes discount factor $\beta_A = 0.999$ which differs slightly from our fixed value of $\beta = 0.99$. The priors and posteriors of the parameters σ_A, ρ_A and κ_A in the posterior case of "Instrument policy rule without policy break" of Adolfson et al. (2007) are reported in the Table 4. We estimate our model with fixed $\beta = 0.999$ and the results are reported bottom of the Table 4. The difference in the model structure is that in our model in equation (30), the capital accumulation equation, the random shock is taken inside the adjustment cost function $\Phi()$ but the work of Adolfson et al. the random shock is left outside as a multiplier.[7]

Table 4: Comparison to Adolfson et al. (2007)

	Prior distributions			Posterior	
		mean	st.error	median	st.error.
σ_A	inv gamma	0.2	2	0.410	0.067
ρ_A	beta	0.85	0.10	0.691	0.081
κ_A	normal	7.694	1.5	8.593	1.306
$\sigma_{\beta=0.999}$	inv gamma	0.1	2	0.065	0.0075
$\rho_{\beta=0.999}$	beta	0.85	0.10	0.485	0.1181
$\kappa_{\beta=0.999}$	normal	4	1.5	6.606	1.11

The Table 4 shows that the standard error of investment shock σ_A is relatively high compared to the value estimated using Finnish data, σ, or to the value estimated using the euro area data σ_{SW}. One reason for this might be the structural differences of the models explained above. Adolfson et al. (2007) also report higher persistence in investment shocks than in Finland but the persistence is smaller than in the euro area. The size of the investment adjustment parameter κ_A is greater than estimated for the Finnish or the euro area data. The reason for this difference is the difference in the prior distributions. Adolfson et al. (2007) use a prior distribution with significantly higher expected value than we do. The estimated persistence parameter value ρ_A is roughly between the estimates of the euro area and Finland. Generally the results using Swedish data differs from the euro area estimates but they differs also from the estimates using the Finnish data. The estimated persistence parameter ρ is smaller than ρ_A and ρ_{SW}. Note that ρ_A is smaller than ρ_{SW} so the estimated investment shock persistence for Finnish data ρ is supported by findings using the Swedish data. The difference in the model structure might have impact on persistence parameter value as well.

In conclusion, one could say that the derived model works rather well in the estimation. Firstly, technically the estimation works fine since the posterior chains

[7]See Adolfson et al. (2007), page 7, equation (10).

converge. Second the model simulation gives a rather nice fit of the model to the data. Third the estimation results do not differ "too much" from the selected studies in the field, but they differ enough to say that there is something unique in the way private investment behaves in Finland. One could thus conclude that the model derived works pretty well and the way investment is modelled in papers like Smets and Wouters (2003) or Adolfson (2007) produces a good model for Finnish data.

3.4.4 The test of robustness, the estimation with the priors of β

It is interesting to see, whether the results change if β in the model (43) is not treated fixed. We test the model with 8 different prior distributions. All the distributions are beta distributions with different parameters chosen so that the expected value of the prior varies from the fixed value of 0.99 to lower values and the standard error of the prior is fixed at 0.28 if possible. The prior no. 2 is the prior distribution used for β as in Canova (2007, 445) with mean of $\frac{99}{101}$ and standard deviation of 0.0137. Other priors in cases 1-4 are chosen such that the expected value of β varies from 0.99 to 0.95 and the standard error is as large as possible. In cases 4-8 the prior mean varies from 0.9 to 0.5 and the prior standard deviation is around 0.287. The results of the estimations are reported in the Table 5. For each estimation case and structural parameter posterior median with the 90% Bayes interval in parenthesis and posterior standard error is reported.

Table 5: The test of robustness of β

	Prior statistics			
Prior #	1	2	3	4
mean (β)	0.99	0.98	0.98	0.95
st.error (β)	0.099	0.014	0.029	0.199
	Posterior statistics			
median (β)	0.987 (0.977 ,1)	0.987 (0.964 , 0.997)	0.997 (0.967 , 1)	0.978 (0.910 , 0.998)
st.error (β)	0.008	0.011	0.012	0.030
median (κ)	6.04 (4.57, 7.68)	6.05 (4.55 , 7.71)	5.36 (3.82 , 7.07)	6.07 (4.57 , 7.67)
st.error (κ)	0.942	0.954	0.992	0.942
median (ρ)	0.51 (0.33,0.71)	0.51 (0.33,0.72)	0.50 (0.32,0.72)	0.51 (0.33,0.72)
st.error (ρ)	0.117	0.119	0.120	0.119
median (σ)	0.066 (0.055 , 0.079)	0.066 (0.056 , 0.081)	0.083 (0.07 , 0.10)	0.067 (0.056 , 0.082)
st.error (σ)	0.0075	0.0076	0.0094	0.0078

The 90% Bayes interval in parenthesis

Table 5 (Continued)				
	Prior statistics			
Prior #	5	6	7	8
mean (β)	0.9	0.8	0.75	0.5
st.error (β)	0.286	0.286	0.287	0.288
	Posterior statistics			
median (β)	0.978 (0.91 , 0.998)	0.999 (0.968 , 1)	1 (0.997 , 1)	0.978 (0.910 , 0.998)
st.error (β)	0.029	0.014	0.012	0.029
median (κ)	6.06 (4.56 , 7.66)	6.05 (4.57 , 7.69)	6.03 (4.59 , 7.66)	6.07 (4.61 , 7.69)
st.error (κ)	0.941	0.949	0.939	0.935
median (ρ)	0.51 (0.33,0.72)	0.51 (0.33,0.71)	0.51 (0.33,0.72)	0.51 (0.33,0.72)
st.error (ρ)	0.119	0.117	0.118	0.118
median (σ)	0.067 (0.056 , 0.72)	0.066 (0.055 , 0.08)	0.066 (0.055 , 0.08)	0.067 (0.056 , 0.081)
st.error (σ)	0.0078	0.0075	0.0077	0.0078
The 90% Bayes interval in parenthesis				

The most significant result is that the posterior median of β is not greatly dependent on the prior distribution. The posterior medians of β change only little from prior to prior and most importantly the value of β remains close to 1. Even in the case number 8, which is a uniform prior distribution the posterior median of β is 0.978. The changes in other structural variables ρ, σ, and κ are very small. Only in case number 3 the median of κ is smaller and the median of σ is higher than in other cases. The value of ρ is not affected by the change of the β prior.

3.4.5 The test of robustness, the estimation with the priors of κ

The literature offers only limited discussion about parameter κ that is the second derivative of the investment adjustment cost function. As in Christiano et al. (2001) the inverse of the parameter κ can be interpreted as the elasticity of investment with respect to one percent change in the current price of installed capital. Christiano et al. (2001) reports that permanent one unit change in the value of installed capital causes 0.38 unit change in investment. Christiano et al. (2001) uses the US data and their estimate for variable $\kappa_{CEE} = 3.60$. Christiano et al. (2001) uses the following formula to calculate the change: $\frac{1}{\kappa(1-\beta)}$. Smets and Wouters (2003) report higher estimate for κ_{SW} than in reported in Christiano et al. (2001) and thus Smets & Wouters (2003) have smaller, about 0.20 unit response in investment. Our estimate of κ is close to the estimate of Smets and Wouters (2003) and thus the change in investment with respect to permanent one unit change in the value of installed capital is somewhat the same in Finland as reported for the euro area.

Another aspect to the parameter κ is offered in Harrison et al. (2005). As in Harrison et al. (2005), the parameter κ can be used to calculate an estimate of investment costs in the steady state when investment is increased 1%. The formula is

$$\frac{\kappa}{2}\left[\frac{\triangle I}{I}\right]^2 = \frac{\kappa}{2}\left[\frac{0.01I}{I}\right]^2 = \frac{\kappa}{2}0.01^2,$$

where κ is our estimate. With the value $\kappa = 6.617$ the 1% increase in investment from the steady state means 0.033% increase in the adjustment costs of the investment. Again because Smets and Wouters (2003) value of κ is close to ours, the conclusion is close to one based on our estimate. Using Adolfson et al. results, the increase in the adjustment costs due to 1% increase in the investment is 0.043%.

We continue to explore the properties of κ by conducting a test of robustness of the results by changing the prior distribution of the parameter κ and while holding other priors and parameters as they are in our base case. Smets and Wouters (2003) chooses the prior $N(4, 1.5)$ that is close to empirical findings in Christiano et al. (2001) and Adolfson et al. (2007) follows the empirical findings in Altig, Christiano, Eichenbaum and Lindé (2002) and chooses the prior $N(7.694, 1.5)$. Both the empirical estimations mentioned uses the US data. In the Table 6 the case and prior choice number 1 is the base case of our paper and the case number 2 uses the same prior distribution as in Adolfson et al. (2007). The test continues in the cases $3-8$ by increasing the standard error of the prior distribution and keeping the expectation of the prior distribution constant thus we move from informative prior to less and less informative prior. The prior parameters are presented at the top of the panel in the Table 6.

Table 6: The test of robustness of κ

	Prior statistics			
Prior #	1	2	3	4
$mean\ (\kappa)$	4	7.694	4	4
$st.error\ (\kappa)$	1.5	1.5	2	3
	Posterior statistics			
$median\ (\kappa)$	6.617 (4.92, 8.53)	9.019 (6.95 , 11.20)	7.561 (5.43 , 10.03)	9.256 (6.194 ,12.78)
$mean(\kappa)$	6.653	9.037	7.624	9.255
$st.error\ (\kappa)$	1.107	1.288	1.404	2.426
$median\ (\rho)$	0.485 (0.309,0.698)	0.426 (0.263,0.633)	0.458 (0.286,0.671)	0.425 (0.260,0.640)
$st.error\ (\rho)$	0.118	0.1128	0.112	0.116
$median\ (\sigma)$	0.065 (0.054 , 0.079)	0.061 (0.051 , 0.074)	0.063 (0.053 , 0.077)	0.061 (0.051 , 0.075)
$st.error\ (\sigma)$	0.008	0.007	0.007	0.007

	Prior statistics			
Prior #	5	6	7	8
$mean\ (\kappa)$	4	4	4	4
$st.error\ (\kappa)$	4	10	15	50
	Posterior statistics			
$median\ (\kappa)$	10.65 (−4.15 ,15.37)	13.08 (−20.47,27.42)	11.35 (−29.37,36.05)	−16.84 (−89.7,94.6)
$mean(\kappa)$	10.05	5.525	3.207	−0.232
$st.error\ (\kappa)$	4.791	17.19	23.21	60.18
$median\ (\rho)$	0.40 (0.246,0.627)	0.381 (0.226,0.595)	0.370 (0.219,0.579)	0.350 (0.208,0.550)
$st.error\ (\rho)$	0.116	0.113	0.111	0.105
$median\ (\sigma)$	0.060 (0.050 , 0.073)	0.057 (0.048 , 0.070)	0.057 (0.047 , 0.069)	0.055 (0.046 , 0.067)
$st.error\ (\sigma)$	0.007	0.007	0.007	0.006

The 90% Bayes interval in parenthesis

The Table 6 reports the posterior statistics of the three structural parameters κ, ρ and σ case by case. The change of the standard error of the prior distribution has a minor impact on the persistence variable ρ and the standard error of the shock σ. The values of both the variables tend to decrease as the standard error of the prior rises. Note that the case no. 2 is a special case. Also there is no notable change in the standard errors of posterior chain of ρ and σ from case to case.

If the parameters ρ and σ behave steadily the posteriors of κ need to be discussed in more detail. As we loosen the prior the distribution the 90% Bayes posterior interval becomes wider which is expected. In the case no. 5 the 90% Bayes interval has negative lower bound as the 5% of the posterior values are less than -4.15 and 5%

are larger than 15.37. The lower bound of the 90% Bayes interval is negative in all the cases $5 - 8$ and the negative values of the posterior of κ appear also in the case no. 4. In the cases $1 - 7$ the posterior median of κ is higher than the expectation of the prior distribution. The median of κ becomes larger as the standard error of the prior rises except in the cases 7 and 8. The standard error of the posterior chain of κ in the cases $1 - 4$ is smaller than the standard error of the prior, but in cases $5 - 8$ it is larger which might imply that the posterior chain is not stabile. The convergence of the chains is tested as above, and the chains do converge (the results not reported here).

The mean of κ is also reported in each case. One can note that in the cases $1 - 5$ the mean and median are close to each other and that the mean is larger than the median in the cases $1 - 3$ and 8. The mean is smaller than the median in the cases $4 - 7$ and in the cases $6 - 8$ the difference of the mean and the median is large. From the Table 6 one could conclude that as the standard error of the prior of κ is large enough and gets even larger the simulated posterior chain starts to alternate between positive and negative values. As the posterior chain alternates the mean of κ is likely to converge towards zero, but the chain itself does not converge.

The cases and priors $1 - 4$ seem more plausible than the cases $5 - 8$ because in the latter cases, zero is included in the 90% Bayes interval. Also in the cases $5 - 8$ the standard error of κ is higher in the simulated posterior than in the prior which would mean that after the inference and adding the data, we would have less information about the parameter of interest. In the cases $1 - 4$ of this test and in all other cases of this paper, the posterior standard error is smaller than the prior standard error. In all the cases $1 - 4$ the posterior median of κ using the Smets & Wouters (2003) prior is included in the 90% Bayes interval. Thus we could conclude that although the prior distribution could be something else, there is no serious doubts concerning suitability of the Smets & Wouters (2003) prior. As noted above, the priors in Smets & Wouters (2003) and Adolfson et al. (2007) both are based on empirical findings in previous literature. Since the model in this paper is close to Smets & Wouters (2003) thus the suitable prior is also the one used in that paper.

4 CONCLUSION

In this paper we have studied private investment in the fashion of recent developments of the neoclassical theory and DSGE modelling conventions. We develop a model that contains rational expectations about future and adjustment costs of the investment. The driving force of the investment equation is the Tobin Q variable. The Tobin Q theory suggests that when the market valuation of the capital used in production is higher than the repurchase price of that capital, firms invest in new physical capital.

The theoretical investment equation derived in the paper (38) is estimated using the Bayesian estimation techniques. The Bayesian method is a standard way in DSGE model estimations but for a standard economist audience the method may not yet be as familiar as other methods in the literature. However, the method suits well in this estimation task and -to be honest- there were failing efforts to use other methods such as GMM before turning to Bayesian methods. The estimated model is linear where the coefficients depend on the structural parameters of the model and the error term of the linear model is assumed i.i.d. zero mean error term. The equation (45) is the estimated model and (44) is the mapping between the structural parameters and the estimated model coefficients. In the base case of the paper we fix the discount factor β at 0.99 as commonly done in the literature. The prior distributions for the standard deviation of investment shock term, σ, the persistence term of the investment shock ρ and the investment adjustment cost parameter κ are chosen as in Smets and Wouters (2003).

The estimation results show that investment shocks are less persistent in Finland than in Sweden or in the euro area. Our estimate for the median of ρ is 0.485 which means that a shock whose size is normalized to one is more than halved to the subsequent period. The persistence parameter value for Finland differs greatly from the value estimated for the euro area by Smets and Wouters (2003). The possible reason for the difference is the difference in the data sets. The persistence of investment shock in Sweden is also less than in the euro area (Adolfson et al. 2007), so there is evidence from other studies that the persistence should be lower for single country. The estimated standard error of the investment shock σ for Finland is less than its euro area counterpart thus the random shocks to investment equation are smaller in Finland than in the euro area.

The estimation results in the posterior value of the adjustment cost parameter κ that is very close value of the parameter estimated by Smets and Wouters (2003). We conduct a test of robustness of the parameter κ by increasing the standard deviation of the prior distribution step by step. The conclusion of the tests is that the posterior values given by the Smets & Wouters (2003) prior is included to results given by other prior distributions. We exclude from the reasoning the priors that give negative simulated values of κ. The posterior mean of κ in Adolfson et al. (2007) is larger than the posterior mean of κ in our results. Using Finnish data and the same prior as Adolfson et al. (2007) for κ the posterior mean of κ increases significantly. Thus choice of the prior of κ plays critical role. To our knowledge, studies defining the more suitable prior information for a small open economies such as Finland are not available thus we need to use the priors that are based on empirical findings on US data (Altig et al. 2003 and Christiano et al. 2005). It is beyond the scope of this paper to dive into capital account tables trying to define reasonable prior distribution, although finding a informative prior might be possible.

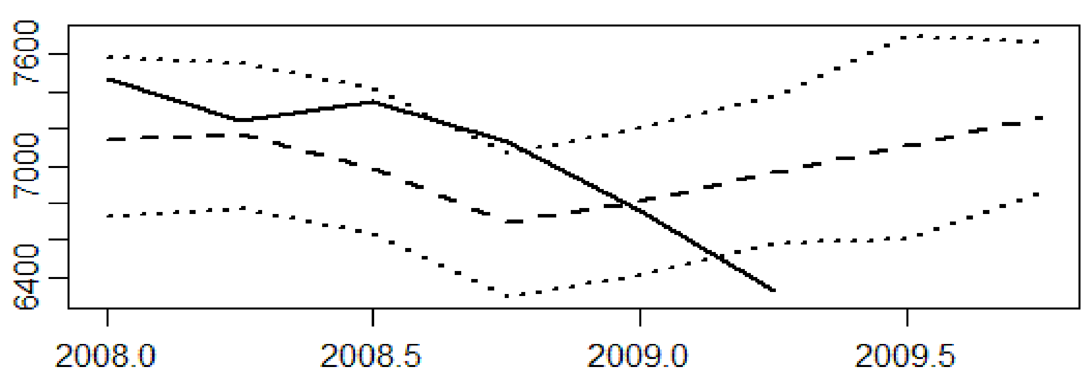
A clear limitation of the econometric exercise of this paper is the Tobin Q proxy. In this paper the theoretical Tobin Q variable is the ratio of the shadow value of capital to household to the shadow value of a utility of household when stock of capital changes one unit. The shadow values are the Lagrangian multipliers of the capital and budget constraints respectively, see the Lagrangian of the optimization problem, equation (31). We construct the empirical Tobin Q proxy as in Takala (1995). Also Kajanoja (1995) estimates investment equations with Finnish data and uses Tobin Q models. Kajanoja (1995) suggests that the Tobin Q proxy could be modelled as done in our paper, but uses other proxy variable because the variable as in this paper did not appear to be stationary (Kajanoja 1995). In this paper we use the investment and Tobin Q series differently and it is argued that the series of the deviations from the steady state is stationary and the estimation can be done. The dominance of the stock price index in Tobin Q proxies is recognized in the literature, for example Barro (1990) and in the same way our proxy for Tobin Q follows very closely to the three-month averages of OMX Helsinki Cap index. That stock index might be a good guess of the market valuation of the installed capital but is not likely to be the most comprehensive proxy for the market value of installed capital in Finland. The empirical applications of the Tobin Q theory are sometimes forced to make compromises due to data availability as mentioned in Kajanoja (1995) and also noticed in this paper. The better proxy for Tobin Q variable of aggregate data is thus a good subject for future research.

The most of the investment literature for Finnish data includes applications of the Tobin Q theory using firm level data. The frequently used data set is the one that includes the 500 largest companies in Finland. This data set is provided by The Research Institute of the Finnish Economy. Some examples of the studies using the firm level data are Brunila (1994) and Vilmunen (2002), see also Vanhala (2006). In this paper we have no possibility to use such a detailed data and instead we use a proxy for Q that is somewhat rough and is dominated by the stock market index. Internationally, the research about Tobin Q and the proxies is lively. The measurement of the Q variable is somewhat hot topic, see for example Erickson and Whited (2000) or Bond and Cummins (2001) for discussion. Also the empirical papers compare different regressors of investment. These papers include Tobin Q as well as well as cash flow, sales, finance restrictions, stock prices and investment plans as regressors, see for example Lamont (2000) or Cummins, Hasset and Oliner (2006). Chirinko (1993) provides a review of the investment literature.

Given that the driving variable of the simulation of the predictive series is likely to incomplete regressor of investment, it seems to produce satisfactory results. The 90% predictive belt in the Figure (6) is not bad. The model works generally well: there are no problems in convergence of the posterior chains of the variables and the results are robust to loosening the strict prior of the discount factor $\beta = 0.99$. The estimated parameter values are also in line with other studies. So we could conclude that the investment equation derived in the paper and the estimation of the equation produces satisfactory results. This paper shows also that Tobin Q model works well, at least this time.

If we now assume that the model is pretty good so why not make a forecast of investment for this year. The forecast is done as the simulation of the model above but now we use the information about the market valuation of capital that is already available for year 2009. If we assume that there has not been change in the price of the physical capital, the series of Tobin Q for first three quarters of 2009 is the series of quarterly averages of the OMX Helsinki Cap index. The value of Tobin Q of the fourth quarter is average of the daily values of the stock price index from beginning of October to the date (18th November). The forecast is plotted in the Figure (7), where the actual series from Statistics Finland is the solid line and the forecast is the dashed line. There is also the 90% Bayes interval for the forecast with dotted lines in the Figure (7). The greatest difference between the forecast and the actual series is obvious: The actual series is decreasing whereas our forecast anticipates growth in private investment towards the end of the year. Well, the forecast is promising but the official data does not seem to agree. The preliminary data of Statistics Finland is subject to revision thus the forecast might not be that bad.

Figure 7: The forecast of private investment for 2009 with 90% Bayes interval.



REFERENCES

Abel, B.A. & Blanchard, O. J. 1983. An Intertemporal Model of Saving and Investment.Econometrica 51 (3) 675-692.

Adolfson M., Laséen S., Lindé J., and Villani M. 2007. Evaluating An Estimated New Keynesian Small Open Economy Model. Sveriges Riksbank Working Paper Series 203.

Altig, D., Christiano, L., Eichenbaum, M., & Lindé, J. 2003. Technology Shocks and Aggregate Fluctuations. Manuscript, Northwestern University.

Bond, S. and Cummins, J. 2001. Noisy Share Prices and the Q Model of Investment. Institute for Fiscal studies.

Brunila, A. 1994. Investment and Financing Considerations: Evidence from Finnish Panel Data. Bank of Finland Discussion papers 4/1994

Cavendish, H. 1798. Experiments to Determine the Density of the Earth. Philosophical Transactions of the Royal Society of London 88, 469-526.

Christiano, L. J., Eichenbaum, M. & Evans, C. L. 2001. Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy. NBER Working Paper 8403.National Bureau of Economic Research.

Christiano, L. J., Eichenbaum, M. & Evans, C. L. 2005. Nominal Rigidities and the Dynamic Effects of a Shock to Monetary Policy. Journal of PoliticalEconomy 113 (1) 1-45.

Cummins, J.G., Hasset, K. A. and Oliner, S.D. 2006. Investment Behavior, Observable Expectations, and Internal Funds. The American Economic Review 96 (3) 796-810.

Erickson, T. and Whited T. M. 2000. Measurement Error and the Relationship between Investment and q. Journal of Political Economy 108 (5) 1027-1057

Geweke J. 1992. Evaluating the Accuracy of Sampling-Based Approaches to the Calculation of Posterior Moments., in Bernardo, J.M., Berger, J.O., Dawid, A.P.and Smith, A.F.M. (eds.). Bayesian Statistics 4. Oxford University Press, Oxford.

Harrison, R., Nikolov K., Quinn, M., Ramsay G., Scott, A. and Thomas, R. 2005. The Bank of England Quarterly Model. Bank Of England.

Hayashi, K. 1982. Tobin's Marginal q and Average q: A Neoclassical Interpretation. Econometrica 50 (1) 213-224.

Jorgenson, D.W. 1963. Capital Theory and Invesment Behavior. The American Economic Review 53 (2) 247-259.

Kajanoja, L. 1995. Aggregate Investmnet and Corporate Indebtedness: Some Empirical Evidence from Finland. Bank of Finland Discussion papers 10/1995

Koop, G. 2003. Bayesian Econometrics. JohnWiley & Sons Ltd. Chichester.

Lamont, O. A. 2000. Investment Plans and Stock Returns. Journal of Finance 55 (6) 2719-2745.

Lindé, J. 2005. Estimating New-Keynesian Phillips curves: A full information maximum likelihood approach. Journal of Monetary Economics 52 (2005) 1135–1149

Lucas R.E. & Prescott E.C. 1971. Investment under Uncertainty. Econometrica 39 659-682.

Penttinen, A. 2009. Bayes-tilastotiede. Lecture course notes spring 2009. Department of Mathematics and Statistics. University of Jyväskylä.

Pekkarinen J. & Sutela P. (2002) Kansantaloustiede. WSOY. Helsinki.

Romer, D. 2006. Advanced Macroeconomics, 3rd edition. McGraw-Hill/Irwin, New York.

Smets, F. & Wouters R. 2003. An Estimated Dynamic Stochastic General Equilibrium Model of the Euro Area. Journal of the European Economic Association 1 (5)

Takala, K. 1995. The Interest Rate Sensitivity of Output in Finland. Bank of Finland Discussion Papers 11/95

Tobin, J. 1969. A General Equilibrium Approach To Monetary Theory. Journal of Money Credit and Banking 1 (1) 15-29.

Vanhala, S. 2006. Odotettu kannattavuus ja rahoitustekijät yrityksen investointikäyttäytymisen selittäjinä. Ekonometrinen tutkimus paneeliaineistolla. Master's Thesis Department of Economics. Helsinki School of Economics.

Vilmunen, J. 2002. Dynamics of investment behaviour in Finland: aggregate and firm level evidence. Bank of Finland Discussion Papers 22/2002

A THE MODEL OF INVESTMENT OF THIS PAPER

In this paper, the work of Smets & Wouters (2003) is followed in setting up the model in which the behaviour of investment is studied. The Smets & Wouters (2003) model is very much like Christiano et al. (2001). It brings together the neoclassical profit maximizing firms, the utility maximizing households, the investment that is costly to adjust and the idea of Tobin Q theory. The derivation of the model and linearization of the mode are done in this section. The Smets & Wouters (2003) model is a comprehensive macroeconometric model for the euro area, but in this paper only the investment section is of interest, so we concentrate on that and leave other features aside.

Assume that there is continuum of households $z \in [0, 1]$. The Households' instantaneous utility depends on level of consumption C and labour supply L. The households own the physical capital and supply capital services as in Christiano et al. (2001) to the firms. The households also supply differentiated labour services to the firms and act as wage setters in the monopolistically competitive markets. The factor market for the physical capital is competitive but subject to real frictions in the form of costly changes of capital utilization rate and costly changes in capital accumulation. The households decide each period how much to consume, accumulate capital and how much to supply capital services. Thus the market equilibrium in final goods market is that all production at time t is consumed, invested or spend on adjusting capital utilization rate. The household optimization problem is exposed to random shocks, ϵ, that follow first order autoregressive progress with i.i.d. zero mean normal error term. That is $\epsilon_t = \rho\, \epsilon_{t-1} + \eta_t$, where $\eta_t \backsim N(0, 1)$ and $|\rho| < 1$. The shocks are associated with preferences, the supply of labour or capital adjustment costs.

The utility function of a household z is familiar constant relative risk aversion form utility function:

$$U_{t,z}(C_{t,z}) = \frac{1}{1-\delta_c} \epsilon_t^C \left(C_{t,z}\right)^{1-\delta_c} - \frac{1}{1+\delta_L} \left(L_{t,z}\right)^{1+\delta_L}, \tag{47a}$$

where $C_{t,z}$ and $L_{t,z}$ are the consumption and the labour supply of the household z at time t,respectively. The variable ϵ_t^C is preference shock of consumption, δ_c is the risk aversion term and δ_L is the inverse of the labour supply elasticity. Assume homogenous households and drop z. Thus household's preferences are presented as

$$\max \sum_{j=0}^{\infty} \beta^j \epsilon_{t+j}^C \left[\frac{1}{1-\sigma_c} (C_{t+j})^{1-\sigma_c} - \frac{\epsilon_{t+j}^L}{1+\sigma_L} {L_{t+j}}^{1+\sigma_L} \right], \tag{48a}$$

where

$$\begin{array}{ccc} \beta & = & \text{subjective discount factor} \\ \sigma_c & = & \text{risk aversion} \\ h & = & \text{degree of habits} \\ L_t & = & \text{labour supply} \\ \sigma_L & = & \text{inverse of the labour supply elasticity} \\ \epsilon^C & = & \text{preference shock} \\ \epsilon^L & = & \text{labour supply shock.} \end{array}$$

Households' maximize their utility subject to budget and capital accumulation constraints. The households' sources of income in nominal terms are wages paid by firms, income from renting capital, dividends from firms and transfers from monetary authority:

$$INC_t = W_t(1-\tau_t^A)L_t + R_t^K(1-\tau_t^K)u_tK_{t-1} + DIV_t + OTT_t,$$

where

$$\begin{array}{ccc} W_t & = & \text{nominal wage rate} \\ \tau_t^A & = & \text{tax rate on labour income} \\ \tau_t^K & = & \text{tax rate on capital income} \\ \tau_t^I & = & \text{tax on investment} \\ u_t & = & \text{utilization rate of capital} \\ R_t^K & = & \text{nominal rental rate on capital} \\ K_t & = & \text{physical capital stock} \\ P_t & = & \text{unit price of final good} \\ DIV_t & = & \text{dividends from firms} \\ OTT_t & = & \text{transfers from monetary authority.} \end{array}$$

Budget constraint contains wealth of households at time t. Household's budget constraint:

$$\begin{array}{ccc} \left[\frac{E_tB_{t+1}^F}{R_t^F(1-\Gamma)} - E_tB_t^F\right] + & & INC_t + \\ + \left[\frac{B_{t+1}^H}{R_t^H} - B_t^H\right] & = & -P_t\left[(1+\tau_t^C)C_t + (1+\tau_t^I)I_t + \Psi(u_t)(1+\tau_t^I)K_{t-1}\right] \end{array}, \tag{49a}$$

where

$$\begin{array}{ccc} E_t & = & \text{exchange rate} \\ B_t^F & = & \text{foreign bonds} \\ B_t^H & = & \text{domestic bonds} \\ R_t^F & = & \text{discount factor on foreign bonds} \\ R_t^H & = & \text{discount factor on domestic bonds} \\ \Gamma & = & \text{risk premium} \\ \tau_t^C & = & \text{consumption value added tax} \\ \Psi(u_t) & = & \text{cost function of varying capital utilization rate} \\ P_t^C & = & \text{price of consumption per unit} \\ I_t & = & \text{investment.} \end{array}$$

Budget constraint (49a) contains returns on holding foreign and domestic bonds (first and second square brackets, respectively). There is a risk premium associated to foreign bonds. Households at time t spend income INC_t on consumption, investment of adjustment of capital utilization. Excess income is used to buy bonds. Also financing consumption from bond market is allowed.

Capital accumulation is the second constraint to the maximization problem. Capital is accumulated according to

$$K_t = (1-\delta)K_{t-1} + \left[1 - \Phi\left(\epsilon_t^I, I_t, I_{t-1}\right)\right] I_t, \tag{50a}$$

where

$$\begin{array}{ccc} \delta & = & \text{depreciation rate} \\ \Phi & = & \text{adjustment cost function} \\ \epsilon_t^I & = & \text{shock to investment adjustment function.} \end{array}$$

Assume that investment adjustment cost function Φ takes as arguments investment I_t in two successive periods. When writing Lagrangian of the optimization problem one must use deflator on budget constraint to get the optimization problem in real terms. Some capital letters are replaced with small letters denoting the change from nominal to real terms. Lagrangian is thus

$$\begin{aligned} L \; = \; & \sum_{j=0}^{\infty} \beta^j \left\{ \epsilon_{t+j}^C \left[\frac{1}{1-\sigma_c} (C_{FL,t+j}(z) - hC_{t+j-1})^{1-\sigma_c} - \frac{\epsilon_{L,t+j}}{1+\sigma_L} L_{t+j}(z)^{1+\sigma_L} \right] \right. \\ & -\Lambda_{t+j} \left[\begin{array}{c} \frac{E_t B_{t+1}^F}{r_t^F(1-\Gamma(\cdot))} - \frac{E_t B_t^F}{P_t} + \frac{B_{t+1}^H}{r_t^H} - \frac{B_t^H}{P_t} + \\ -INC_t + (1+\tau_t^C)C_t + (1+\tau_t^I)I_t + \Psi(u_t)(1+\tau_t^I)K_{t-1} \end{array} \right] \\ & \left. -\Xi_{t+j} \left[K_t - (1-\delta)K_{t-1} - \left[1 - \Phi\left(\epsilon_t^I \frac{I_t}{I_{t-1}}\right)\right] I_t \right] \right\}. \end{aligned} \tag{51a}$$

To find the optimum, we need first order conditions for investment I_t and capital K_t. Also first order condition of capital utilization u_t rate is presented

$$\frac{\partial L}{\partial K_t} = -\Xi_t + \beta \left\{ \begin{array}{c} (-\Lambda_{t+1}) \left[\begin{array}{c} -r_{t+1}^K (1-\tau_{t+1}^K) u_{t+1} + \\ +\Psi(u_{t+1})(1+\tau_{t+1}^I) \end{array} \right] + \\ -\Xi_{t+1}(-(1-\delta) \end{array} \right\} \equiv 0, \tag{52a}$$

$$\iff \Xi_t = \beta(\Lambda_{t+1}) \left[r_{t+1}^K (1-\tau_t^K) u_{t+1} - \Psi(u_{t+1})(1+\tau_{t+1}^I) \right] + \beta \Xi_{t+1}(1-\delta) \tag{53a}$$

$$
\begin{aligned}
\frac{\partial L}{\partial I_t} &= -\Lambda_t[(1+\tau_t^I)] - \Xi_t \left[\begin{array}{c} (-)\left[1-\Phi\left(\epsilon_t^I \frac{I_t}{I_{t-1}}\right)\right]+ \\ +(-1)(-\Phi'(\epsilon_t^I \frac{I_t}{I_{t-1}})(\frac{\epsilon_t^I}{I_{t-1}})I_t \end{array} \right] \\
&\quad +\beta\left(-\Xi_{t+1}\right)(-)\left[-\Phi'\left(\epsilon_{t+1}^I \frac{I_{t+1}}{I_t}\right)\left(\epsilon_{t+1}^I \frac{-I_{t+1}}{(I_t)^2}\right) I_{t+1}\right] \\
&\equiv 0
\end{aligned} \tag{54a}
$$

$$
\begin{aligned}
\Lambda_t(1+\tau_t^I) &= \Xi_t\left[1-\Phi\left(\epsilon_t^I \frac{I_t}{I_{t-1}}\right) - \Phi'(\epsilon_t^I \frac{I_t}{I_{t-1}})(\epsilon_t^I \frac{I_t}{I_{t-1}})\right] \\
&\quad +\beta\left(\Xi_{t+1}\right)\Phi'\left(\epsilon_{t+1}^I \frac{I_{t+1}}{I_t}\right)\left[\epsilon_{t+1}^I\left(\frac{I_{t+1}}{I_t}\right)^2\right]
\end{aligned} \tag{55a}
$$

$$
\frac{\partial L}{\partial u_t} = -\Lambda_t\left[(-r_t^K(1-\tau_t^K)K_{t-1}) + \Psi'(u_t)(1+\tau_t^I)K_{t-1}\right] \equiv 0
$$

$$
r_t^K(1-\tau_t^K) = \Psi'(u_t)(1+\tau_t^I) \tag{56a}
$$

The first order condition for capital utilization is obvious. Function Ψ is increasing, so capital utilization rate u_t is raised up to the level where change of costs when rising utilization one unit equals real return on capital. The Lagrangian multipliers Λ_t and Ξ_t contain relevant information for optimization problem. Λ_t can be interpreted as marginal cost of using one unit of final good on capital in stead of consumption. In other words, Λ_t is the cost of acquiring one unit of capital. Ξ_t is the Lagrangian multiplier that indicates the present value of additional unit of capital. Now one can define Tobin's Q:

$$
Q_t = \frac{\Xi_t}{\Lambda_t}. \tag{57a}
$$

Use definition (57a), (53a) to get

$$
\begin{aligned}
Q_t &= \frac{\beta(\Lambda_{t+1})}{\Lambda_t}\left[\begin{array}{c} r_{t+1}^K(1-\tau_t^K)u_{t+1}+ \\ -\Psi(u_{t+1})(1+\tau_{t+1}^I) \end{array} \right] + \beta\frac{\Lambda_{t+1}}{\Lambda_t}\frac{\Xi_{t+1}}{\Lambda_{t+1}}(1-\delta) \\
&= \frac{\Lambda_{t+1}}{\Lambda_t}\beta\left[\begin{array}{c} Q_{t+1}(1-\delta) + r_{t+1}^K(1-\tau_t^K)u_{t+1}+ \\ -\Psi(u_{t+1})(1+\tau_{t+1}^I) \end{array} \right].
\end{aligned}
$$

Apply in the same way (57a) to (55a) to get

$$
\begin{aligned}
(1+\tau_t^I) &= \begin{array}{c} \frac{\Xi_t}{\Lambda_t}\left[1-\Phi\left(\epsilon_t^I \frac{I_t}{I_{t-1}}\right) - \Phi'(\epsilon_t^I \frac{I_t}{I_{t-1}})(\epsilon_t^I \frac{I_t}{I_{t-1}})\right]+ \\ +\beta\frac{\Lambda_{t+1}}{\Lambda_t}\frac{\Xi_{t+1}}{\Lambda_{t+1}}\Phi'\left(\epsilon_{t+1}^I \frac{I_{t+1}}{I_t}\right)\left[\epsilon_{t+1}^I\left(\frac{I_{t+1}}{I_t}\right)^2\right] \end{array} \\
&= \begin{array}{c} Q_t\left[1-\Phi\left(\epsilon_t^I \frac{I_t}{I_{t-1}}\right) - \Phi'(\epsilon_t^I \frac{I_t}{I_{t-1}})(\epsilon_t^I \frac{I_t}{I_{t-1}})\right]+ \\ +\beta\frac{\Lambda_{t+1}}{\Lambda_t}Q_{t+1}\Phi'\left(\epsilon_{t+1}^I \frac{I_{t+1}}{I_t}\right)\left[\epsilon_{t+1}^I\left(\frac{I_{t+1}}{I_t}\right)^2\right] \end{array}
\end{aligned}
$$

One can show that term that contains investment tax τ_t^I has no impact on result of the optimization it is just a scale factor. To get analysis more tractable, assume from now on that $\tau_t^K = \tau_t^I = 0$ and thus conditions of optimum for capital, investment and capital utilization are

$$Q_t = \frac{\Lambda_{t+1}}{\Lambda_t}\beta[Q_{t+1}(1-\delta) + r_{t+1}^K u_{t+1} - \Psi(u_{t+1})] \tag{58a}$$

$$\begin{array}{c} Q_t\left[1 - \Phi\left(\epsilon_t^I \frac{I_t}{I_{t-1}}\right) - \Phi'(\epsilon_t^I \frac{I_t}{I_{t-1}})(\epsilon_t^I \frac{I_t}{I_{t-1}})\right] + \\ +\beta\frac{\Lambda_{t+1}}{\Lambda_t}Q_{t+1}\Phi'\left(\epsilon_{t+1}^I\frac{I_{t+1}}{I_t}\right)\left[\epsilon_{t+1}^I\left(\frac{I_{t+1}}{I_t}\right)^2\right] \end{array} = 1 \tag{59a}$$

$$r_t^K = \Psi'(u_t).$$

The behaviour of variables in steady state is derived next. Drop time subscripts and random terms and remember that adjustment cost function and its first derivative equals zero in steady state. Solve (59a) for $\bar{Q}$:

$$\begin{aligned} \bar{Q}\left[1 - \Phi\left(\frac{\bar{I}}{\bar{I}}\right) - \Phi'(\frac{\bar{I}}{\bar{I}})(\frac{\bar{I}}{\bar{I}})\right] + \beta\frac{\bar{\Lambda}}{\bar{\Lambda}}\bar{Q}\Phi'\left(\frac{\bar{I}}{\bar{I}}\right)\left[\left(\frac{\bar{I}}{\bar{I}}\right)^2\right] &= 1 \\ \bar{Q}\left[1 - \Phi(1) - \Phi'(1)(1)\right] + \beta\,\bar{Q}\,\Phi'(1)\left[(1)^2\right] &= 1 \\ \bar{Q} &= 1. \end{aligned}$$

Then assume that in steady state capital utilization rate $\bar{u}$ equals 1 and there are no adjustment cost of utilization rate. Solve (58a) for r^K :

$$\begin{aligned} \bar{Q} &= \frac{\bar{\Lambda}}{\bar{\Lambda}}\beta[\bar{Q}(1-\delta) + r^K\bar{u} - \Psi(\bar{u})] \\ 1 - \beta(1-\delta) &= \beta r^K \\ r^K &= \frac{1-\beta(1-\delta)}{\beta} \end{aligned}$$

Then take a look at motion in capital in steady state. Remember, there is no investment adjustment cost in steady state.

$$\begin{aligned} K_t &= (1-\delta)K_{t-1} + \left[1 - \Phi\left(\epsilon_t^I, I_t, I_{t-1}, K_{t-1}\right)\right] I_t \\ K &= (1-\delta)K + I \\ K - (1-\delta)K &= I \\ I &= \delta \end{aligned}$$

In steady state, only replacement investment is done to keep K_t unchanged.

To get equation for investment, one must use technics of log-linearization to equation (59a). Manipulate it to get

$$\begin{array}{c} Q_t\Phi'(\epsilon_t^I \frac{I_t}{I_{t-1}})(\epsilon_t^I \frac{I_t}{I_{t-1}}) + 1 + \\ -\beta \frac{\Lambda_{t+1}}{\Lambda_t} Q_{t+1}\Phi'\left(\epsilon_{t+1}^I \frac{I_{t+1}}{I_t}\right)\left[\epsilon_{t+1}^I \left(\frac{I_{t+1}}{I_t}\right)^2\right] \end{array} = Q_t\left[1-\Phi\left(\epsilon_t^I \frac{I_t}{I_{t-1}}\right)\right] \tag{60a}$$

Log-linearization uses the following equation

$$x_t = \log X_t - \log \bar{X}, \tag{61a}$$

where log means natural logarithm, x_t is defined as logarithmic deviation of X_t from its steady state value $\bar{X}$. Solve equation (61a) for $\log X_t$:

$$\log X_t = \log \bar{X} + x_t$$

Use formula $e^x = e^y, if\ x = y$ to get

$$e^{\log X_t} = e^{\log \bar{X} + x_t} = e^{\log \bar{X}} \times e^{x_t}$$

then use formula $a^{\log_a x} = x$ to get

$$X_t = \bar{X}e^{x_t}. \tag{62a}$$

Now we can write (60a) in in following form

$$\begin{array}{rl} \bar{Q}e^{q_t}\Phi'(\bar{\epsilon}e^{\epsilon_t}\frac{\bar{I}e^{i_t}}{\bar{I}e^{i_{t-1}}})\left(\bar{\epsilon}e^{\epsilon_t}\frac{\bar{I}e^{i_t}}{\bar{I}e^{i_{t-1}}}\right)+1+ & \\ -\beta E_t\left[\begin{array}{c} \frac{\bar{\Lambda}e^{\Lambda_{t+1}}}{\bar{\Lambda}e^{\Lambda_t}}\bar{Q}e^{q_{t+1}}\Phi' \times \\ \times\left(\bar{\epsilon}e^{\epsilon_{t+1}}\frac{\bar{I}e^{i_{t+1}}}{\bar{I}e^{i_t}}\right)\left(\bar{\epsilon}e^{\epsilon_{t+1}}\left(\frac{\bar{I}e^{i_{t+1}}}{\bar{I}e^{i_t}}\right)^2\right)\end{array}\right] & = \\ & = \quad \bar{Q}e^{q_t}\left[1-\Phi\left(\bar{\epsilon}e^{\epsilon_t}\frac{\bar{I}e^{i_t}}{\bar{I}e^{i_{t-1}}}\right)\right]. \end{array} \tag{63a}$$

Notice that in steady state $\frac{\bar{\Lambda}e^{\Lambda_{t+1}}}{\bar{\Lambda}e^{\Lambda_t}} = 1$ and $\bar{Q} = 1$. Also $\bar{\epsilon}$ as well as ratios $\frac{\bar{I}}{\bar{I}}$ equal 1. From now on these terms that equal one are unwritten. (63a) turns to

$$\begin{array}{c} e^{q_t}\Phi'(e^{\epsilon_t}\frac{e^{i_t}}{e^{i_{t-1}}})\left(e^{\epsilon_t}\frac{e^{i_t}}{e^{i_{t-1}}}\right)+1+ \\ -\beta E_t\left[e^{q_{t+1}}\Phi'\left(e^{\epsilon_{t+1}}\frac{e^{i_{t+1}}}{e^{i_t}}\right)\left(e^{\epsilon_{t+1}}\left(\frac{e^{i_{t+1}}}{e^{i_t}}\right)^2\right)\right] \end{array} = e^{q_t}\left[1-\Phi\left(e^{\epsilon_t}\frac{e^{i_t}}{e^{i_{t-1}}}\right)\right] \tag{64a}$$

Divide both sides now with e^{q_t} to get

$$\begin{array}{c} \Phi'(e^{\epsilon_t}\frac{e^{i_t}}{e^{i_{t-1}}})\left(e^{\epsilon_t}\frac{e^{i_t}}{e^{i_{t-1}}}\right)+\frac{1}{e^{q_t}}+ \\ -\beta E_t\left[\frac{e^{q_{t+1}}}{e^{q_t}}\Phi'\left(e^{\epsilon_{t+1}}\frac{e^{i_{t+1}}}{e^{i_t}}\right)\left(e^{\epsilon_{t+1}}\left(\frac{e^{i_{t+1}}}{e^{i_t}}\right)^2\right)\right] \end{array} = 1-\Phi\left(e^{\epsilon_t}\frac{e^{i_t}}{e^{i_{t-1}}}\right) \tag{65a}$$

One can show, that in steady state $\frac{e^{q_{t+1}}}{e^{q_t}}$ equals one, so from now on that term is unwritten. Move $\frac{1}{e^{q_t}} = e^{-q_t}$ to right hand side. Also write fractions using e's exponents

$$\begin{array}{c}\Phi'\left(e^{\epsilon_t+i_t-i_{t-1}}\right)\left(e^{\epsilon_t+i_t-i_{t-1}}\right)+\\-\beta E_t\left[\Phi'\left(e^{\epsilon_{t+1}+i_{t+1}-i_t}\right)\left(e^{\epsilon_{t+1}}\left(e^{i_{t+1}-i_t}\right)^2\right)\right]\end{array} = 1-e^{-q_t}-\Phi\left(e^{\epsilon_t+i_t-i_{t-1}}\right) \tag{66a}$$

Equation (66a) is still an exact form of (59a), but now we need to approximate (66a). Taylor series of $F(x)$ around x_0 is defined:

$$F(x) = \frac{F(x_0)}{0!} + \frac{F'(x_0)}{1!}(x-x_0) + \frac{F''(x_0)}{2!}(x-x_0)^2 + \frac{F^3}{3!}(x-x_0)^3 + \cdots \tag{67a}$$

In our case, we use only first order approximation, so we need only first two terms of the sum in (67a). In steady state (variables denoted with superscript ss) there is no stochastic component of investment and it does not change from period to period, so $i_t^{ss} - i_{t-1}^{ss} = 0$, $\epsilon_t^{ss} = 0$ and remember that $\bar{Q} = 1 \implies q_t^{ss} = 0$ in steady state. Taylor approximations are

$$\begin{array}{cccc}
e^{\epsilon_t} & \approx & e^{\epsilon_t^{ss}} + e^{\epsilon_t^{ss}}(\epsilon_t - \epsilon_t^{ss}) & = & 1+\epsilon_t \\
e^{\epsilon_t+i_t-i_{t-1}} & \approx & e^{ss} + e^{ss}(\epsilon_t + i_t - i_{t-1}) & = & 1+\epsilon_t+i_t-i_{t-1} \\
e^{-q_t} & \approx & e^{-q_t^{ss}} + e^{-q_t^{ss}}(-q_t) & = & 1-q_t \\
\\
\Phi(e^{\epsilon_t+i_t-i_{t-1}}) & \approx & \begin{array}{c}\Phi(e^{ss})+\\+\Phi'(e^{ss})(e^{\epsilon_t+i_t-i_{t-1}}-e^{ss})\end{array} & = & \Phi'(e^0)(e^{\epsilon_t+i_t-i_{t-1}}-1)=0 \\
\\
\Phi'(e^{\epsilon_t+i_t-i_{t-1}}) & \approx & \begin{array}{c}\Phi'(e^{ss})+\\+\Phi''(e^{ss})(e^{\epsilon_t+i_t-i_{t-1}}-e^{ss})\end{array} & = & \text{A} \\
\text{A} & \approx & \Phi''(e^0)(\epsilon_t+i_t-i_{t-1}) & = & \equiv \kappa(\epsilon_t+i_t-i_{t-1})
\end{array} \tag{68a}$$

where $e^{ss} = e^{\epsilon_t^{ss}+i_t^{ss}-i_{t-1}^{ss}}$ and A is a constant for technical reasons. Use results in (68a) to obtain an approximation of (66a):

$$\begin{array}{c}\kappa(\epsilon_t+i_t-i_{t-1})\left(1+\epsilon_t+i_t-i_{t-1}\right)+\\-\beta E_t\left[\kappa(\epsilon_{t+1}+i_{t+1}-i_t)\left(1+\epsilon_{t+1}\right)+\left(1+i_{t+1}-i_t\right)^2\right]\end{array} = q_t. \tag{69a}$$

Then we need to multiply out the products in (69a). Note that product of two small numbers, for example $i_t \times i_{t-1}$, equals approximately zero.

$$\kappa(\epsilon_t+i_t-i_{t-1}) - \beta E_t\left[\kappa(\epsilon_{t+1}+i_{t+1}-i_t)\right] = q_t \tag{70a}$$

The last task to do is solve i_t from (70a):

$$\begin{aligned}
\epsilon_t+i_t-i_{t-1}-\beta E_t\left[\epsilon_{t+1}+i_{t+1}-i_t\right] &= \frac{1}{\kappa}q_t \\
\epsilon_t+i_t-i_{t-1}+\beta i_t-\beta E_t\left[\epsilon_{t+1}\right]-\beta E_t[i_{t+1}] &= \frac{1}{\kappa}q_t
\end{aligned}$$

$$
\begin{aligned}
(1+\beta)(i_t) &= \frac{1}{\kappa}q_t + i_{t-1} + \beta E_t\left[\epsilon_{t+1}\right] + \beta E_t[i_{t+1}] - \epsilon_t \\
i_t &= \frac{1}{1+\beta}i_{t-1} + \frac{\beta}{1+\beta}E_t[i_{t+1}] + \frac{1}{1+\beta}\frac{1}{\kappa}q_t + \frac{\beta E_t[\epsilon_{t+1}] - \epsilon_t}{1+\beta}. \quad (71a)
\end{aligned}
$$

Equation (71a) is investment function of the model. At time t investment is a function of lagged investment i_{t-1} and expected investment i_{t+1}. The investment depends also on variable q_t that contains information about capital and its valuation. The adjustment costs have impact on the investment through second derivative of the adjustment cost function Φ.The investment equation (71a) also contains a random shock term ϵ_t, originally modelled to cause randomness in the investment adjustment cost function.

Since in the investment i_t in (71a) depends on the second derivative of adjustment cost function, it clear that adjustment cost function Φ must be a function of minimum second order, otherwise cost adjustment term disappears and causes trouble over all in this approach.

B LINEAR INTERPOLATION

In order to get quarterly data from annual series, the capital time series must be interpolated. The source of the capital time series is the capital account tables of Statistics Finland. Here we use the series of the total of fixed assets series in real and nominal terms of the Finnish economy. The source is the tables of the gross stock of fixed capital. Since the annual series states the amount of capital at the end of the year t, the fourth quarter of year t in the quarterly series equals the value of annual series that year. The interpolation is done in 3 steps. The calculate the difference of successive years K_{t-1} and K_t and set the fourth quarter of year $t-1$ equal to K_{t-1}. Then the first quarter of year t equals the sum of K_{t-1} and one fourth of the difference $K_t - K_{t-1}$. The second quarter of year t equals the sum of K_{t-1} and two fourths of the difference $K_t - K_{t-1}$. The idea of linear interpolation is formulated below:

$$\begin{aligned}
Q4_{t-1} &= K_{t-2} + 4 \times \frac{K_{t-1} - K_{t-4}}{4} = K_{t-1} \\
Q1_t &= K_{t-1} + \frac{K_t - K_{t-1}}{4} \\
Q2_t &= K_{t-1} + 2 \times \frac{K_t - K_{t-1}}{4} \\
Q3_t &= K_{t-1} + 3 \times \frac{K_t - K_{t-1}}{4} \\
Q4_t &= K_{t-1} + 4 \times \frac{K_t - K_{t-1}}{4} = K_t.
\end{aligned}$$

C DATA APPENDIX

The investment series is the seasonally adjusted time series published by Statistics Finland in the quarterly national accounts. In the Table 7a the series used in estimation is presented.

Table 7a: The Investment Series

Year	Q1	Q2	Q3	Q4
1995	3715	3698	3621	3722
1996	3875	3860	3909	3973
1997	4113	4396	4398	4705
1998	4754	4806	5204	5232
1999	5259	5096	5093	5244
2000	5387	5525	5567	5821
2001	5965	5853	5819	5553
2002	5466	5630	5526	5406
2003	5568	5521	5798	5843
2004	5874	5901	5868	5894
2005	5979	6173	6341	6509
2006	6620	6571	6809	6671
2007	6855	7532	7160	7503
2008	7469	7245	7341	7132
	millions of year 2000 euros			

The proxy of Tobin Q variable is presented in Table 8a. The series is calculated as described in section 3.1 and as in Takala (1995).

Table 8a: The Tobin Q Proxy Series

Year	Q1	Q2	Q3	Q4
1995	1787	1764	1912	1723
1996	1743	1918	2038	2220
1997	2611	2765	3051	3052
1998	3331	3917	3542	2981
1999	3326	3617	3773	4232
2000	5452	4843	4457	3858
2001	3354	3134	2668	2731
2002	3014	2882	2447	2430
2003	2325	2297	2550	2839
2004	3008	2895	2908	3168
2005	3364	3427	3785	3855
2006	4364	4367	4221	4623
2007	4953	5255	5155	4978
2008	4266	4116	3446	2431

D MANIPULATION OF THE ESTIMATION EQUATION

The theoretical investment equation is

$$i_t = \frac{1}{1+\beta} i_{t-1} + \frac{\beta}{1+\beta} E_t[i_{t+1}] + \frac{\varphi}{1+\beta} q_t + \frac{\beta}{1+\beta}(E_t[\epsilon_{t+1}] - \epsilon_t), \tag{72a}$$

where $\varphi = \frac{1}{\kappa}$. It is assumed that the investment adjustment shock term ϵ_t in (72a) follows a stationary $AR(1)$ process $\epsilon_t = \rho\,\epsilon_{t-1} + \eta_t$, where $\eta_t \sim N(0,1)$. Thus the conditional expectation $E_t[\epsilon_{t+1}] = \rho\,\epsilon_t$. Using these properties one can solve (72a) for ϵ_t to get

$$\epsilon_t = -\frac{1+\beta}{\beta(1-\rho)} i_t + \frac{1}{\beta(1-\rho)} i_{t-1} + \frac{1}{(1-\rho)} E_t[i_{t+1}] + \frac{\varphi}{\beta(1-\rho)} q_t. \tag{73a}$$

Replace $E_t[\epsilon_{t+1}]$ and ϵ_t in (72a) with $\rho\,\epsilon_t$ and $\rho\epsilon_{t-1} + \eta_t$ respectively and simplify

$$\begin{aligned}
i_t &= \frac{1}{1+\beta} i_{t-1} + \frac{\beta}{1+\beta} E_t[i_{t+1}] + \frac{\varphi}{1+\beta} q_t + \frac{\beta}{1+\beta}(E_t[\epsilon_{t+1}] - \epsilon_t) \\
i_t &= \frac{1}{1+\beta} i_{t-1} + \frac{\beta}{1+\beta} E_t[i_{t+1}] + \frac{\varphi}{1+\beta} q_t + \frac{\beta}{1+\beta}\,(\rho\,\epsilon_t - \epsilon_t) \\
i_t &= \frac{1}{1+\beta} i_{t-1} + \frac{\beta}{1+\beta} E_t[i_{t+1}] + \frac{\varphi}{1+\beta} q_t + \frac{\beta\,(\rho-1)}{1+\beta}\,\epsilon_t \\
i_t &= \frac{1}{1+\beta} i_{t-1} + \frac{\beta}{1+\beta} E_t[i_{t+1}] + \frac{\varphi}{1+\beta} q_t + \frac{\beta\,(\rho-1)}{1+\beta}\,(\rho\,\epsilon_{t-1} + \eta_t) \\
i_t &= \frac{1}{1+\beta} i_{t-1} + \frac{\beta}{1+\beta} E_t[i_{t+1}] + \frac{\varphi}{1+\beta} q_t + \frac{\beta\,(\rho-1)}{1+\beta}\,\rho\,\epsilon_{t-1} + \\
&\quad + \frac{\beta\,(\rho-1)}{1+\beta}\,\eta_t
\end{aligned}$$

Continue by plugging lagged error term in (73a) into equation.

$$
\begin{aligned}
i_t &= \frac{1}{1+\beta} i_{t-1} + \frac{\beta}{1+\beta} E_t[i_{t+1}] + \frac{\varphi}{1+\beta} q_t - \frac{\beta\ (\rho\ -1)}{1+\beta}\ \rho \times \\
&\left(-\frac{1+\beta}{\beta(1-\rho)} i_{t-1} + \frac{1}{\beta(1-\rho)} i_{t-2} + \frac{1}{(1-\rho)} i_t + \frac{\varphi}{\beta(1-\rho)} q_{t-1} \right) + \\
&+ \frac{\beta\ (\rho\ -1)}{1+\beta}\ \eta_t \\
i_t &= \frac{1}{1+\beta} i_{t-1} + \frac{\beta}{1+\beta} E_t[i_{t+1}] + \rho i_{t-1} - \frac{\rho}{1+\beta}\ i_{t-2} + \\
&+ \frac{\varphi}{1+\beta} q_t - \frac{\rho\,\varphi}{1+\beta}\ q_{t-1} - \frac{\beta\,\rho}{1+\beta}\ i_t + \\
&+ \frac{\beta\ (\rho\ -1)}{1+\beta}\ \eta_t \\
i_t + \frac{\beta\,\rho}{1+\beta}\ i_t &= \frac{1}{1+\beta} i_{t-1} + \rho i_{t-1} - \frac{\rho}{1+\beta}\ i_{t-2} + \frac{\beta}{1+\beta} E_t[i_{t+1}] + \\
&+ \frac{\varphi}{1+\beta} q_t - \frac{\rho\,\varphi}{1+\beta}\ q_{t-1} + \\
&+ \frac{\beta\ (\rho\ -1)}{1+\beta}\ \eta_t \\
\frac{1+\beta+\beta\rho}{(1+\beta)} i_t &= \frac{1+\rho+\rho\beta}{1+\beta}\ i_{t-1} - \frac{\rho}{1+\beta} i_{t-2} + \frac{\beta}{1+\beta} E_t[i_{t+1}] + \\
&+ \frac{\varphi}{1+\beta} q_t - \frac{\rho\,\varphi}{(1+\beta)} q_{t-1} + \frac{\beta\ (\rho\ -1)}{1+\beta}\ \eta_t
\end{aligned}
$$

$$
\begin{aligned}
i_t &= \frac{1+\rho+\rho\beta}{1+\beta+\beta\rho}\ i_{t-1} - \frac{\rho}{1+\beta+\beta\rho} i_{t-2} + \frac{\beta}{1+\beta+\beta\rho} E_t[i_{t+1}] + \\
&+ \frac{\varphi}{1+\beta+\beta\rho} q_t - \frac{\rho\,\varphi}{1+\beta+\beta\rho}\ q_{t-1} + \\
&+ \frac{\beta\ (\rho\ -1)}{1+\beta+\beta\rho}\ \eta_t.
\end{aligned}
$$

or

$$
i_t = \gamma_1\, i_{t-1} - \gamma_2 i_{t-2} + \gamma_3 E_t[i_{t+1}] + \gamma_4\ q_t - \gamma_5\ q_{t-1} + \gamma_6 \eta_t, \tag{74a}
$$

where

$$\begin{array}{lllllll} \gamma_1 & = & \frac{1+\rho+\rho\beta}{1+\beta+\beta\rho} & \quad & \gamma_4 & = & \frac{\varphi}{1+\beta+\beta\rho} \\ \gamma_2 & = & \frac{\rho}{1+\beta+\beta\rho} & \quad & \gamma_5 & = & \frac{\rho\,\varphi}{1+\beta+\beta\rho} \\ \gamma_3 & = & \frac{\beta}{1+\beta+\beta\rho} & \quad & \gamma_6 & = & \frac{\beta\,(\rho-1)}{1+\beta+\beta\rho} \end{array} \tag{75a}$$

One more manipulation is needed to make the estimation easier and example of Lindé (2005) is followed. Consider the term $\gamma_3 E_t[i_{t+1}]$ with expectation in equation (74a). Under rational expectations $E_t[i_{t+1}]$ equals $i_{t+1} + \xi_{t+1}$, where ξ_{t+1} is an independently and identically distributed (i.i.d.) zero mean error with finite variance σ_ξ. Thus one can write

$$\gamma_3 E_t[i_{t+1}] = \gamma_3 i_{t+1} + \gamma_3 \xi_{t+1}. \tag{76a}$$

Replace then the expectation term in (74a) with expression in line (76a), solve for i_{t+1} and lag one period to get

$$\begin{aligned} i_t &= \gamma_1\, i_{t-1} - \gamma_2 i_{t-2} + \gamma_3 i_{t+1} + \gamma_3 \xi_{t+1} + \gamma_4\, q_t - \gamma_5\, q_{t-1} + \gamma_6 \eta_t \\ \gamma_3 i_{t+1} &= i_t - \gamma_1\, i_{t-1} + \gamma_2 i_{t-2} - \gamma_4\, q_t + \gamma_5\, q_{t-1} - \gamma_6 \eta_t - \gamma_3 \xi_{t+1} \\ i_{t+1} &= \frac{1}{\gamma_3} i_t - \frac{\gamma_1}{\gamma_3}\, i_{t-1} + \frac{\gamma_2}{\gamma_3} i_{t-2} - \frac{\gamma_4}{\gamma_3}\, q_t + \frac{\gamma_5}{\gamma_3}\, q_{t-1} - \frac{\gamma_6}{\gamma_3} \eta_t - \xi_{t+1} \\ i_t &= \frac{1}{\gamma_3} i_{t-1} - \frac{\gamma_1}{\gamma_3}\, i_{t-2} + \frac{\gamma_2}{\gamma_3} i_{t-3} - \frac{\gamma_4}{\gamma_3}\, q_{t-1} + \frac{\gamma_5}{\gamma_3}\, q_{t-2} + v_t, \end{aligned}$$

where $v_t = -\frac{\gamma_6}{\gamma_3}\eta_{t-1} - \xi_t$ is an i.i.d. zero mean error term with finite variance σ_v. Using the mapping (75a) for $\gamma_1, ..., \gamma_5$ equation to be estimated can be written as

$$i_t = \frac{1+\beta+\beta\rho}{\beta}\, i_{t-1} - \frac{1+\rho+\rho\beta}{\beta}\, i_{t-2} + \frac{\rho}{\beta}\, i_{t-3} - \frac{\varphi}{\beta}\, q_{t-1} + \frac{\rho\,\varphi}{\beta}\, q_{t-2} - v_t, \tag{77a}$$

Henceforth, the following mapping is used:

$$\begin{array}{lllllll} \theta_1 & = & \frac{1+\beta+\beta\rho}{\beta} & \quad & \theta_4 & = & \frac{\varphi}{\beta} \\ \theta_2 & = & \frac{1+\rho+\rho\beta}{\beta} & \quad & \theta_5 & = & \frac{\rho\,\varphi}{\beta} \\ \theta_3 & = & \frac{\rho}{\beta} & & & & \end{array} \tag{78a}$$

and thus (79a) is written

$$i_t = \theta_1\, i_{t-1} - \theta_2\, i_{t-2} + \theta_3\, i_{t-3} - \theta_4\, q_{t-1} + \theta_5\, q_{t-2} - v_t. \tag{79a}$$

E SIMULATION OF THE 90% POSTERIOR PREDICTIVE BELT

The Figure (6) shows the 90% posterior predictive belt of the model. We simulate the equation (39) in the following way. We need to manipulate the equation (39) into the form

$$(1 - \Phi L)(1 - \gamma L^{-1})\, y_t = e_t$$

where L is the lag-operator and L^{-1} is the lead-operator. Expand the product on the left hand side and solve for y_t.

$$\begin{aligned} (1 - \Phi L)(1 - \gamma L^{-1})\, y_t &= e_t && \text{(80a)} \\ (1 - \Phi L - \gamma L^{-1} + \Phi L\, \gamma L^{-1})\, y_t &= e_t \\ (1 + \Phi\, \gamma)\, y_t &= \Phi L + \gamma L^{-1} + e_t \\ (1 + \Phi\, \gamma)\, y_t &= \Phi\, y_{t-1} + \gamma\, y_{y+1} + e_t \end{aligned}$$

$$y_t = \frac{\Phi}{1 + \Phi\, \gamma}\, y_{t-1} + \frac{\gamma}{1 + \Phi\, \gamma}\, y_{y+1} + \frac{1}{1 + \Phi\, \gamma}\, e_t \tag{81a}$$

The last line (81a) looks very close to investment equation of this paper

$$i_t = \frac{1}{1+\beta} i_{t-1} + \frac{\beta}{1+\beta} E_t[i_{t+1}] + \frac{\varphi}{1+\beta} q_t + \frac{\beta}{1+\beta} (E_t[\epsilon_{t+1}] - \epsilon_t). \tag{82a}$$

Then, solve Φ and γ as functions of β equalizing the the coefficients of y_{t-1} and y_{t+1} with i_{t-1} and i_{t+1} respectively.

$$\begin{aligned} &\begin{cases} \frac{\gamma}{1+\Phi\,\gamma} = \frac{\beta}{1+\beta} \\ \frac{\Phi}{1+\Phi\,\gamma} = \frac{1}{1+\beta} \end{cases} && \text{(83a)} \\ \Leftrightarrow\ &\begin{cases} \Phi_1 = 1 \\ \gamma_1 = \beta \end{cases} \text{ or } \begin{cases} \Phi_2 = \frac{1}{\beta} \\ \gamma_2 = 1 \end{cases} \end{aligned}$$

To continue with the simulation assume now that everything else except investment terms in investment equation (82a) is exogenous. According to the theory Tobin Q variable is endogenous, but to make the simulation, q_t is assumed exogenous as well as the error term $\frac{\beta}{1+\beta}(E_t[\epsilon_{t+1}] - \epsilon_t)$. One can choose either of the pair of the values that solve the pair of equations (83a). If one plugs (Φ_1, γ_1) into (81a), uses the definition that $e_t = \varphi\, q_t + \beta\, (E_t[\epsilon_{t+1}] - \epsilon_t)$ and replaces y_t with i_t one gets

$$i_t = \frac{1}{1+\beta} i_{t-1} + \frac{\beta}{1+\beta} i_{t+1} + \frac{1}{1+\beta} \left[\varphi\, q_t + \beta\, (E_t[\epsilon_{t+1}] - \epsilon_t)\right], \tag{84a}$$

where expectation of the investment term is left unwritten to make the notation more readable. The expectation in the error term is replaced with the best forecast available at time t, that is $E_t[\epsilon_{t+1}] = \rho\,\epsilon_t$ and $\varphi = \frac{1}{\kappa}$. One can derive this equation back to the form (80a):

$$\begin{aligned} i_t &= \frac{1}{1+\beta} i_{t-1} + \frac{\beta}{1+\beta} i_{t+1} + \frac{1}{1+\beta}\left[\frac{1}{\kappa}\,q_t + \beta\,(\rho\,\epsilon_t - \epsilon_t)\right] \\ &\Leftrightarrow\ (1-L)(1-\beta L^{-1})\,i_t = \frac{1}{\kappa}\,q_t + \beta\,(\rho\,\epsilon_t - \epsilon_t) \end{aligned}$$

Now, the form (80a) is simulated in two phases. First, use notation $V_t = (1-\beta L^{-1})\,i_t$ and write

$$V_t = V_{t-1} + \frac{1}{\kappa}\,q_t + \beta\,(\rho\,\epsilon_t - \epsilon_t).$$

To simulate one series $V_1 ... V_{56}$, we use the time series of the Tobin Q proxy from 1995Q1 to 2008Q4 in place of q_t, given κ, ρ and the starting value $V_0 = 0$. The initial investment shock ϵ_0 is sampled from $N(0, \sigma)$ distribution and the subsequent value is a sum of the preceding value and a draw from the distribution $N(0, \sigma)$. The values of σ are estimated above. Then manipulate $V_t = (1-\beta L^{-1})\,i_t$ to get $i_t = \beta\,i_{t+1} + V_t$ and assign a starting value $i_{57} = 0$. With the series $V_{56} ... V_1$ solve backwards the investment series $\mathbf{i}_t$.

Above we used only given values of κ, ρ and σ. To use the posterior chains of the parameters, every 80th iterated value is picked up and stored and thus we have sample of 1000 iterated and independent values from the posterior chains. With the samples of κ, ρ and σ, one can simulate 1000 different investment chains as explained above. Then at each period of time $t = 1...56$ store the 5%, 50% and 95% quantiles of the simulated values.