

DEPARTMENT OF PHYSICS
UNIVERSITY OF JYVÄSKYLÄ
RESEARCH REPORT No. 4/2008

DARK ENERGY, EXTENDED GRAVITY, AND SOLAR SYSTEM CONSTRAINTS

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Academic Dissertation
for the Degree of
Doctor of Philosophy

*To be presented, by permission of the
Faculty of Mathematics and Natural Sciences
of the University of Jyväskylä,
for public examination in Auditorium FYS-1 of the
University of Jyväskylä on May 2, 2008
at 12 o'clock noon*



Jyväskylä, Finland
April 2008

Preface

I wish to thank my supervisor Doc. Kimmo Kainulainen for giving me the opportunity to work in cosmology and for his strong emphasis on the actual physics of a given problem. I am also very happy to acknowledge the cheerful atmosphere always present in his office. Working together has been a pleasure.

I would also like to express my gratitude to Prof. emer. Vesa Ruuskanen for making me feel very welcome when I first set my foot at the physics department in Jyväskylä, and to Doc. Hannu Kurki-Suonio and Doc. Tuomas Multamäki for their valuable comments to this manuscript written four years later. I would like to thank my collaborators Johanna Piilonen and Vappu Reijonen, my friends and colleagues in the Theoretical Particle Physics and Cosmology group, and the whole Department of Physics at the University of Jyväskylä for providing such a pleasant and relaxed working atmosphere.

Finally, I thank my wonderful family and my dear Stina. Your support is invaluable.

This thesis has been supported by the Graduate School in Particle and Nuclear Physics at the University of Jyväskylä, the Centre for International Mobility (CIMO), the Finnish Cultural Foundation, and the Helsinki Institute of Physics.

Abstract

This thesis considers extended theories of gravity as a possible solution to the dark energy problem and in particular studies the impact of Solar System constraints on scalar-tensor theory and $f(R)$ gravity. The present observational status in cosmology and the basic properties of scalar-tensor theory and $f(R)$ gravity are reviewed. The main work is then presented in four appended papers. In summary, Solar System observations put strong constraints on both scalar-tensor theory and $f(R)$ gravity, in particular via the post-Newtonian parameter γ_{PPN} which is the main focus of this thesis.

The scalar-tensor theory discussed in the first paper is a model inspired by large extra dimensions. Here, two large extra dimensions offer a possible solution to the hierarchy problem and the effective four-dimensional theory is a dilatonic scalar-tensor theory exhibiting a cosmological behaviour similar to quintessence. It was shown that this model can also give rise to other types of cosmologies, some more akin to k -essence and possibly variants of phantom dark energy. The observational limits on γ_{PPN} strongly constrain the scalar field coupling to matter, which together with the cosmological constraints nearly determine the model parameters.

The work presented in the three latter papers considered static, spherically symmetric spacetimes in $f(R)$ gravity. The generalized Tolman-Oppenheimer-Volkoff equations were derived both in the metric and in the Palatini formalism of $f(R)$ gravity. By solving these equations for the configuration corresponding to the Sun, it was shown that metric $f(R)$ gravity will in general fail the strong constraint on γ_{PPN} , whereas Palatini $f(R)$ gravity will yield the observed value, $\gamma_{\text{PPN}} \approx 1$. However, the non-standard relation between the gravitational mass and the density profile of a star in $f(R)$ gravity will constrain the allowed forms of the function $f(R)$ also in the Palatini formalism. Although solutions corresponding to $\gamma_{\text{PPN}} \approx 1$ do exist in the metric formalism, a study of the stability properties of the spherically symmetric solutions reveals a necessity for extreme fine tuning, which affects all presently known metric $f(R)$ models in the literature.

List of publications

This thesis is based on the work contained within the following publications:

I Dark energy, scalar-tensor gravity, and large extra dimensions

K. Kainulainen and D. Sunhede,
Phys. Rev. D **73** (2006) 083510.

II The interior spacetimes of stars in Palatini $f(R)$ gravity

K. Kainulainen, V. Reijonen and D. Sunhede,
Phys. Rev. D **76** (2007) 043503.

III Spherically symmetric spacetimes in $f(R)$ gravity theories

K. Kainulainen, J. Piilonen, V. Reijonen and D. Sunhede,
Phys. Rev. D **76** (2007) 024020.

IV On the stability of spherically symmetric spacetimes in metric $f(R)$ gravity

K. Kainulainen and D. Sunhede,
arXiv:0803.0867 [gr-qc].

The majority of the calculations in Paper I-III and all calculations in Paper IV was done by the author of this thesis. He made a major contribution in the writing process (most of which was done together with K. Kainulainen) and produced all figures, excluding Fig. 7 in Paper III which was done by V. Reijonen.

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Chapter 1

Introduction

Einstein's theory of General Relativity is one of the greatest successes in 20th century physics. From theoretical considerations alone, it famously predicted gravitational red shift, bending of light by the Sun and the precession of the perihelion of Mercury. Moreover, it forecasted the expansion of the Universe before it was observed by Hubble in 1929 [1]. General Relativity is so rich in applications that modern astrophysics and cosmology would not exist without it. However, the discovery that the expansion rate of the Universe seems to be accelerating [2,3] leaves standard cosmology with a serious problem. What is the origin of the perceived dark energy component which drives the acceleration? For a homogeneous and isotropic Universe containing only baryons, dark matter and radiation, the only possible solution in General Relativity is to invoke a nonzero cosmological constant Λ . While this so-called Λ CDM scenario is in very good agreement with current observations, the theoretical difficulties associated with a pure cosmological constant makes it an unappealing solution. Instead, one often adds some new exotic component of matter, such as quintessence, which only behaves like a cosmological constant close to present times. An alternative explanation could be that we are using an overly simplistic description of the Universe and that we need to further take into account the effects from inhomogeneities at late times.

In this thesis we will follow a different approach and instead examine extensions to General Relativity. We will consider a flat, homogeneous and isotropic Universe containing only baryons, dark matter and radiation, but where modifications to gravity at large scales will yield the observed accelerating expansion at present. In particular, we will focus on scalar-tensor theory and $f(R)$ gravity and study how such theories are constrained by Solar System measurements. The thesis is divided into two parts, where Chapters 2-6 review the background to the research presented in the second part, Papers I-IV. We begin with a general overview of cosmology in Chapter 2 where we also discuss the Solar System constraints relevant for the work in Papers I-IV. Chapter 3 discusses the basic features of scalar-tensor theory and reviews how they affect cosmology and the Solar System. The relation between scalar-tensor theory and extra-dimensional

scenarios is discussed in Chapter 4, where we in particular focus on the possibility of solving the hierarchy problem via large extra dimensions (LED). This provides further background to the research in Paper I, where we study a scalar-tensor theory for dark energy motivated by LED. Chapter 5 continues by discussing $f(R)$ gravity theories, both in the metric and in the Palatini formalism. We review both cosmological implications and the Solar System constraints which we explore in detail in Papers II-IV. We also review the connection between $f(R)$ gravity and Jordan-Brans-Dicke theory. Finally, Chapter 6 contains our conclusions and provides a summary of Papers I-IV.

On notation: our metric has signature $(-, +, +, +)$. All equations are written in natural units ($\hbar = c = k_B = 1$) unless the fundamental constants are explicitly displayed, and we employ the reduced planck mass $M_{\text{Pl}} \equiv (8\pi G)^{-1/2}$ where G is Newton's constant.

Chapter 2

Background and motivation

2.1 Basic cosmology

The framework for modern cosmology is the Hot Big Bang scenario, a picture which arose when Hubble discovered the expansion of the Universe [1]. Since the Universe is expanding today, becoming colder and less dense, reversing time will make it hotter and more dense. Extrapolated far enough back in time we will ultimately reach an extremely small, dense and hot state. The event which at this point set off the expansion is simply referred to as the Big Bang.

The expansion of the Universe is only one of many observations that makes us believe in the Hot Big Bang model. Indeed the main observations in favour of Big Bang are the following:

- The Universe is expanding. Or more specifically stated: Redshift of galaxies are proportional to their distance away from us (Hubble's law).
- The sky is filled with uniform electromagnetic radiation. This cosmic microwave background is isotropic and has a black-body spectrum with temperature $T \approx 2.73$ K.
- Relative abundances of light elements; 99% of all baryonic matter is made up of hydrogen and helium. The mass fraction of He:H ≈ 0.25 .

In addition, the Hot Big Bang model also provides a good framework for explaining the baryon asymmetry in the Universe and the observed number of baryons to photons $n_b/n_\gamma \sim 10^{-9}$ agrees well with nucleosynthesis. The well-known Standard Model of particle physics and its various extensions make the history of the early Universe a very rich field of study and it provides us with concrete evidence for the Hot Big Bang model dating back to 1–100 s after the Big Bang (see *e.g.* [4]). In particular, nucleosynthesis provides both the earliest and perhaps most convincing test of the Hot Big Bang model (for a review see Ref. [5]).

While the overall evolution of the Universe is described by gravitation, the dynamics of the matter content can roughly be divided into four different eras, each one dominated by specific types of interactions:

- 0-1 min. – Electroweak and strong interactions.
- 1-30 min. – Nuclear interactions.
- 30 min. - 100,000 years – Electromagnetic interactions.
- 100,000 years to present – Gravitational interactions.

In this thesis we will mainly be concerned with late times where the overall dynamics of the Universe is determined by gravity alone. Although earlier epochs provide us with important observational constraints, they will not play an active role in our discussion. The canonical theory of gravity is General Relativity (GR), which can be defined via the following action:

$$S_{\text{GR}} \equiv \frac{1}{2\kappa} \int d^4x \sqrt{-g} [R - 2\Lambda] , \quad (2.1)$$

where $\kappa \equiv 8\pi G$, g is the determinant of the metric tensor $g_{\mu\nu}$, $R \equiv g^{\mu\nu} R_{\mu\nu}$ is the Ricci scalar, and $R_{\mu\nu}$ is the Ricci tensor built from $g_{\mu\nu}$. From a classical point of view, the cosmological constant Λ is not expected, but it can be added and indeed shows up in the most general formulation of GR. When Einstein formulated the theory of General Relativity, the Universe was thought to be static. Therefore, he eventually included the cosmological constant, making static solutions possible by tuning a repulsive Λ so that it exactly balances the gravitational attraction of matter. This idea along with the constant was however dropped with the discovery of the expanding Universe. Nevertheless, since the cosmological constant shows up in the most general formulation, a complete cosmological theory must either explain its value or show why it should be exactly zero.

Taking the action (2.1) together with the matter action S_{m} and varying with respect to the metric, we obtain the equations of motion for $g_{\mu\nu}$ in the presence of matter:

$$G_{\mu\nu} \equiv R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \kappa T_{\mu\nu} - \Lambda g_{\mu\nu} , \quad (2.2)$$

where $G_{\mu\nu}$ is the Einstein tensor and the stress-energy tensor $T_{\mu\nu}$ is defined via $\sqrt{-g} T_{\mu\nu} \equiv -2\delta S_{\text{m}}/\delta g^{\mu\nu}$. These are of course the familiar Einstein equations whose phenomenology is quite simple; spacetime (represented by $G_{\mu\nu}$) gets curved by matter ($T_{\mu\nu}$) and possibly a cosmological constant (Λ). The bare value of the cosmological constant is identified as the vacuum energy density. Since vacuum energies show up over and over again in quantum field theory, the Einstein equations would be in fact somewhat unsatisfactory if they did not allow for a corresponding term Λ .

2.1.1 The cosmological principle

Relativistic cosmology is based on two assumptions, where the first one is the *cosmological principle*. The cosmological principle states that the Universe is both homogeneous and isotropic. It is based on the compelling idea that we should not live in a special place in the Universe and it greatly simplifies the highly nonlinear structure of the Einstein equations (2.2). Observations have now given strong evidence that the cosmological principle indeed holds and it has come to serve as the central premise of modern cosmology. The nearly identical temperature of the cosmic microwave background (CMB) across the sky tells us that the Universe was extremely homogeneous at earlier times and observations such as the 2dF Galaxy Redshift Survey show that it still holds at large scales today. See Figs. 2.1 and 2.2.

For a homogeneous and isotropic space the most general spacetime interval is the Friedmann-Robertson-Walker line element. It can be written in the form

$$ds^2 \equiv g_{\mu\nu} dx^\mu dx^\nu = -dt^2 + a(t)^2 \left[\frac{dr^2}{1 - kr^2} + r^2 d\Omega^2 \right], \quad (2.3)$$

where the curvature $k = -1, 0, +1$, represents a hyperbolic, Euclidean and spherical geometry, respectively, more commonly referred to as the open, flat and closed Universe. The most striking property of the Friedmann-Robertson-Walker line element is that the bracketed part is *independent* of time. These comoving coordinates remain constant for a specific point in space, while the expansion of the Universe is encoded in one single parameter, the scale factor $a(t)$.

2.1.2 Matter as a perfect fluid

The second assumption typically made in relativistic cosmology is that the matter in the Universe can be treated as a *perfect fluid*. The stress-energy tensor for a perfect fluid is given by

$$T_{\mu\nu} = (\rho + p)u_\mu u_\nu + pg_{\mu\nu}, \quad (2.4)$$

where ρ represents its energy density, p is the pressure and u_μ is the normalized four-velocity of the fluid. The equation of state for a perfect fluid can in many cases be described by $p = w\rho$ where w is independent of time. From the conservation of energy (more precisely the $\nu = 0$ component of the conservation of stress-energy, $\nabla_\mu T^{\mu\nu} = 0$), it is straightforward to show that this gives the following density evolution

$$\rho \propto \frac{1}{a^{3(1+w)}}. \quad (2.5)$$

Obviously w is zero for pressureless matter (dust), corresponding to $\rho \propto a^{-3}$. For relativistic matter (radiation) w equals one third, corresponding to $\rho \propto a^{-4}$.

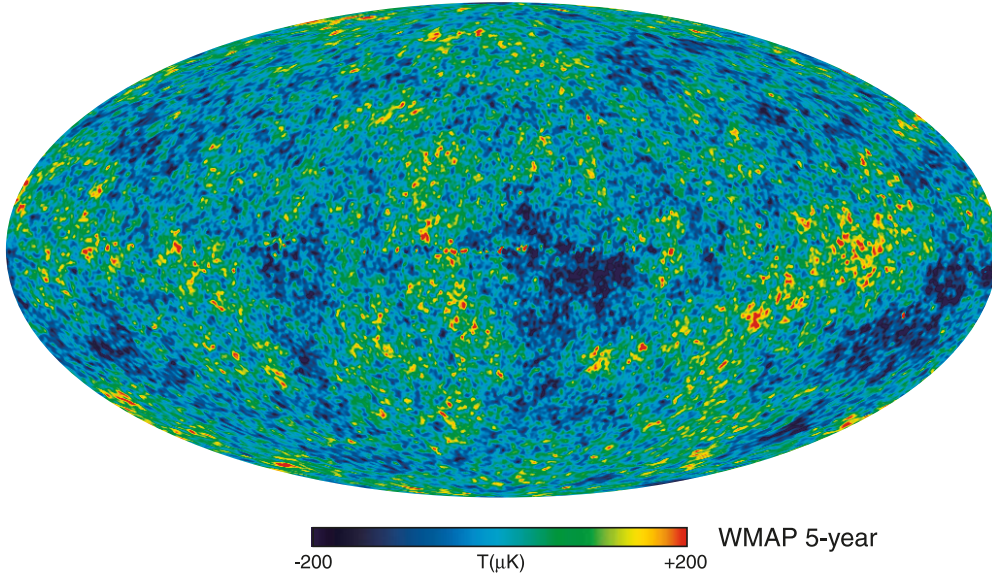


Figure 2.1: Foreground-reduced temperature fluctuations in the CMB across the sky as measured by WMAP (displayed in Galactic coordinates) [6]. Note that the amplitude of the temperature fluctuations is very small $\Delta T/T \sim 10^{-4}$.

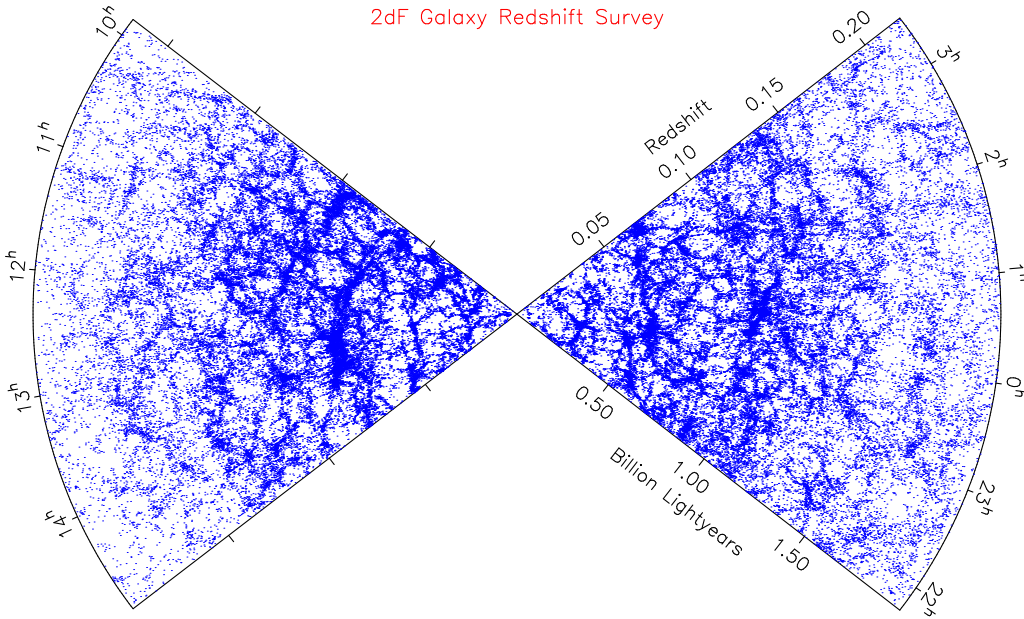


Figure 2.2: Coneplot of the full 2dF Galaxy Redshift Survey [7], illustrating the cosmological principle at present. The figure shows the distribution of galaxies across the sky, where the radial coordinate is distance from earth and the angular coordinate is angle on the sky.

Hence, even though the Universe is matter dominated at present, it must have been radiation dominated at sufficiently early times.

From Eqn. (2.5) it also follows that $w = -1$, corresponding to negative pressure, is an interesting special case. All states of matter with $w > -1$ gets diluted when the Universe expands, but for $w = -1$ the energy density remains constant. This peculiar value does of course correspond to a cosmological constant which we will discuss further below.

2.1.3 The Friedmann equations

The Einstein equations together with the cosmological principle and the assumption that matter behaves as a perfect fluid yield the Friedmann equations:

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} = \frac{8\pi G}{3}\rho + \frac{\Lambda}{3}, \quad (2.6)$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) + \frac{\Lambda}{3}, \quad (2.7)$$

where a dot represents a derivative with respect to time t . Here, terms containing ρ and p correspond to regular matter (which we define as matter with $p > 0$), whereas Λ corresponds to a matter state with negative pressure $p = -\rho$ trying to expand the Universe. The discovery that the expansion of the Universe seems to be accelerating (for a recent update see Ref. [8]), have thus once again put Λ in the limelight. However, one should note that all states of matter with $w < -1/3$, *i.e.* densities which dilute slower than curvature, act to accelerate the expansion.

The expansion rate of the Universe is measured via the Hubble parameter $H \equiv \dot{a}/a$. In general, $H \propto 1/t$ and sets the timescale for the evolution of a . The Universe roughly doubles its size during the time H^{-1} . Although the parameter is not constant in time, its present day value H_0 is generally referred to as the Hubble constant. The actual value of H_0 isn't known very well and it is usually written as $H_0 = 100h$ km/s Mpc $^{-1}$. One of the most reliable results for the parameter h comes from direct measurement via the Hubble Space Telescope Key Project, giving $h = 0.72 \pm 0.08$ [9].

By defining a total energy density $\rho_{\text{tot}} \equiv \rho + \Lambda/8\pi G$, the first Friedmann equation (2.6) yields the following expression for the curvature

$$\frac{k}{a^2 H^2} = \frac{\rho_{\text{tot}}}{3H^2/8\pi G} - 1 = \Omega_{\text{tot}} - 1, \quad (2.8)$$

where

$$\Omega_{\text{tot}} \equiv \frac{\rho_{\text{tot}}}{\rho_{\text{crit}}}, \quad \rho_{\text{crit}} \equiv \frac{3H^2}{8\pi G}. \quad (2.9)$$

Observations have shown that the present value of the density parameter Ω_{tot} is very close to 1 [8]. This is also a prediction from inflation in the very early

Universe (see *e.g.* Refs. [11, 12] and [13]). If $k = 0$ then by definition $\Omega_{\text{tot}} = 1$. In other words, if the present Universe is flat, its total energy density is very close to the critical density ρ_{crit} .

The acceleration/deceleration of the Universe is measured via the deceleration parameter $q \equiv -\ddot{a}/(aH^2)$. Putting this into Eqn. (2.7) and using the fact that the present Universe is dominated by pressureless matter ($p \ll \rho$), one obtains the following expression for the present deceleration:

$$q_0 = \frac{1}{2}\Omega_{\text{m}} - \Omega_{\Lambda} , \quad (2.10)$$

where Ω_{m} is the present value of the matter density parameter and $\Omega_{\Lambda} \equiv \rho_{\Lambda}/\rho_{\text{crit}} \equiv (\Lambda/8\pi G)/\rho_{\text{crit}}$. Hence, whether the present Universe is accelerating or decelerating is completely determined by Ω_{m} and Ω_{Λ} .

2.2 Cosmological observations

We will in this section briefly review the present observational status in cosmology, with focus on matter content and the observational evidence for dark energy. The accelerating expansion of the Universe was discovered only in the late nineties [2, 3], but the evidence has been steadily growing during the past decade. Today, the observed acceleration has become one of the most important problems in modern cosmology.

2.2.1 Type Ia supernovae

The accelerating expansion of the Universe was first discovered by two independent collaborations studying type Ia supernovae (SN Ia) [2, 3]. A large subsample of the type Ia supernovae are (at least close to) good standard candles since they have large and nearly uniform intrinsic luminosity, $M \sim -19.5$ [14]. This extreme brightness allow them to be detectable at high redshifts, up to $z \sim 1$, which is needed for observing deviations from a linear Hubble flow. A type Ia supernova is generally believed to be a white dwarf, accreting mass from a companion star until it reaches the Chandrasekhar mass of $1.4M_{\odot}$. After this the white dwarf collapses and thermonuclear pressure in the core rips it apart in the observed explosion. Since an accreting white dwarf always explodes at roughly $1.4M_{\odot}$, most type Ia supernovae have very similar characteristics.

A convenient measure for astronomical distances is the distance modulus $m - M$, where m is the apparent magnitude and M is the absolute magnitude. The distance modulus is related to the luminosity distance d_L via

$$m - M = 5 \lg d_L + 25 , \quad (2.11)$$

where d_L is measured in units of Mpc. Additionally, the luminosity distance depends on cosmological parameters, $d_L = d_L(z, \Omega_{\text{m}}, \Omega_{\Lambda}, H_0)$. For example, in a

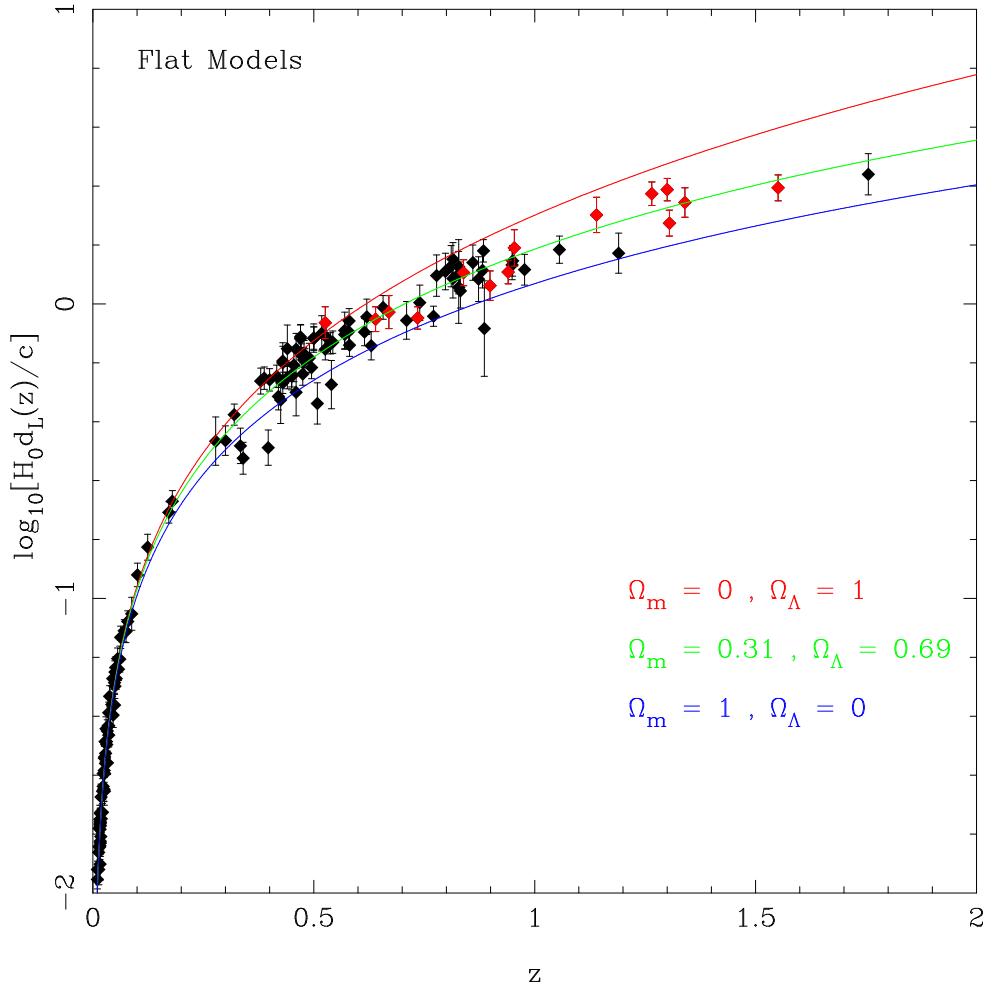


Figure 2.3: The luminosity distance d_L measured in units c/H_0 versus the redshift z in a flat cosmological model. The black data points correspond to the Riess *et al.* “gold” data set and red points show data from the Hubble Space Telescope. Solid lines show theoretical curves for different matter compositions in the Universe where $(\Omega_m, \Omega_\Lambda) = (0.31, 0.69)$ is the best fit and $(\Omega_m, \Omega_\Lambda) = (1, 0)$ and $(0, 1)$ for the lower and upper curve, respectively. From Ref. [15].

flat Universe one has

$$d_L = \frac{1+z}{H_0} \int_0^z \frac{dz'}{\sqrt{\sum_i \Omega_i (1+z')^{3(1+w_i)}}}, \quad (2.12)$$

where Ω_i is the present relative amount of matter with an equation of state w_i . In summary, mapping the magnitude-redshift relation for distant supernovae will effectively measure the energy content of the Universe, albeit with some obvious degeneracy between different species. Under the assumption that the Universe is flat, supernovae measurements points towards a matter density $\Omega_m \approx 0.3$ and favour a positive cosmological constant [3, 15, 16]. Fig 2.3 shows the luminosity distance versus redshift for the Riess *et al.* “gold” data sets [16]

which strongly disfavor the traditional flat Universe filled with ordinary matter, $(\Omega_m, \Omega_\Lambda) = (1, 0)$. The best fit value of Ω_m in the joint analysis performed in Ref. [15] is $\Omega_m = 0.31 \pm 0.04$. One should however note that the systematic errors in supernovae cosmology are still under debate and we refer to Ref. [17] and references therein for further discussion.

2.2.2 The cosmic microwave background

Along with the expansion of the Universe, the observation of the cosmic microwave background [18] is the most important evidence for the standard Hot Big Bang model. The CMB is almost perfectly isotropic, but small anisotropies were detected in 1992 by the COBE satellite [19], later by several balloon experiments and most recently by the WMAP satellite [20]. These small temperature variations are caused by primordial density fluctuations and acoustic oscillations between baryons and photons at recombination (*i.e.* when atoms form and the Universe becomes transparent for electromagnetic radiation). Moreover, the detailed form of these fluctuations depend intricately on the cosmological parameters. By measuring the CMB anisotropies to a high accuracy it is thus possible to obtain very detailed information about almost all of the fundamental cosmological parameters.

The anisotropies in the CMB are closely related to the origin of structure in the Universe. Indeed, the primordial density perturbations causing the temperature fluctuations in the CMB will continue to evolve and form the galaxies, clusters and superclusters observed today. This also means that in order to give a theoretical description of the measured CMB anisotropies, we first need to determine the primordial perturbations and then evolve them to present times by taking into account the specific matter content of the Universe. In the simplest model, one assumes that the primordial density fluctuations are described by Gaussian statistics so that their properties are completely described by the power spectrum Δ^2 . It is convenient to describe the perturbations in terms of the curvature perturbations \mathcal{R}_k , which measure the spatial curvature of a co-moving slice of spacetime, and one often approximates the corresponding power spectrum with a power law:

$$\Delta_{\mathcal{R}}^2(k) = \Delta_{\mathcal{R}}^2(k_*) \left(\frac{k}{k_*} \right)^{n_s-1}, \quad (2.13)$$

where the constant k_* is an arbitrarily chosen scale and n_s is the spectral index. This spectrum is in accordance with most inflationary models which predict a nearly scale-invariant power spectrum, $n_s \approx 1$. The constant power spectrum, $n_s = 1$, is known as the Harrison–Zel’dovich spectrum. Although the above power spectrum of the primordial perturbations is remarkably simple, it will undergo a complex evolution to present times and give rise to a highly non-trivial power spectrum for the CMB. Moreover, the fact that this evolution is very sensitive to the underlying cosmological model and specific matter content

of the Universe, is what makes the CMB such a rich source for observational constraints.

A map of the temperature fluctuations across the sky was shown in Fig. 2.1. In this picture one has already removed a lot of foreground contamination coming in particular from the Galaxy. This is possible since the spectra of foreground sources differs from the CMB signal, and can be removed by use of several independent maps from measurements at different frequencies. The resulting map of temperature fluctuations can then be expanded in spherical harmonics Y_{lm} :

$$\frac{\Delta T}{T}(\theta, \phi) = \sum_{lm} a_{lm} Y_{lm}(\theta, \phi) . \quad (2.14)$$

The most commonly studied quantity is however the angular two-point correlation function:

$$C(\theta) = \left\langle \frac{\Delta T(\hat{\mathbf{m}})}{T} \frac{\Delta T(\hat{\mathbf{n}})}{T} \right\rangle = \sum_l \frac{2l+1}{4\pi} C_l P_l(\cos \theta) , \quad (2.15)$$

where θ is the angle between the unit vectors $\hat{\mathbf{m}}$ and $\hat{\mathbf{n}}$, and P_l are the Legendre polynomials. Note that physics is independent of m since there is no preferred direction in the Universe. The angular power spectrum C_l is usually plotted with $l(l+1)C_l$ as a function of l , where l roughly corresponds to an angular scale $\Delta\theta \simeq \pi/l$.

The angular scale for the largest anisotropies are determined by the size of the largest connected regions at recombination, *i.e.* the horizon. This roughly corresponds to an angle of one degree in the sky today. Thus, at $l \simeq 10^2$ the angular power spectrum starts to show Doppler peaks corresponding to the density fluctuations in the early Universe (for a detailed account on the underlying mechanisms see *e.g.* Ref. [21]). However, the apparent angular size of the above regions is naturally affected by the geometry of the Universe. Compared to a flat Universe, the angle appears larger for a closed geometry and smaller for an open geometry. Hence, the first peak of the spectrum will appear at different values of l depending on the curvature of the Universe.

Fig. 2.4 shows the angular power spectrum for the WMAP five-year data. The insert illustrates the difference in the spectrum for one flat and one open Universe, with otherwise identical parameters. It is clear that the first Doppler peak is situated around $l \simeq 200$ for a flat geometry, while it is shifted towards larger values for an open Universe. Data from the Boomerang [22, 23], MAXIMA [24, 25] (for a joint analysis see Ref. [26]), and WMAP [20] experiments have in this manner shown that the Universe is very close to being flat. By combining CMB data from WMAP together with distance measurements from type Ia supernovae and the baryon acoustic oscillations in the distribution of galaxies, one gets [8]

$$-0.0175 < 1 - \Omega_{\text{tot}} < 0.0085 \text{ (95\% CL)} , \quad (2.16)$$

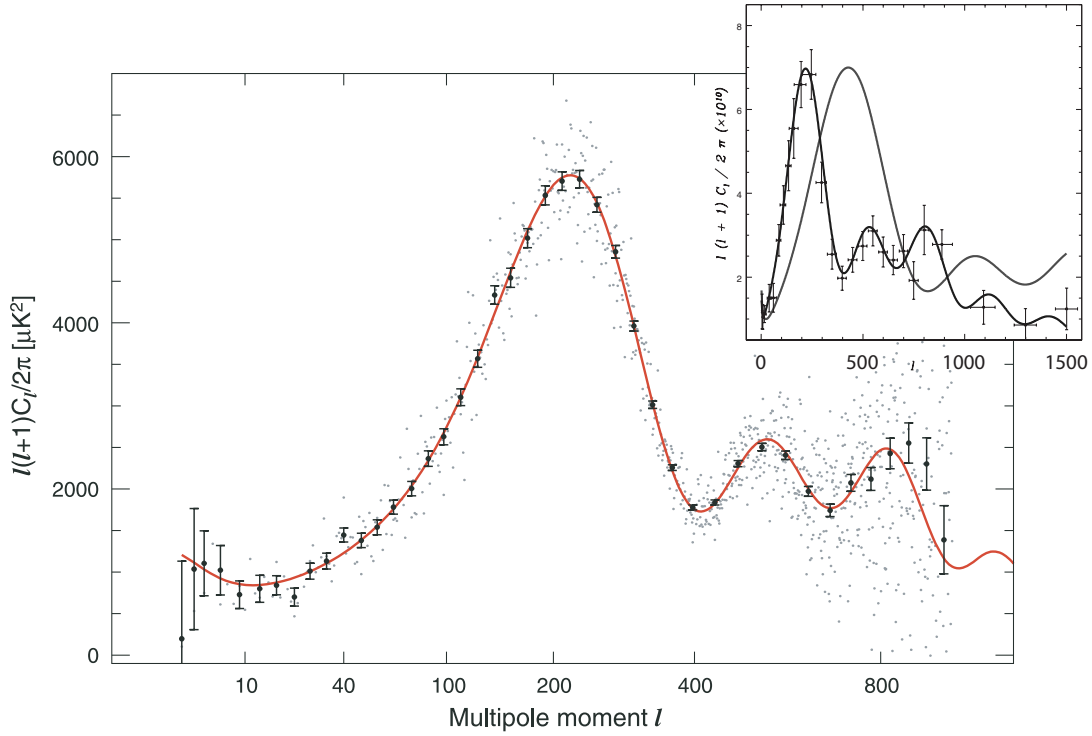


Figure 2.4: The angular power spectrum for the WMAP five-year data with a theoretical curve corresponding to the WMAP-only best-fit Λ CDM model. The black dots correspond to the binned data with 1σ error bars whereas grey dots represent the unbinned data. From Ref. [20]. The insert shows the angular power spectrum for a flat Universe, $\Omega_m = 0.3$ and $\Omega_\Lambda = 0.7$, and an open Universe, $\Omega_m = 0.3$ and $\Omega_\Lambda = 0$. We see that the open Universe with first Doppler peak around $l \simeq 400$ is conclusively ruled out by the data. Adapted from Ref. [28].

lending very strong support to the standard inflationary Hot Big Bang model.

Other cosmological parameters influence the height and positions of various peaks in characteristic ways. By making a fit to the observed angular power spectrum one obtains the preferred values for a specific set of parameters. The WMAP best-fit cosmological model gives [27]:

$$\begin{aligned}\Omega_m h^2 &= 0.1326 \pm 0.0063, \\ \Omega_b h^2 &= 0.02273 \pm 0.00062,\end{aligned}\tag{2.17}$$

where $h = 0.719^{+0.026}_{-0.027}$. That is, in summary, the Universe is flat and, according to the standard Λ CDM fit, composed of roughly 4.4% baryons, 21% non-baryonic matter (*i.e.* dark matter) and 74% dark energy. Note that the above value of h is in perfect agreement with the value measured by the Hubble Space Telescope [9].

2.2.3 Other observations

While high precision measurements of the CMB alone put strong constraints on cosmological parameters, there is a wealth of data from other recent cosmological observations. These can either be used for consistency tests or in order to break the many degeneracies in the parameter space of the CMB angular power spectrum. We have already discussed the type Ia supernovae which measure the accelerating expansion of the Universe, proportional to $\frac{1}{2}\Omega_m - \Omega_\Lambda$. Since the CMB effectively measures the total amount of energy $\Omega_{\text{tot}} \approx \Omega_m + \Omega_\Lambda$ via the position of the first peak in the the angular power spectrum, these methods are highly complementary in the $(\Omega_m, \Omega_\Lambda)$ -plane. This feature pointed towards a positive cosmological constant already before CMB data was accurate enough to independently measure the cosmological parameters. Today, CMB data alone finds $\Omega_\Lambda = 0.742 \pm 0.030$ (within Λ CDM) [20] which is in good agreement with SN Ia observations.

Cold dark matter. The discrepancy between the amount of matter Ω_m and the amount of baryons Ω_b necessitates a large amount of cold dark matter in the Universe. Dark matter can be defined as matter which does not emit, absorb or scatter light and its presence is hence inferred from gravitational effects on regular baryonic matter. The traditional method is to measure the luminosity of a galaxy cluster which corresponds to the total amount of baryons in the system. By comparing the baryonic mass to the total mass needed for obtaining the observed gravitational dynamics of the cluster (using the virial theorem), one receives an estimate on the amount of dark matter which is roughly five times larger than the baryonic mass. The existence of dark matter is also supported by the observed rotation curves of individual galaxies, which revolve much faster than expected from estimates based on visible baryonic matter alone. In summary, measurements have long indicated that $0.1 \lesssim \Omega_m \lesssim 0.4$ [29], which is notably larger than the amount of baryons inferred from nucleosynthesis, $\Omega_b h^2 = 0.017\text{--}0.024$ [30]. Recently, dark matter has also been observed separate from ordinary matter through measurements of two colliding galaxy clusters, the so-called Bullet Cluster. Here gravitational lensing provides compelling evidence for that the majority of the mass is in the form of collisionless dark matter [31]. The exact nature of dark matter is unknown, but the majority of current candidates are elementary particles such as axions and weakly interacting massive particles (WIMPS). Analysis and simulation of structure formation shows that most dark matter should be non-relativistic, cold dark matter (CDM) in order for galaxies to form. A small fraction of relativistic, hot dark matter is allowed however [32, 33].

Galaxy distribution. In addition to their impact on the CMB, density fluctuations can also be measured by mapping the large scale structures in the Universe. The baryon acoustic oscillations (BAO) in the distribution of galaxies are of particular notice for breaking degeneracy in the CMB parameter space. BAO occur during structure formation when pressure terms dominate the ef-

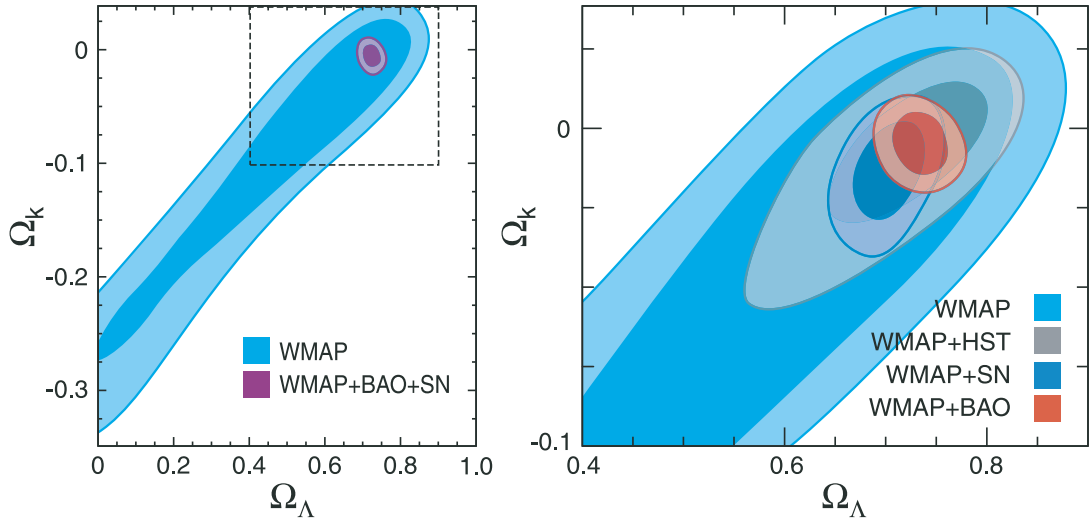


Figure 2.5: Constraints on the vacuum energy density Ω_Λ and the curvature Ω_k of the Universe. The contours show the 68% and 95% confidence levels. **Left:** the WMAP-only constraint (light blue) compared with WMAP+BAO+SN (purple). This figure shows how powerful the extra distance information is for constraining Ω_k . **Right:** a blow-up of the region within the dashed lines in the left panel, showing WMAP-only (light blue), WMAP+HST (Hubble Space Telescope) (gray), WMAP+SN (dark blue), and WMAP+BAO (red). The BAO provides the most stringent constraint on Ω_k . Adapted from Ref. [8].

fect of gravity on baryons. The corresponding scale is given by the Jeans length $\lambda_J \equiv 2\pi c_s / \sqrt{4\pi G \rho}$, where c_s is the baryonic sound speed. Above the Jeans scale, the baryon density contrast grows to match cold dark matter, but below this scale it will instead oscillate as a standing wave. These “acoustic” oscillations show up in the galaxy distribution power spectrum and were first detected by the Sloan Digital Sky Survey (SDSS) [34]. They are particularly useful for breaking the degeneracy between curvature Ω_k and Ω_Λ . Fig. 2.5 shows constraints in the $(\Omega_\Lambda, \Omega_k)$ -plane from CMB data alone and together with BAO data. It is clear that the data sets are highly complimentary, giving $(\Omega_\Lambda, \Omega_k) \approx (0.7, 0.0)$. The best fit model from CMB data alone is also in good agreement with both the BAO and general features in the shape of the galaxy power spectrum [20].

Weak lensing. Images of galaxies get distorted via gravitational lensing from mass fluctuations along the line of sight. For a flat Universe, the probability for a source to be redshifted increases dramatically when Ω_Λ gets close to 1 so that absence of lensing will place an upper bound on the amount of dark energy [29]. The map of deformed galaxy shapes can also be analyzed to yield the matter power spectrum, which amplitude σ_8 and corresponding value of Ω_m provide a good complement to the constraints given by CMB data [20]. The parameter σ_8 is defined as the root mean square of the linear density fluctuations in the mass distribution on a scale $8h^{-1}$ Mpc. This is the characteristic length scale

for galaxy clustering and the measured root mean square fluctuation in galaxy *numbers* within a sphere of $8h^{-1}$ Mpc is close to unity [35]. Therefore, σ_8 is often taken as a measure of the amplitude of the density fluctuations.

Age of the Universe. Let us finally mention an interesting aspect concerning the age of the Universe. Some of the oldest objects in the Universe are globular clusters. For example, the age of globular clusters in the Milky Way has been determined as 13.5 ± 2 Gyr [36] and the age of the Messier 4 globular is constrained to 12.7 ± 0.7 Gyr [37, 38]. This puts a conservative lower bound on the age of the Universe $t_0 > 11$ –12 Gyr. Now, for a flat Universe which contains only pressureless matter it is straightforward to show that $t_0 = 2/3H_0$. However, since direct measurement of the Hubble parameter yields $H_0^{-1} \approx 12$ –15 Gyr [9], this would imply that t_0 is only 8–10 Gyr which is in serious conflict with the age of the globular clusters. A simple cure would be to assume that we live in an open Universe $\Omega_{\text{tot}} \approx \Omega_m < 1$, since in this case it would take longer time for the gravitational attraction to slow down the expansion rate to its present value. However, given that the Universe is flat, it is remarkable that the simplest solution to the age problem is instead to add a positive cosmological constant Ω_Λ which will also counteract the decelerating effect of regular matter.

2.2.4 The Λ CDM model

In summary, observations has led to a cosmological concordance model known as the Λ CDM model, according to which

$$\begin{aligned}\Omega_m &\approx 0.28, \\ \Omega_b &\approx 0.046, \\ \Omega_\Lambda &\approx 0.72,\end{aligned}\tag{2.18}$$

and $h \approx 0.70$ [39]. That is, the Universe is flat, contains a lot of non-baryonic cold dark matter and is dominated by a dark energy component, causing the Universe to accelerate:

$$q_0 \approx \frac{1}{2}0.28 - 0.72 < 0.\tag{2.19}$$

Given that the Universe is homogeneous and isotropic, this dark energy is either in the form of a true cosmological constant or some exotic matter with an equivalent equation of state. The minimal Λ CDM model with adiabatic and nearly scale-invariant Gaussian density fluctuations, fits current observations with only six parameters [8]: $\Omega_b h^2$, $\Omega_{\text{CDM}} h^2$, Ω_Λ , n_s , τ , and $\Delta_{\mathcal{R}}^2$, where n_s and $\Delta_{\mathcal{R}}^2$ describe the tilt and overall shape of the power spectrum of primordial curvature perturbations, and τ is the optical depth to reionization. From this set one can derive for instance the bare values of Ω_b , Ω_{CDM} and h , as well as the age of the Universe t_0 and the amplitude of the matter power spectrum σ_8 .

2.3 The cosmological constant problem

The present observations leave us with an unsolved problem: why and how is $\Omega_\Lambda > 0$? The simplest solution would be a bare cosmological constant Λ_0 corresponding to true vacuum,

$$\rho_{\Lambda_0} \approx 0.72\rho_{\text{crit}} \sim (10^{-3} \text{ eV})^4. \quad (2.20)$$

This approach has two serious problems. First of all, estimating the vacuum energy expected from quantum field theory yields

$$\rho_{\text{vac}} \sim (10^{18} \text{ GeV})^4. \quad (2.21)$$

Comparing this density to Eqn. (2.20) results in the infamous discrepancy of about 120 orders of magnitude between the theoretical and the observed value of the cosmological constant. Although this discrepancy more conservatively becomes of 45 order of magnitude if one replaces M_{Pl} by the QCD scale ~ 150 MeV, it is needless to say that this is unacceptable.

The theoretical vacuum energy density is a sum of contributions from the potential energy of possible scalar fields in the very early Universe. One could imagine these terms to cancel and just leave the tiny observed value, but this seems, to put it mildly, unlikely. Even if one did believe this to be the case, one still has to face the second problem. While regular matter density has decreased many orders of magnitude since early times, a bare cosmological constant Λ_0 is truly constant. For them to be of the same order today is an amazing coincidence that would require an extreme amount of fine-tuning. Although string theory has recently begun to address these questions (for a review see Ref. [17]), the magnitude of the bare cosmological constant remains an unsolved problem.

2.4 Dark energy

The minimal Λ CDM model is in very good agreement with current observations but the problems associated with a pure cosmological constant makes it an unappealing solution from the theoretical point of view. Furthermore, although the high accuracy of present data allows us to put constraints on a model within a certain paradigm, one can not stress enough that changing the underlying assumptions might point at an entirely different cosmological scenario. However, the Λ CDM model remains a useful tool in that it provides us with a baseline from which we can search for anomalies. It is the simplest approximation we can get away with at the present level of cosmological tests.

Barring a true solution to the cosmological constant problem, one can divide the possible ways to solve the dark energy problem into three categories, each one associated with one of the main underlying assumptions in section 2.1. From now on, we will simply assume that the bare cosmological constant is set to zero (perhaps by some unknown symmetry principle), but we offer no insight as to why this should be the case.

2.4.1 Exotic fluids

The last main assumption made in section 2.1 concerned the matter content of the Universe. So far, we have only considered constant equations of state corresponding to radiation and pressureless matter, but it is entirely possible that the Universe also contains some more exotic component. Suggestions for such fluids include quintessence [40–43], k -essence [44–48], phantom matter [49–58], and chaplygin gases [59–66]. Although some of these scenarios may be motivated by higher-dimensional physics and/or extensions to gravity, the majority of the models are described as a fluid with only minimal coupling to gravity. A non-trivial equation of state makes it possible for the exotic fluid to emulate a cosmological constant at late times.

By analogy with inflation, a tempting solution for the dark energy problem would be a scalar field χ with a suitable potential causing an effective cosmological constant at present. This is what we refer to as quintessence. The corresponding Lagrangian is given by

$$\mathcal{L}_Q = -\frac{1}{2}g^{\mu\nu}\partial_\mu\chi\partial_\nu\chi - V(\chi) , \quad (2.22)$$

which for a Friedman-Robertson-Walker background leads to the familiar equation of motion

$$\ddot{\chi} + 3H\dot{\chi} + \frac{\partial V}{\partial\chi} = 0 . \quad (2.23)$$

Using the definition of the stress-energy tensor of the quintessence field, $\sqrt{-g} T_{\mu\nu}^Q \equiv -2\delta S_Q/\delta g^{\mu\nu}$, it is straightforward to obtain the equation of state:

$$w_\chi \equiv \frac{p_\chi}{\rho_\chi} = \frac{\frac{1}{2}\dot{\chi}^2 - V(\chi)}{\frac{1}{2}\dot{\chi}^2 + V(\chi)} , \quad (2.24)$$

with the field acting as an effective cosmological constant when $\dot{\chi}^2 \ll V(\chi)$. Note however that all states with $w_\chi < -\frac{1}{3}$ act to accelerate the Universe. While the matter density ρ_m is steadily decreasing, the quintessence field χ is slowly rolling down its potential. Eventually ρ_χ will begin to dominate and start to accelerate the Universe.

In its basic form, the quintessence scenario does not avoid the problem of fine-tuning. We still have to adjust initial conditions so that ρ_χ end up slightly larger than ρ_m at present. However, it has been shown that a wide range of potentials exhibit so-called tracker solutions where the quintessence field tracks the dominant energy density component [67,68], thus alleviating the fine-tuning in the initial conditions. Nevertheless, the model still needs to be adjusted so that the quintessence field starts to dominate only at very late times. This is related to the problem of explaining why $\rho_\chi \sim \rho_{\text{crit}}$ today, which typically translates into explaining the presence of an extremely small mass scale $\sim \rho_{\text{crit}}^{1/4}$ in the potential $V(\chi)$. A possible way to address this problem is to consider

exponential forms of the potential,

$$V(\chi) \propto \exp(-\lambda\chi), \quad (2.25)$$

and these have been investigated to great extent in the literature [40, 42, 69–75]. A desirable feature of exponential potentials is that small deviations in χ can lead to a substantial change in the energy density of the field. In other words, χ does not need to change many orders of magnitude for ρ_χ to evolve in a cosmologically interesting way. For potentials of the above form, the evolution of the field will often approach a *scaling solution*, defined as a solution where the kinetic and potential energy of χ maintain a fixed ratio. That is, $w_\chi = \text{const.}$ and the energy density of the scalar field scales exactly as a power of the scale factor, $\rho_\chi \propto a^{-n}$.

The naturalness of quintessence models has been discussed further in Refs. [76, 77], which also considered the possibilities of a non-canonical kinetic term in the Lagrangian of the quintessence field: $k^2(\chi)(\partial\chi)^2$. Given a suitable choice of the function $k(\chi)$, it was shown that together with an exponential potential, such a model can create the desired amount of dark energy at present without the need for a small mass scale. In this “natural quintessence” scenario, all parameters are instead of the order of the Planck mass. Note however that this model is different from the aforementioned k -essence scenario, where the potential only plays a negligible role.

2.4.2 Non-homogeneous cosmologies

The second possible way to solve the dark energy problem is to abandon the cosmological principle and also take into account effects from nonlinear inhomogeneities. As exhibited by the CMB, the early universe was very close to homogeneous and isotropic. However, although this remains true as a first approximation also at late times, it is obvious from Fig. 2.2 that the Universe exhibits significant structure also at very large scales. The standard assumption is that nonlinear effects on the detectable light average out at cosmological distances. This assumption was criticized already in the ‘60s [78–81] and it was later suggested that structure formation has effects on the observed distance-redshift relation which the Friedmann-Robertson-Walker description fails to capture [82, 83]. At the time, observations were nevertheless far too inaccurate for distinguishing any features beyond the standard homogeneous and isotropic scenario.

The discovery of the apparent acceleration has once again made the above issues relevant. Two of the more widely considered non-homogeneous scenarios are Lemaître-Tolman-Bondi cosmologies [84–96] and gravitational backreaction [97–111]. Recently, a novel and particularly interesting scenario has been suggested in Ref. [112], which expands on the ideas presented in Refs. [82, 83]. Here one takes into account the opaque lumps in the Universe and that the regions which detectable light traverses become emptier and emptier compared to

the average energy density. Since space expands faster with lower density, the observed expansion is then perceived to accelerate along our line of sight.

2.4.3 Extended gravity

Our final option is to alter the theory of gravity. This may seem to be a radical approach but it is important to remember that, unlike in quantum field theory, the GR gravitational action is not uniquely set by some underlying symmetries and theoretical arguments (renormalizability). Instead, the Einstein-Hilbert action only corresponds to the simplest possible covariant action which can be built from the metric so that it gives non-trivial equations of motion for $g_{\mu\nu}$. This makes extensions to the Einstein-Hilbert action interesting in their own right, since they allow us to explore which features are special to GR and which are more general properties of covariant theories.

Of particular interest as candidates for dark energy are scalar-tensor theories and $f(R)$ gravity. They are the main concern of this thesis and we will discuss these theories in detail in the two upcoming chapters. In addition to the metric, scalar-tensor theories of gravity also include a non-minimally coupled scalar field. Here, unlike in the case of quintessence, the new scalar field is really a part of the gravity sector. Scalar-tensor theories include both extended quintessence [113–120] and chameleon models [121–123], even though these scenarios are sometimes discussed from a fluid description point of view. However, the explicit non-minimal coupling in these theories indeed makes the associated scalar field part of the gravity sector. One can also consider more complicated scalar-tensor theories where the scalar field couples to dark (non-baryonic) matter only [124–126].

Extended theories of gravity come in many different flavours and include for example non-Riemannian cosmologies [127–129], Gauss-Bonnet dark energy [130–136], and Tensor-Vector-Scalar theory (TeVeS) [137–142]. Another scenario which has received much attention (see [143–145] and references therein) is the DGP model (Dvali-Gabadadze-Porrati) [146–148] and a related proposal by Deffayet *et al.* [149–151]. In these models our four-dimensional spacetime, a so-called *brane*, is assumed to be embedded in a higher dimensional Minkowski *bulk* spacetime. All Standard Model fields are confined to the brane whereas gravity propagates also in the Minkowski bulk. Unlike in many other brane-world scenarios, gravity remains four-dimensional at small distance scales but leaks into the bulk at large distances. Gravity may hence become diluted at cosmological scales, leading to the observed accelerating expansion at present.

2.4.4 The dark energy equation of state

Constraining the nature of dark energy is one of the most important challenges in cosmology today. Given the severe problems associated with a cosmological constant, it would be a tremendous success if one would find evidence for that

the dark energy equation deviates from -1 . Indeed, most dark energy models can be parametrized in terms of an effective equation of state, including both scalar-tensor theory and $f(R)$ gravity [152, 153].

The most direct constraints on the dark energy equation of state w_{DE} comes from type Ia supernovae, galaxy redshifts surveys (via BAO), weak lensing effects, and galaxy clustering [154], where CMB data and independent measurements of the Hubble parameter are often used to break the degeneracy between dark energy and other cosmological parameters. Despite recent advances, the quality of the data is nevertheless not yet good enough to warrant any real study of the evolution of w_{DE} . In particular, it is not possible to detect any deviations from a cosmological constant, either at present or in the past. One of the most stringent constraints on w_{DE} comes from the ESSENCE Supernova Survey, which combined with BAO data constrains a constant dark energy equation of state to

$$w_{\text{DE}} = -1.05^{+0.13}_{-0.12} (\text{stat. } 1\sigma) \pm 0.13 (\text{syst.}) \quad (2.26)$$

at present in a flat Λ CDM model [155].

The central value of w_{DE} is slightly smaller than -1 and even more negative values used to be preferred in the past. This has lead several authors to also consider dark energy with an equation of state $w_{\text{DE}} < -1$, so-called phantom matter [49–58]. The most basic approach is to propose a scalar field with negative kinetic term, $-\frac{1}{2}\dot{\chi}^2 + \dots$, but it is perhaps more interesting to note that the same effect can come from interactions in the dark sector. A coupling between dark energy and dark matter, as appears in for example extended quintessence, will alter the redshift dependence of the dark matter density. As a result, an observer who fits data using non-interacting dark matter may indeed obtain $w_{\text{DE}} < -1$ despite that no such exotic matter component is present [156].

2.5 Solar System constraints and the PPN formalism

Let us finally review the constraints on gravity coming from Solar System observations. We will only mention constraints which are relevant for our upcoming discussion and we refer to Ref. [157] for further details. Some aspects related to scalar-tensor theories and $f(R)$ gravity will be discussed in the corresponding chapters.

The high precision of Solar System measurements allows us to put strong constraints on the relativistic corrections to Newtonian gravity. The standard treatment is the parametrized post-Newtonian (PPN) formalism containing 10 parameters describing the possible deviations from Newtonian gravity, all of which are constrained to be very close to the values predicted by GR [157]. For the extensions to gravity discussed in this thesis, only two of these param-

eters may differ from their corresponding GR values.¹ These are the famous Eddington parameters γ_{PPN} and β_{PPN} , which are both identical to 1 in GR.

The PPN formalism is valid for any gravity theory which respects the Einstein equivalence principle. It is defined in the slow motion, weak-field limit where the metric can be expanded around a Minkowski background $\eta_{\mu\nu}$. To obtain the Newtonian limit for a particle moving on a time-like geodesic one only needs to know the term $\mathcal{O}(1/c^2)$ in the g_{00} component of the metric. Consequently, the post-Newtonian limit is given by expanding g_{00} to $\mathcal{O}(1/c^4)$, g_{0i} to $\mathcal{O}(1/c^3)$, and g_{ij} to $\mathcal{O}(1/c^2)$. For a spherical and non-rotating body centered in an otherwise empty space, g_{0i} will vanish and the PPN metric reduces to

$$g_{00} = -1 + \frac{2GM}{r_N c^2} - \beta_{\text{PPN}} \frac{1}{2} \left(\frac{2GM}{r_N c^2} \right)^2 + \mathcal{O} \left(\frac{1}{c^6} \right), \quad (2.27)$$

$$g_{ij} = \delta_{ij} \left(1 + \gamma_{\text{PPN}} \frac{2GM}{r_N c^2} \right) + \mathcal{O} \left(\frac{1}{c^4} \right), \quad (2.28)$$

where M is the gravitational mass of the body and r_N belongs to a nearly globally Lorentz covariant coordinate system². Described in words, γ_{PPN} measures the amount of spatial curvature (g_{ij}) produced by a unit rest mass and β_{PPN} measures the amount of nonlinearity in the superposition law for gravity (g_{00}). The strongest constraint on γ_{PPN} comes from the Cassini spacecraft [160],

$$\gamma_{\text{PPN}} - 1 = (2.1 \pm 2.3) \times 10^{-5}, \quad (2.29)$$

and the parameter β_{PPN} is constrained via lunar laser ranging, which together with the Cassini data yields $\beta_{\text{PPN}} - 1 = (1.2 \pm 1.1) \times 10^{-4}$ [161].

A related constraint concerns the gravitational field strength and the Newtonian potential. For a test particle following a metric geodesic given by $g_{\mu\nu}$, it is straightforward to show that in the slow motion ($v \ll 1$), weak-field limit, $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$ with $|h_{\mu\nu}| \ll 1$, the spatial part of the geodesic equation reduces to

$$\frac{d^2 \vec{x}}{dt^2} = \frac{1}{2} \vec{\nabla} h_{00} \quad (2.30)$$

in Galilean coordinates. Thus, we see that the geodesic world line of a test particle is identical to the Newtonian equation of motion if one makes the identification $h_{00} = -2\Phi_N$, where Φ_N is the Newtonian gravitational potential. In other words, the temporal component of the metric is given by

$$g_{00} = -(1 + 2\Phi_N), \quad (2.31)$$

in the slow motion, weak-field limit in Galilean coordinates. Now, Birkhoff's theorem states that the exterior vacuum solution for a spherically symmetric

¹Please note that we are referring to the set of parameters defined in Ref. [157], which differs from the one used in for example Ref. [158]

²Note that all arbitrariness in the coordinates has been removed by choosing the so-called PPN gauge [159].

star in GR is given by the Schwarzschild metric, for which $g_{00} = -(1 - 2GM/r)$ in Schwarzschild coordinates. The mass parameter M is determined by matching with the interior solution and one obtains

$$\Phi_N = -\frac{GM}{r_N} \approx -\frac{GM}{r} = -\frac{G}{r} \int_0^{r_\odot} dr 4\pi r^2 \rho, \quad (2.32)$$

where we have neglected pressure ($p \ll \rho$), and r_\odot is the radius of the star, *e.g.* the Sun. The last step in the above equation is crucial. Matching with the interior solution tells us that the gravitational mass M of an object is simply given by the flat spatial volume integral over its density profile ρ , just like in Newtonian theory. Note that this is an exact result, which has nothing to do with either the approximations related to obtaining the Newtonian potential, or the uniqueness of the vacuum solution. It simply follows from the particular form of the 00 component of the Einstein equations in the spherically symmetric case. Thus, any extended theory of gravity which alters this form may exhibit a non-standard relation between the density profile and the gravitational mass of the Sun. As pointed out in Papers II, III, and IV, this will put strong constraints on the allowed $f(R)$ models, both in metric $f(R)$ gravity (which suffers from a non-unique vacuum) and in Palatini $f(R)$ gravity (where the vacuum solution is unique).

2.6 Concluding remarks

The standard Hot Big Bang scenario is a remarkably successful description of our Universe. However, present observations indicate that the expansion of the Universe is accelerating, implying the existence of a dark energy component with negative pressure. Although a pure Λ CDM model gives a very good fit to current data, the severe problems associated with a true cosmological constant encourages us to seek for a dynamical explanation of dark energy. We reviewed the possible inclusion of exotic fluids such as quintessence, which may act as a cosmological constant at late times. Abandoning the cosmological principle may also cause effects that will be perceived as an accelerating expansion.

In this thesis we will follow a different approach. We will study a flat ($k = 0$), homogeneous and isotropic Universe filled with radiation and regular matter only, but where extensions to the theory of gravity may alter the behaviour at cosmological scales, causing the observed acceleration at present. In particular, we will consider scalar-tensor theory and $f(R)$ gravity. Both of these scenarios can be constructed in such a way that they yield the desired acceleration. For example, scalar-tensor theories can give rise to a behaviour very similar to quintessence, but also k -essence and possibly phantom matter [I]. In $f(R)$ gravity theories the accelerating expansion is instead caused by an inherent positive curvature of spacetime, $R_0 \sim \Lambda$, obtained via the (relaxed) vacuum solution to the gravitational field equations. However, any extended gravity

theory which has a modified behaviour at cosmological scales can potentially yield non-standard dynamics also at much smaller distances. Some of the most accurate measurements of gravity come from the Solar System and these observations are highly useful when testing alternative theories of gravity. The tight limits on the post-Newtonian parameter γ_{PPN} is particularly restrictive and it will be the main constraint studied in this thesis. In summary, we will see that after fixing the desired cosmological behaviour, Solar System observations puts very strong constraints on the models, both in scalar-tensor theory [I] and in $f(R)$ gravity [II, III, IV].

Chapter 3

Scalar-tensor theory

Scalar-tensor theories of gravity are perhaps the most widely considered extension to General Relativity. Here the gravitational Lagrangian also includes a scalar field φ with non-minimal coupling, *i.e.* it couples explicitly to the metric via the Ricci scalar R . Thus, the scalar field does not describe gravitating matter, but is instead an additional degree of freedom for the gravitational field. The gravitational sector hence contains both a spin 2 field (the metric $g_{\mu\nu}$) and a spin 0 field (the scalar φ), where the latter influences only the coupling strength between matter and the space-time geometry is described by $g_{\mu\nu}$.

The prototype scalar-tensor theory is the Jordan-Brans-Dicke theory of gravity [162, 163], defined via the action

$$S_{\text{JBD}} \equiv \frac{1}{2\kappa} \int d^4x \sqrt{-g} \left[\phi R - \frac{\omega}{\phi} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - U(\phi) \right] + S_{\text{m}}[g_{\mu\nu}, \psi_{\text{m}}], \quad (3.1)$$

where ω is a constant, dimensionless parameter and ψ_{m} corresponds to standard model matter fields. Although the original scenario did not include a potential $U(\phi)$, most modern versions use the above form. One should nevertheless be aware of the fact that the presence of a potential will usually have significant impact on the theory, and results derived in the trivial case $U(\phi) \equiv 0$ are often no longer valid.

A more general scalar-tensor theory can be written in the following commonly used form for the gravitational part of the action:

$$S_{\text{STG}} = \frac{1}{2\kappa} \int d^4x \sqrt{-g} [F(\varphi)R - Z(\varphi)g^{\mu\nu} \partial_\mu \varphi \partial_\nu \varphi - U(\varphi)], \quad (3.2)$$

where $F(\varphi)$, $Z(\varphi)$, and the potential $U(\varphi)$ are arbitrary functions of φ . Note however that these functions are not truly independent since it is always possible to redefine the scalar field via either $F(\varphi) \rightarrow \varphi$, $|Z(\varphi)|(\partial\varphi)^2 \rightarrow (\partial\varphi)^2$, or $U(\varphi) \rightarrow \varphi$. Although more general forms of scalar-tensor theories exist (see *e.g.* Ref. [164]), the action (3.2) describes a very large class of models, including the one considered in Paper I. Below, we will review some basic properties of scalar-tensor theories and discuss how they relate to cosmology and Solar System measurements. More thorough expositions can be found in Refs. [165, 166].

3.1 Conformal frames

Conformal transformations provide a convenient technique for studying singularities and the global structure of spacetime via Penrose diagrams [167]. In scalar-tensor theory they are also related to the concept of conformal frames. A conformal transformation, also referred to as Weyl rescaling, is a local rescaling of the metric tensor:

$$g_{\mu\nu} \rightarrow \Omega^2(x)g_{\mu\nu} . \quad (3.3)$$

This transformation is said to bring us from one conformal frame (or gauge) to another. However, most theories of gravitation with massive matter fields are *not* invariant under the transformation (3.3) and the above notation and terminology can be somewhat confusing. It is often more convenient to explicitly keep track of the metrics associated with different conformal frames:

$$\hat{g}_{\mu\nu} \equiv \Omega^2(x)g_{\mu\nu} , \quad (3.4)$$

which by definition is just a simple change of variables. It corresponds to a local change of length scales (*i.e.* units):

$$ds^2 \equiv g_{\mu\nu}dx^\mu dx^\nu = \Omega^{-2}(x)\hat{g}_{\mu\nu}dx^\mu dx^\nu \equiv \hat{g}_{\mu\nu}d\hat{x}^\mu d\hat{x}^\nu , \quad (3.5)$$

where $d\hat{x}^\mu \equiv dx^\mu/\Omega(x)$. At the classical level, different conformal frames describe the same physics albeit in different manners [168, 169].

The so-called Jordan frame can be defined as the conformal frame where matter fields couple only to the metric defining the invariant volume element in the action. For a scalar-tensor theory of the Jordan-Brans-Dicke type, the action in the Jordan frame is indeed of the form given in Eqn. (3.1):

$$S_{\text{JBD}} = \int d^4x \sqrt{-g} [\mathcal{L}_G(g_{\mu\nu}, \phi) + \mathcal{L}_m(g_{\mu\nu}, \psi_m)] , \quad (3.6)$$

$$\mathcal{L}_G \equiv \frac{1}{2\kappa} \left(\phi R_g - \frac{\omega}{\phi} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - U(\phi) \right) , \quad (3.7)$$

where $R_g \equiv R$, *i.e.* the Ricci scalar defined in terms of the metric $g_{\mu\nu}$. Since the matter Lagrangian depends on the matter fields ψ_m and the metric $g_{\mu\nu}$ only, the Jordan frame corresponds to a choice of units where the rate of clocks and length of rods (made of matter), remain constant in spacetime so that all observations have their standard interpretation. That is, \mathcal{L}_m is indeed of the canonical form used in quantum field theory.

Although the rate of clocks and length of rods are constant in the Jordan frame, the gravitational coupling may vary in spacetime. Indeed, in a JBD type theory the Ricci scalar comes multiplied with a scalar field ϕ so that one has an effective gravitational coupling $G_{\text{eff}} \equiv G/\phi$. It may hence be convenient to define a conformal frame where the effective gravitational coupling remains constant.

This is the Einstein frame.¹ Any scalar function multiplying a term linear in the Ricci scalar can always be absorbed by defining a metric $\hat{g}_{\mu\nu}$ conformally related to $g_{\mu\nu}$. For a JBD theory this corresponds to a conformal change of variables

$$\hat{g}_{\mu\nu} \equiv \Omega^2 g_{\mu\nu} = \phi g_{\mu\nu} , \quad (3.8)$$

which results in the following form of the action:

$$S_{\text{JBD}} = \int d^4x \sqrt{-\hat{g}} \left[\mathcal{L}_{\text{G},\phi}(\hat{g}_{\mu\nu}, \phi) + \frac{\mathcal{L}_{\text{m}}(\phi^{-1}\hat{g}_{\mu\nu}, \psi_{\text{m}})}{\phi^2} \right] , \quad (3.9)$$

$$\mathcal{L}_{\text{G},\phi} \equiv \frac{1}{2\kappa} \left(R_{\hat{g}} - \frac{3+2\omega}{2\phi^2} \hat{g}^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - \frac{U(\phi)}{\phi^2} \right) , \quad (3.10)$$

where $R_{\hat{g}}$ is the Ricci scalar defined in terms of the metric $\hat{g}_{\mu\nu}$. The apparent strength of gravity is now constant and the action takes the form of an Einstein-Hilbert term with matter and a scalar field. However, the matter Lagrangian is no longer of the canonical form since it explicitly couples to the scalar field ϕ . Hence, the corresponding system of units now depends on the scalar field so that a change in ϕ will also change the rate of clocks and length of rods. Although this does not pose any problems in principle, observables require special attention when ϕ changes during the course of an experiment. For example, when working in Einstein frame units, the mass of a Hydrogen atom on Earth is not necessarily the same as the mass of a Hydrogen atom in a distant Cepheid if ϕ varies on cosmological scales. Let us nevertheless once again stress that the actions given in Eqn. (3.6) and Eqn. (3.9) correspond to the exact same physics and that they are completely equivalent at the classical level. The Einstein frame coincides with the Jordan frame in General Relativity.

Although different conformal frames only correspond to different choices of units, the form of the gravitational action in a particular frame obviously matters. That is, if we for example choose the system of units where the rate of clocks and length of rods remain constant so that our matter Lagrangian is of the canonical form, an action corresponding to $\mathcal{L}_{\text{G},\phi}(g_{\mu\nu}, \phi) + \mathcal{L}_{\text{m}}(g_{\mu\nu}, \psi_{\text{m}})$ will of course give different physics than the action (3.6). Nevertheless, this has been the source of some confusion and we refer to the original debate for details [170–173].

3.2 Cosmology in scalar-tensor theory

Adding a scalar field to the gravity sector introduces a host of novel features relevant for astrophysics and cosmology. For example, gravitational waves in scalar tensor-theories contain monopole radiation whereas in GR the lowest order is a quadrupole [174]. The main departures in cosmology originate in the

¹A more strict definition of the Einstein frame is to say that it is the conformal frame where the action contains a pure Einstein-Hilbert term $\int d^4x \sqrt{-g} R_g$.

modified expansion rate of the Universe. Nucleosynthesis in particular puts strong constraints on the time variation of the effective gravitational coupling $G_{\text{eff}} \equiv G/F(\varphi)$, since a non-standard expansion rate would change the produced amounts of different elements during nucleosynthesis [166].² A modified expansion rate can naturally affect both supernovae measurements and the CMB, but it is also possible for the effects to remain hidden from standard cosmological observations while still having an impact on the dark matter relic abundance [175]. Below, we will review some more general properties and focus on the possibility of obtaining dark energy via extended quintessence.

3.2.1 Equations of motion

The general scalar-tensor theory defined via Eqn. (3.2) gives the following equations of motion in the Jordan frame:

$$F(\varphi)G_{\mu\nu} = 8\pi GT_{\mu\nu} + (\nabla_\mu \nabla_\nu - g_{\mu\nu} \square)F(\varphi) + Z(\varphi) \left(\partial_\mu \varphi \partial_\nu \varphi - \frac{1}{2} g_{\mu\nu} (\partial\varphi)^2 \right) - \frac{1}{2} g_{\mu\nu} U(\varphi), \quad (3.11)$$

$$2Z(\varphi)\square\varphi = \frac{\partial U}{\partial\varphi} - \frac{\partial Z}{\partial\varphi}(\partial\varphi)^2 - \frac{\partial F}{\partial\varphi}R_g, \quad (3.12)$$

where $T_{\mu\nu}$ is the canonical stress-energy tensor and all quantities are defined in terms of the Jordan frame metric $g_{\mu\nu}$, *e.g.* $\square \equiv g^{\mu\nu} \nabla_\mu \nabla_\nu$. The last term in Eqn. (3.12) manifests the scalar field coupling to matter and the Ricci scalar R_g can be replaced by $T \equiv g^{\mu\nu} T_{\mu\nu}$ via the trace of Eqn. (3.11).

Although the stress-energy tensor $T_{\mu\nu}$ has the canonical form in the Jordan frame, the complicated structure of the equations of motion makes it more convenient to study cosmology in the Einstein frame. By employing the conformal transformation $\hat{g}_{\mu\nu} \equiv F(\varphi)g_{\mu\nu}$ and defining

$$\kappa \left(\frac{d\chi}{d\varphi} \right)^2 \equiv \frac{3}{2} \left(\frac{\partial \log F(\varphi)}{\partial\varphi} \right)^2 + \frac{Z(\varphi)}{F(\varphi)}, \quad (3.13)$$

$$2\kappa V(\chi) \equiv \frac{U(\varphi)}{F^2(\varphi)}, \quad (3.14)$$

it is straightforward to show that the action (3.2) together with matter is equivalent to the following form in the Einstein frame:

$$S = \int d^4x \sqrt{-\hat{g}} \left[\frac{1}{2\kappa} R_{\hat{g}} - \frac{1}{2} \hat{g}^{\mu\nu} \partial_\mu \chi \partial_\nu \chi - V(\chi) + \hat{\mathcal{L}}_{\text{m}}(\hat{g}_{\mu\nu}, \chi, \psi_{\text{m}}) \right], \quad (3.15)$$

where the Einstein frame matter Lagrangian is defined via

$$\hat{\mathcal{L}}_{\text{m}}(\hat{g}_{\mu\nu}, \chi, \psi_{\text{m}}) \equiv \frac{\mathcal{L}_{\text{m}}(F^{-1}\hat{g}_{\mu\nu}, \psi_{\text{m}})}{F^2}. \quad (3.16)$$

²Note however that the strength of gravity measured in a Cavendish type experiment is different: $G_{\text{Cav}} = G_{\text{eff}}(1 + \alpha^2)$. The factor α^2 is defined in Eqn. (3.18) and can be interpreted as the exchange of scalar field particles between two bodies.

That is, the Lagrangian reduces to the familiar Einstein-Hilbert form with a canonical scalar field χ . One important difference from GR remains however: the matter Lagrangian in the Einstein frame explicitly couples to χ . Thus, although the above action will yield the regular Friedmann equations in the Einstein frame, the corresponding stress-energy tensor and hence density and pressure, will not only depend on the Einstein frame scale factor, but also on the scalar field χ . Furthermore, the equation of motion for the scalar field will explicitly couple to the trace of the stress-energy tensor since $\hat{\mathcal{L}}_{\text{m}}$ is not constant under variation with respect to χ . Despite these complications, the Einstein frame remains advantageous in many applications and we refer to Paper I for further details.

3.2.2 Extended quintessence

Non-minimally coupled scalar fields as a possible solution to the dark energy problem has been studied to great extent in the literature [I, 57, 58, 113–120, 175–182] (see Ref [17] for a review). In general, the scenario behaves very similar to regular quintessence, exhibiting both tracking and scaling solutions for a suitable choice of the potential. The main advantage of extended quintessence is that it can provide a natural origin for the scalar field, with strong motivation from higher-dimensional theories. The perhaps best developed cosmology along these lines is the SLED scenario [183–190] which we will discuss further in Chapter 4. The scenario was first introduced in the model by Albrecht, Burgess, Ravndal & Skordis [191, 192], and it was also the subject of our study in Paper I.

Although the detailed cosmological evolution in extended quintessence will of course depend on the specific choice of the coupling function and the potential, the overall behaviour remains the same for most models. Fig. 3.1 shows a typical evolution of the energy density in such a scenario. The particular model displayed here corresponds to the scalar-tensor theory explored in Paper I, where we also discuss many of the relevant cosmological constraints. However, in this study we did not include the possible effects on the CMB and there exist at least three characteristic features of non-minimally coupled models in the resulting angular power spectrum [116, 117, 119, 166]. First of all, the non-minimal coupling will lead to a curvature/matter dependent effective potential in the equation of motion for the scalar field. The additive contribution of this term will alter the cosmic equation of state and enhance the Integrated Sachs-Wolfe effect. That is, the amplitude for low multipoles l will be enhanced in the CMB angular power spectrum. Second, since a dark energy model based on extended quintessence will in general expand faster than GR at early times, the Hubble length is smaller in the past so that perturbations enter the horizon at comparatively later times. This will slightly reduce the amplitude of the acoustic oscillations. Finally, since the reduced Hubble length at decoupling corresponds to smaller angular scales, the Doppler peaks in the angular power spectrum will be shifted towards larger multipoles l . Altogether, these features might make

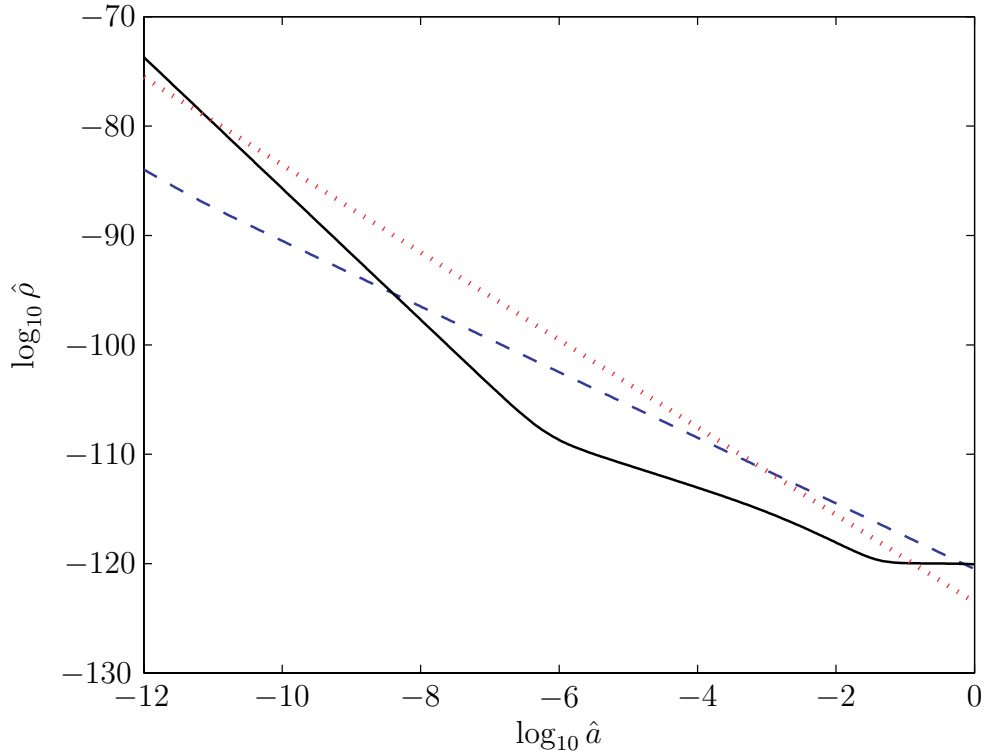


Figure 3.1: Cosmological evolution in the scalar-tensor theory studied in Paper I, where the presence of large extra dimensions leads to extended quintessence and also solves the hierarchy problem. Shown are different components of the Einstein frame energy density $\hat{\rho}$ (in Planck units) as a function of the Einstein frame scale factor \hat{a} (normalized to one at present). The various curves represent pressureless matter (dashed), radiation (dotted) and scalar field energy density (solid). Note that since the conformal factor $\Omega^2 = F(\varphi)$ evolves very little after $\log_{10} \hat{a} \approx -10$ in the above solution, the Einstein frame simply corresponds to a constant scaling of units compared to the Jordan frame for these times [I].

it possible to distinguish coupled dark energy from regular quintessence or a cosmological constant. However, one should remember that quantitatively these effects depend on the particular model. Furthermore, the majority of all current cosmological data is interpreted in the Λ CDM framework, and given an extended theory of gravity not only CMB aspects of cosmology needs to be reevaluated.

3.3 Solar System constraints in scalar-tensor theory

Solar system constraints on scalar-tensor theories are conveniently studied via the equation of motion for the Einstein frame scalar field:

$$\hat{\square}\chi = \frac{\partial V}{\partial \chi} - \sqrt{\kappa/2} \alpha(\chi) \hat{T}, \quad (3.17)$$

where a hat indicates that the quantity is defined in terms of the Einstein frame metric $\hat{g}_{\mu\nu}$, *e.g.* $\hat{T} \equiv \hat{g}^{\mu\nu} \hat{T}_{\mu\nu}$ is the trace of the Einstein frame stress-energy $\sqrt{-\hat{g}} \hat{T}_{\mu\nu} \equiv -2 \delta S_m / \delta \hat{g}^{\mu\nu}$. The scalar field coupling to matter is given by

$$\alpha(\chi) \equiv -\frac{1}{\sqrt{2\kappa}} \frac{d \log F}{d\chi}. \quad (3.18)$$

The impact of the above coupling becomes apparent in the conservation law for stress-energy. While the canonical Jordan frame stress-energy is indeed conserved in scalar-tensor theory, $\nabla_\mu T^\mu_\nu = 0$, the corresponding conservation law does not hold in the Einstein frame: $\hat{\nabla}_\mu \hat{T}^\mu_\nu = \sqrt{\kappa/2} \alpha(\chi) \hat{T} \partial_\nu \chi \neq 0$. This is a direct consequence of the fact that Einstein frame particle masses depend on the scalar field χ .

The coupling α is strongly constrained by Solar System measurements since it is in direct correspondence with the Eddington parameters in the PPN formalism:

$$\begin{aligned} \gamma_{\text{PPN}} &= 1 - \frac{2\alpha^2}{1 + \alpha^2} \\ &\stackrel{*}{=} \frac{1 + \omega}{2 + \omega}, \end{aligned} \quad (3.19)$$

$$\begin{aligned} \beta_{\text{PPN}} &= 1 + \frac{1}{\sqrt{2\kappa}} \frac{\alpha^2}{(1 + \alpha^2)^2} \frac{d\alpha}{d\chi} \\ &\stackrel{*}{=} 1, \end{aligned} \quad (3.20)$$

where the second steps hold for the Jordan-Brans-Dicke field only, for which $\alpha^2 = 1/(3 + 2\omega)$. In other words, the only parameter which is different from GR in JBD theory is γ_{PPN} and we obtain the following constraint from Eqn. (2.29):

$$\frac{1}{\omega} \sim \alpha^2 \lesssim 10^{-5}. \quad (3.21)$$

However, one should note that the above expressions for the Eddington parameters are valid only in the massless limit $V(\chi) = U(\varphi) \equiv 0$, where the “fifth force” associated with the scalar field has infinite range. For a massive field, $m_\chi^2 \sim \partial^2 V / \partial \chi^2$, the effect of the scalar field coupling to matter is effectively cut off at distances $\gtrsim 1/m_\chi$ and the above bound may no longer be applicable.

Furthermore, it is also possible for the scalar field to gain additional mass via an effective potential arising from the local matter distribution; the chameleon effect [121–123]. In such a case, one needs to obtain at least an approximate solution to the full field equations in order to determine if the theory is compatible with Solar System constraints.

Note that the modified source term for the 00 component of the Einstein tensor in the Jordan frame, Eqn. (3.11), indeed leads to a non-standard relation between the gravitational mass and the density profile of a star in scalar-tensor theory. In other words, the perceived gravitational mass is in general different from the total baryonic rest mass also in the Newtonian limit. It should nevertheless be stressed that scalar-tensor theories do respect the equivalence principle, the above effect is just a consequence of the fact that the strength of gravity is environment dependent.

3.4 Concluding remarks

This chapter reviewed some basic properties of scalar-tensor theory and how they relate to the type of cosmologies studied in this thesis. In particular, we stressed the equivalence of different conformal frames and how conformal transformation provides a powerful tool for simplifying the equations of motion. This was especially useful for our study in Paper I. We also discussed the impact of extended quintessence models on the cosmic microwave background and finally reviewed Solar System constraints in terms of the PPN parameters.

As mentioned, scalar-tensor theories typically arise in the context of extra dimensions and we will discuss such scenarios further in the next chapter. In the final chapter of this thesis, we will also see that the Jordan-Brans-Dicke theory is mathematically equivalent to $f(R)$ gravity.

Chapter 4

Brane-worlds, the hierarchy problem, and dark energy

The idea that we live on a brane (corresponding to regular four-dimensional space-time) in a higher-dimensional bulk space dates back to the pioneering works of Nordström, Kaluza and Klein [193–196]. In Kaluza-Klein theory one considers General Relativity living in five dimensions where the extra spatial dimension is small and compact. For the effective four-dimensional theory, the additional degrees of freedom in the 5D metric will yield Maxwell’s electromagnetic field and a scalar field corresponding to the size of the extra dimension. The scalar field, commonly known as the radion, is non-minimally coupled and Kaluza-Klein theory provides the prototypical scenario for how extra dimensions may lead to a scalar-tensor theory in four dimensions.

In the modern context, standard model fields are often confined to the four-dimensional brane while gravity (representing the dynamics of spacetime itself) propagates also in the higher-dimensional bulk. These scenarios arise mainly in the context of supergravity and string theory, and brane-worlds have become one of the most active areas of research in modern high energy physics. A very famous example is the Randall-Sundrum model [197, 198] where our brane is embedded in a five-dimensional anti de Sitter bulk. By warping the extra dimension it is possible to create the observed high energy scale of gravity ($\sim M_{\text{Pl}}$) on our brane while the true gravity scale is much lower in the bulk. A review of the present status of brane-world gravity is certainly beyond the scope of this thesis however, and we refer to Refs. [199, 200] for an overview of the subject. Below, we will review some of the more basic concepts which are relevant for our work in Paper I. In particular we will focus on large extra dimensions as a possible solution for the hierarchy problem.

4.1 The hierarchy problem

There seems to exist at least two fundamental scales in nature. The electroweak scale $m_{\text{EW}} \sim 100$ GeV, where particles become massive, and the Planck scale

$M_{\text{Pl}} \sim 10^{18}$ GeV, where gravity has the same strength as the other fundamental interactions. That is, M_{Pl} is the maximum energy scale where the standard model of particle physics could apply. The hierarchy problem arises due to the presence of a scalar field in the standard model, the Higgs boson. Quantum corrections to a scalar mass diverge and therefore the Higgs boson should have mass at the natural cutoff of the theory, M_{Pl} . This is why a fundamental scale much higher than the electroweak scale is problematic.

Explaining the hierarchy between the above scales, *i.e.* why $m_{\text{EW}}/M_{\text{Pl}} \sim 10^{-16}$, has been perhaps the greatest motivation for exploring physics beyond the standard model. The most common way to solve the problem is by postulating supersymmetry (SUSY), where each standard model particle has a superpartner with spin that differs by half a unit. Supersymmetry may solve the hierarchy problem because the non-logarithmic quantum corrections to the scalar mass no longer diverge due to cancellations between fermionic and bosonic Higgs interactions. However, it is only a solution if supersymmetry is broken close to the electroweak scale.

It is worth noticing that while the electroweak interaction has been probed at distances $\sim 1/m_{\text{EW}}$, gravity has not been tested nowhere near $\sim 1/M_{\text{Pl}}$; in fact, Newtonian gravity has only been examined down to about 0.1 mm. Hence, interpreting M_{Pl} as the fundamental scale of gravity is based on the assumption that the interaction remains unchanged for the 30 orders of magnitude down to $1/M_{\text{Pl}} \sim 10^{-31}$ mm.

4.2 Large extra dimensions

Arkani-Hamed *et al.* [201–203] have proposed a solution to the hierarchy problem using large extra dimensions. In this picture gravity is effectively much weaker than the other forces because it gets diluted when propagating in the extra dimensions, and the approach is totally different from considering a new effective field theory being responsible for a connection between the scales. Instead, the problem is solved by postulating that m_{EW} is the only fundamental scale in nature, setting the scale for gravitational interactions as well. This gives a trivial solution to why the electroweak scale is in fact ~ 100 GeV, since this scale now becomes the natural cut-off for the theory.

Suppose there exist n compactified extra dimensions of radius $\sim R$. Further, let all standard model fields be confined to the usual $(3 + 1)$ -dimensional brane, while the graviton is propagating freely in $(4 + n)$ dimensions. Now, the gravitational potential $V(r)$ for two test masses m_1, m_2 follows from Gauss law in $(4 + n)$ dimensions [201]:

$$V(r) \propto \frac{m_1 m_2}{M^{2+n}} \frac{1}{r^{1+n}} \quad \text{for } r \ll R, \quad (4.1)$$

where M is the $(4 + n)$ -dimensional “Planck mass”, *i.e.* the true scale of gravity. However, if the masses are separated by a distance $r \gg R$, their gravitational

flux lines will effectively no longer penetrate the extra dimensions and we obtain the usual, apparently much stronger $1/r$ dependence

$$V(r) \propto \frac{m_1 m_2}{M^{2+n} R^n} \frac{1}{r} \quad \text{for } r \gg R. \quad (4.2)$$

Hence, comparing with the traditional $V(r) \propto m_1 m_2 / (M_{\text{Pl}}^2 r)$ yields the following expression for the effective four-dimensional Planck mass:

$$M_{\text{Pl}}^2 \sim M^{2+n} R^n. \quad (4.3)$$

By assuming that the only fundamental scale in the $(4+n)$ -dimensional theory is roughly the electroweak scale, *i.e.* $M \sim 1$ TeV, one receives the following solution for the radius

$$R \sim 10^{\frac{30}{n}-17} \text{ cm} \times \left(\frac{1 \text{ TeV}}{M} \right)^{1+\frac{2}{n}}. \quad (4.4)$$

From the above relation we see that a single extra dimension is ruled out immediately, since the corresponding radius, $R \sim 10^{13}$ cm, would imply deviations of Newtonian gravity at solar system scales, which obviously is not the case. Nevertheless, for $n \geq 2$ one enters yet unfamiliar territory and the case $n = 2$ is particularly interesting. While a fundamental scale at exactly 1 TeV yields a slightly too large radius for the extra dimensions, an $M \sim 10$ TeV have a corresponding radius $R \sim 1\text{--}10$ μm which is not yet excluded. However, upcoming gravity experiments may soon probe these scales.

The simple scheme above is remarkably successful. By providing a natural cut-off for the electroweak interaction one simultaneously explain the order of the Planck mass M_{Pl} . The observed gravitational strength is just an effective coupling stemming from the natural scale witnessed in $(4+n)$ dimensions. Since particle physics have been probed down to distance scales $\sim 1/m_{\text{EW}} \sim 10^{-16}$ cm, it appears very likely that standard model fields indeed are confined to a $(3+1)$ -dimensional brane (unless of course the number of extra dimensions is very large, $n \gtrsim 30$). However, to realize this framework in terms of localizing the standard model fields onto the brane is a different question. One possibility is to embed the framework in string theory. Additionally, one needs to stabilize the extra dimensions at the desired scale. These issues have been discussed further in for example Refs. [201–206].

4.3 Dark energy and (S)LED

The hierarchy problem and the stabilization of large extra dimensions (LED) has also been considered in a six-dimensional brane-world scenario by Albrecht, Burgess, Ravndal & Skordis [191,192]. Here, the presence of a bulk scalar field with cubic self-interaction (which is renormalizable in 6D) induces a Casimir

potential for the radion. Furthermore, the bulk scalar also induces logarithmic corrections to both the kinetic terms and the potential, which are crucial for the stabilization mechanism. The scale of physics on the brane is imagined to be $M_b \sim 1$ TeV, while the size of the compactified dimensions r can be as large as ~ 1 mm.¹

The corresponding effective four-dimensional action in the LED scenario is given by the following form in the Jordan frame [I, 192]:

$$S_{\text{LED}} = \int d^4x \sqrt{-g} \left[\frac{M_b^2 (M_b r)^2}{2} \left(A(r) R_g + 2B(r) \left(\frac{\partial r}{r} \right)^2 \right) - C(r) \frac{U_0}{r^4} + \mathcal{L}_m(g_{\mu\nu}, \psi_m) \right], \quad (4.5)$$

where U_0 is a dimensionless constant on the order of one and

$$A(r) \approx 1 + a \log M_b r, \quad (4.6)$$

$$B(r) \approx 1 + b \log M_b r, \quad (4.7)$$

$$C(r) \approx 1 + c \log M_b r, \quad (4.8)$$

represent the logarithmic corrections coming from the bulk scalar field. Here a , b , and c are small parameters ($\ll 1$) proportional to the dimensionless coupling constant of the bulk scalar field. In the original scenario, the stabilization of the compactified dimensions is based on the observation that large extra dimensions are obtainable if the potential restraining their radius has a logarithmic correction. This is possible since that for a negative value of c the potential of the radion develops a minimum at exponentially large distances: $r \sim M_b^{-1} \exp(1/|c|)$.

Remarkably, the induced potential will not only stabilize the extra dimensions but it will also be of a form suitable for addressing the dark energy problem. This possibility becomes more apparent if we introduce a new scalar field

$$\varphi \equiv \log M_b r, \quad (4.9)$$

giving the following form of the action (4.5):

$$S_{\text{LED}} = \int d^4x \sqrt{-g} \left[\frac{M_b^2 e^{2\varphi}}{2} \left(A(\varphi) R_g + 2B(\varphi) (\partial\varphi)^2 \right) - M_b^4 U_0 C(\varphi) e^{-4\varphi} + \mathcal{L}_m(g_{\mu\nu}, \psi_m) \right], \quad (4.10)$$

where we have redefined the functions A , B , and C according to $A(\varphi) \equiv 1 + a\varphi$ etc. The above action is indeed of the same form as the general scalar-tensor

¹Note that extra dimensions with a size of 1 mm were not yet excluded when this model was proposed.

theory in Eqn. (3.2) with $F(\varphi) = A(\varphi)e^{2\varphi}$, $Z(\varphi) = -2B(\varphi)e^{2\varphi}$, and $U(\varphi) = 2M_b^2 U_0 C(\varphi)e^{-4\varphi}$. Furthermore, the presence of an exponential potential for the scalar field makes it an ideal candidate for extended quintessence. In particular, the value of $M_b r \sim 10^{15}$ needed in order to produce the observed gravitational strength will naturally yield the correct magnitude of the potential, without the presence of a small energy scale associated with the critical energy density. A typical cosmological solution in this scenario was indeed shown in Fig. 3.1 where the model gives rise to an accelerating expansion at present via a quintessence-like behaviour. As will be discussed in Paper I, it is also possible for the model to give rise to scenarios similar to k -essence and possibly phantom dark energy.

The Solar System constraints on the scalar field coupling to matter result in a strong constraints on the function $A(\varphi)$. Roughly speaking, $A(\varphi) \ll 1$ which requires that $a\varphi \approx -1$. Remarkably, this requirement will also give rise to a new stabilization mechanism for the radius of the extra dimensions. This becomes clear when we consider the potential of the scalar field φ in the Einstein frame:

$$\hat{U}(\varphi) = \frac{U(\varphi)}{F^2(\varphi)} = M_b^4 U_0 \frac{1 + c\varphi}{(1 + a\varphi)^2} e^{-8\varphi}. \quad (4.11)$$

Now, as the scalar field rolls down the potential and gets closer to the value required by the Solar System constraints, $a\varphi \approx -1$, it will also get closer and closer to the singularity in the potential at $\varphi = -1/a$. That is, the potential gets steeper and steeper and it will in practise be impossible for the field pass this point. Indeed, as was shown in Paper I, even if one shoots the scalar field with a huge amount of kinetic energy, the potential will act as an unsurmountable barrier confining the field. That is, the radion cannot evolve past $M_b r \sim 10^{15}$. Fig. 4.1 shows the corresponding potential for the canonically scaled scalar field χ in the Einstein frame, obtained via the definitions in Eqn. (3.13) and Eqn. (3.14). Note that since the scale in φ is very strongly condensed with respect to χ near the singularity, the hill becomes extremely steep in the φ picture. We refer to Paper I for further details.

The Albrecht *et al.* scenario has since received considerable further development by Burgess *et al.* [183–190]. In this improved version, all fields in the bulk are supersymmetric and the corresponding physics is hence described by 6D supergravity. Supersymmetry is of course broken on our brane however. The size of the extra dimensions are imagined to be $\sim 10 \mu\text{m}$ (in good agreement with current constraints, see below), setting the scale of 6D gravitational physics to $\sim 10 \text{ TeV}$. The presence of supersymmetric large extra dimensions (SLED) provides a solid foundation for studying the naturalness issues, and many aspects of the scenario have been studied in detail (see references in Ref. [190] for further details). In particular, the SLED scenario does not set the effective 4D cosmological constant to zero, but *predicts* that it vanishes at tree level with small corrections $\sim \rho_{\text{crit}}/\kappa$. As a consequence, the resulting scalar-tensor theory is of the same form as in the LED scenario above, which is known to give rise to realistic forms of extended quintessence. Finally, the size of the extra

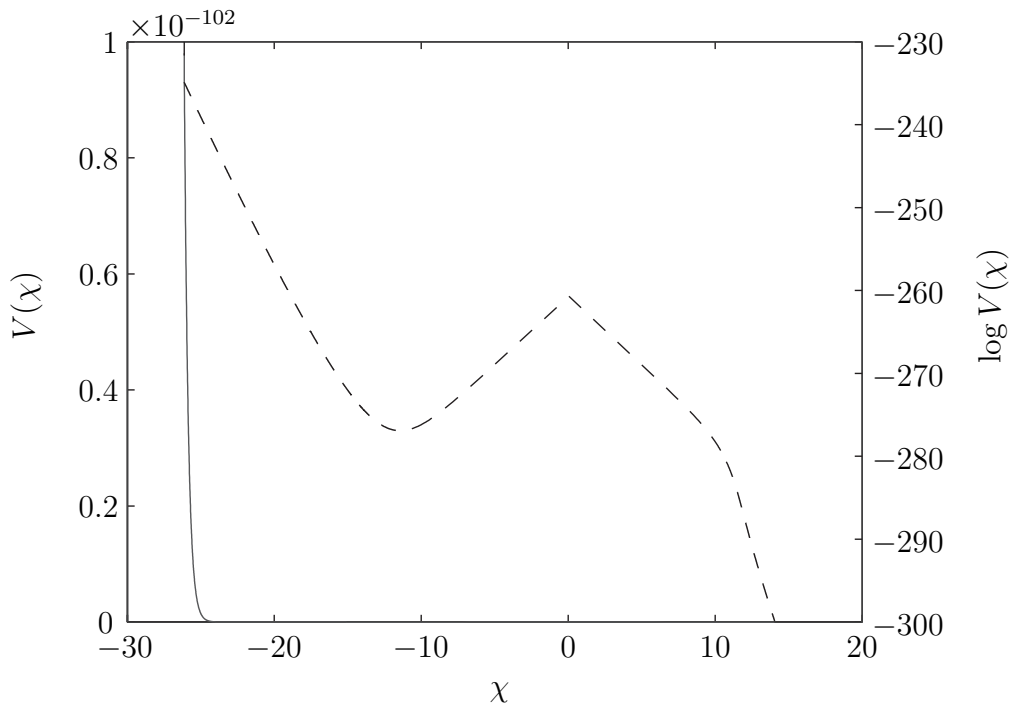


Figure 4.1: The Einstein frame potential $V(\chi)$ as a function of χ , both measured in units of Planck mass. The dashed line represents $\log V(\chi)$ which clearly shows the minimum stabilizing the radius of the extra dimensions. Note that this minimum arises solely from the $A(\varphi)$ correction and that the correction coming from $C(\varphi)$ in the potential (4.11) leaves no significant features in the above plot.

dimensions in the SLED model must be very large, which makes the scenario unusually predictive for future precision tests of gravity and the upcoming high energy experiments at the Large Hadron Collider (LHC).

4.4 Observational constraints on large extra dimensions

Let us finally review the possible ways to constrain the size of large extra dimensions. The obvious way is to make precise measurements of gravity at small distances. Such experiments have improved considerably in recent years and high precision measurements of Newton's square law constrains the size of large extra dimensions to $\lesssim 50 \mu\text{m}$ [207] (see also Ref. [189]).

A complementary way to obtain constraints is via cosmology and astrophysical processes. For each extra dimension, there exist a number of excitations of the graviton, Kaluza-Klein (KK) modes, corresponding to available phase-space in the bulk. A physical process involving the possible emission of a graviton could therefore show significant differences from the standard picture. Nevertheless,

processes involving KK modes are often highly model dependent. Therefore, we will not discuss actual constraints, but instead focus on the ideas behind them. In summary, the obtained upper bounds on the size of the extra dimensions are typically $\sim 100 \mu\text{m}$ and we refer to Ref. [30] for further details.

Cosmological constraints. Cosmology provides several ways to constrain theories with extra dimensions. For example, the successful prediction of light element abundances requires that the expansion rate is very close to the standard one during nucleosynthesis. The extra dimensions must then be stabilized at this time so that the strength of gravity is truly constant.

At early times, the energies are also high enough for creating KK modes in the heat bath. A large number of KK modes are in general produced during reheating after inflation, and these can easily become the dominating component if the extra dimensions are too large.

Constraints from supernovae. Supernovae can also be used to place limits on the size of extra dimensions. Normally, most of the energy from a type II supernova core is carried away by neutrinos, but in the presence of extra dimensions the energy can also be transported away by the KK modes. This picture makes it possible to constrain the size of extra dimensions by using the observed neutrino flux from SN 1987a [208, 209].

Neutron star limits. In connection with supernovae observations, constraints can also be obtained from gamma ray limits for nearby young supernovae remnants and neutron stars. In this case the KK modes are created with relatively small velocities and hence form a halo around the neutron star. An observable flux of gamma rays should then be emanated from neutron stars as these states decay into photons. At present, the EGRET satellite measurements provide limits on the gamma ray flux, but the upcoming GLAST satellite may also detect the actual decay of KK modes.

4.5 Concluding remarks

The possible existence of large extra dimensions offers a very attractive solution to the hierarchy problem. What makes them of interest here is that extra dimensions might naturally give rise to a scalar-tensor theory with suitable properties for a realistic extended quintessence scenario. Although there are a number of observational constraints on extra-dimensional models, they tend to be somewhat model dependent. Moreover, these constraints restrict the upper limit on the scale of the extra dimension which does limit the utility of large extra dimensions as means to cure the hierarchy problem, but not so much their use in obtaining an effective scalar-tensor theory of extended quintessence, which is our main motivation here.

Chapter 5

$f(R)$ gravity

Non-linear modifications to the Einstein-Hilbert action have a long history [210, 211] and have been of interest for a variety of reasons. They are for example encountered in loop calculations in General Relativity (when considered as an effective field theory) [212] and arise in effective actions derived from string theory [213, 214]. More recently, it was discovered that a suitable modification of the Einstein-Hilbert action can result in an accelerating expansion at present without the need for a cosmological constant [215–219]. Although these theories offer no insight to why the pure cosmological constant should vanish, they provide a possible dynamical explanation for the dark energy problem and the scenario has garnered much attention, with several hundred articles appearing during the last few years.

The non-linear theories of gravitation considered in this thesis take the following form in the Jordan frame:

$$\begin{aligned} S &= S_f + S_m \\ &= \int d^4x \sqrt{-g} \left[\frac{1}{2\kappa} f(R) + \mathcal{L}_m(g_{\mu\nu}, \psi_m) \right], \end{aligned} \quad (5.1)$$

which reduce to the familiar Einstein-Hilbert action for $f(R) = R - 2\Lambda$. The underlying idea is that if the function $f(R)$ modifies the behaviour of gravity in the infrared, *i.e.* in the low curvature regime at late times, the above action may provide a possible explanation for the dark energy problem. A simple extension along these lines is the modification presented in the original models [217, 218]: $f(R) - R = -\mu^4/R$ where $\mu^2 \sim \Lambda$ in order to explain the observed acceleration at present.

A non-linear term $f(R)$ in the gravitational Lagrangian introduces an ambiguity in the variation of the action. In GR, one assumes that the affine connection of the spacetime manifold is given by the Levi-Civita connection. This can be justified by the fact that if one varies the Einstein-Hilbert action with the metric $g_{\mu\nu}$ and the affine connection $\Gamma_{\mu\nu}^\rho$ as independent variables, the resulting equation of motion for the affine connection is just the metric compatibility equation. In other words, the affine connection will coincide with the Levi-Civita

connection. However, in $f(R)$ gravity this is no longer the case and one has to make an *a priori* choice of which variational principle to use. Thus, one speaks of metric $f(R)$ gravity where the affine connection is fixed by hand to the Levi-Civita connection, and Palatini $f(R)$ gravity where the affine connection is kept as an independent variable. These two different variational principles will indeed give different equations of motion and thus represent two separate theories. The main difference is that metric $f(R)$ gravity is a fourth order theory while Palatini $f(R)$ gravity remains second order. However, it should be noted that the equations of motion in both metric and Palatini $f(R)$ gravity imply the canonical conservation law for stress-energy in the Jordan frame, $\nabla_\mu T^{\mu\nu} = 0$ [220], where the covariant derivative is indeed defined in terms of the Levi-Civita connection $\{\overset{\rho}{\underset{\mu\nu}{\Gamma}}\}$.

5.1 Metric $f(R)$ gravity

By fixing the affine connection of the spacetime manifold to the Levi-Civita connection,

$$\Gamma_{\mu\nu}^\rho \equiv \{\overset{\rho}{\underset{\mu\nu}{\Gamma}}\} = \frac{1}{2}g^{\rho\sigma}(\partial_\mu g_{\sigma\nu} + \partial_\nu g_{\sigma\mu} - \partial_\sigma g_{\mu\nu}), \quad (5.2)$$

and varying the action (5.1) with respect to $g_{\mu\nu}$, we obtain the equations of motion for metric $f(R)$ gravity:

$$F(R)R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}f(R) - (\nabla_\mu \nabla_\nu + g_{\mu\nu}\square)F(R) = 8\pi GT_{\mu\nu}, \quad (5.3)$$

where $F(R) \equiv \partial f / \partial R$ and all covariant derivatives are defined in terms of the Levi-Civita connection. Since the Ricci scalar contains second order derivatives of $g_{\mu\nu}$, the gradient terms in Eqn. (5.3) will yield fourth order derivatives of the metric. The fact that metric $f(R)$ gravity is a fourth order theory makes it hard to determine the appropriate boundary condition for a given problem, which in particular makes the study of Solar System constraints fairly complicated. The main problem is that Birkhoff's theorem does not hold in metric $f(R)$ gravity so that the spherically symmetric vacuum solution is no longer unique. Furthermore, it is not necessarily static and many configurations exhibit a violent time instability due to the higher order derivatives in the equations of motion [221]. It is nevertheless clear that Eqn. (5.3) does reduce to the familiar second order Einstein equations if (and only if) $f(R) = R - 2\Lambda$.

The possibility for metric $f(R)$ gravity to explain the accelerating expansion stems from the observation that the relaxed vacuum solution to the above equations is of de Sitter (dS) type. That is, for $\nabla_\mu F = 0$ the equations of motion (5.3) reduce to the Einstein equations with an effective cosmological constant λ :

$$G_{\mu\nu} = -g_{\mu\nu} \frac{1}{2} \left(R - \frac{f}{F} \right) \equiv -g_{\mu\nu} \lambda(R). \quad (5.4)$$

Hence, if the trace of the above equation, $F(R)R - 2f(R) = 0$, has a nonzero positive root $R_0 = 4\lambda(R_0) \equiv 4\Lambda_0$, spacetime will have an inherent positive curvature which accelerates the expansion of the Universe. Note that for a negative root $R_0 < 0$, space-time is instead of anti de Sitter (AdS) type.

5.1.1 Cosmology in metric $f(R)$ gravity

Non-linear corrections to the gravitational Lagrangian as a source of acceleration was first discussed in the context of inflation [222–224]. In these scenarios one considered a conformal term R^2 driving the acceleration at high curvatures. Similar mechanisms has also been incorporated in some more recent models which try to address both inflation and the dark energy problem by choosing a function $f(R)$, with corrections becoming important at both high and low curvatures [225–227].

A vast number of models for late time acceleration in metric $f(R)$ gravity have been suggested and studied in the literature [215–217, 219, 226–249, 268]. As discussed above, the origin of the acceleration can be thought of as due to an inherent positive curvature R_0 of spacetime. This picture is nevertheless somewhat misleading since the effective equation of state is not necessarily equal to -1 in an $f(R)$ gravity theory. For example, by assuming that the evolution of the scale factor is given by a power law $a \propto t^m$, one can show that the effective equation of state is given by the following expression in a $f(R) - R = -\mu^{2(n+1)}/R^n$ model ($n \geq 1$) [217]:

$$w_{\text{DE}} = -1 + \frac{2(n+2)}{3(2n+1)(n+1)}, \quad (5.5)$$

giving $w_{\text{DE}} = -2/3$ for $n = 1$. In reality, the dark energy equation of state will of course be time-dependent since the scale factor will have a more general evolution, but the power-law remains a good approximation during a specific era. Furthermore, the above example clearly shows that the effective equation of state will in general differ from that of a pure cosmological constant, and it tells us that the simple original scenario with $n = 1$ is ruled out by constraints on w_{DE} (see Eqn. (2.26)). The constraints on w_{DE} in metric $f(R)$ gravity have been further explored in for example Ref. [250]. Here it was shown that the present values of $f(R)$ and its derivatives $\partial^n f / \partial R^n$ with $n \leq 3$ can be related to the present values of the Hubble rate, the deceleration q , and the jerk, snap and lerk parameters containing the third, fourth and fifth time derivative of the scale factor, respectively. This shows that it is in principle possible to distinguish metric $f(R)$ gravity from a pure cosmological constant even if the scenario predicts an equation of state very close to -1 . Present observational constraints are nevertheless much too weak to discriminate between any such features. Note that the expansion history alone will not determine the function $f(R)$ in metric $f(R)$ gravity, since the fourth order nature of the field equations

requires time derivatives up to order $n+2$ of the scale factor in order to determine $\partial^n f / \partial R^n$.

A generic difficulty with studying cosmology in metric $f(R)$ gravity is that most constraints turn out to be model dependent. However, there exists at least one particular obstacle that all models need to overcome: there must exist a standard matter dominated epoch prior to the late time accelerating era. This may seem like a trivial demand but it has been shown that a large class of models fail this basic requirement, since their scale factor will grow as $t^{1/2}$ instead of the standard law $t^{2/3}$ during the matter dominated phase [236, 238, 244]. Such a behaviour is grossly inconsistent with CMB data and excluded models include for example all functions $f(R) - R = \alpha R^n$ where $n > 1$ or $n < 0$. It should nevertheless be noted that the non-canonical evolution occurs simply due to the fact that we require an accelerating expansion at present. The excluded models can indeed give rise to the standard evolution $t^{2/3}$, but in this case the solution corresponds to a stable point in the phase space so that it will never give away to acceleration [238].

More general properties of cosmology in metric $f(R)$ gravity have been considered in for example Refs. [251, 252]. A recent paper has also pointed out that there exists a curvature singularity appearing at the non-linear level in metric $f(R)$ gravity, which may plague any model which modify Einstein gravity in the infrared [253]. The impact of this discovery on the viability of metric $f(R)$ models is yet to be determined however. In summary, most simple metric $f(R)$ models fail the cosmological tests, but more complicated models have been suggested which claim to obey all observational constraints [226, 227, 239, 245, 247, 249]. However, as we will discuss in the next section, it appears that these scenarios face a naturalness problem when placed under close examination in the Solar System.

5.1.2 Solar System constraints in metric $f(R)$ gravity

Solar System constraints have been a source of great debate in metric $f(R)$ gravity. An early paper by Chiba [254] used the fact that metric $f(R)$ gravity is equivalent to a Jordan-Brans-Dicke theory with $\omega = 0$, giving $\gamma_{\text{PPN}} = 1/2$ which is certainly ruled out by observations. However, this argument assumes that the corresponding potential is negligible and many authors have argued both for [III, 255–262] and against [226, 227, 247, 263–269] Chiba’s result. The ambiguity stems from the fourth order nature of the field equations. As mentioned, the vacuum solution is no longer unique so that even though the Schwarzschild-dS metric is a solution to the field equations, there is no guarantee that it is the exterior solution obtained in the Solar System. On the contrary, matching with the interior of the Sun typically gives a solution corresponding to $\gamma_{\text{PPN}} = 1/2$ [III, IV, 257, 260] and we refer to Paper III for further details. Fig. 5.1 shows a typical solution for the metric components g_{00} and g_{11} in the Solar System and the corresponding value of γ_{PPN} is shown in Fig. 5.2.

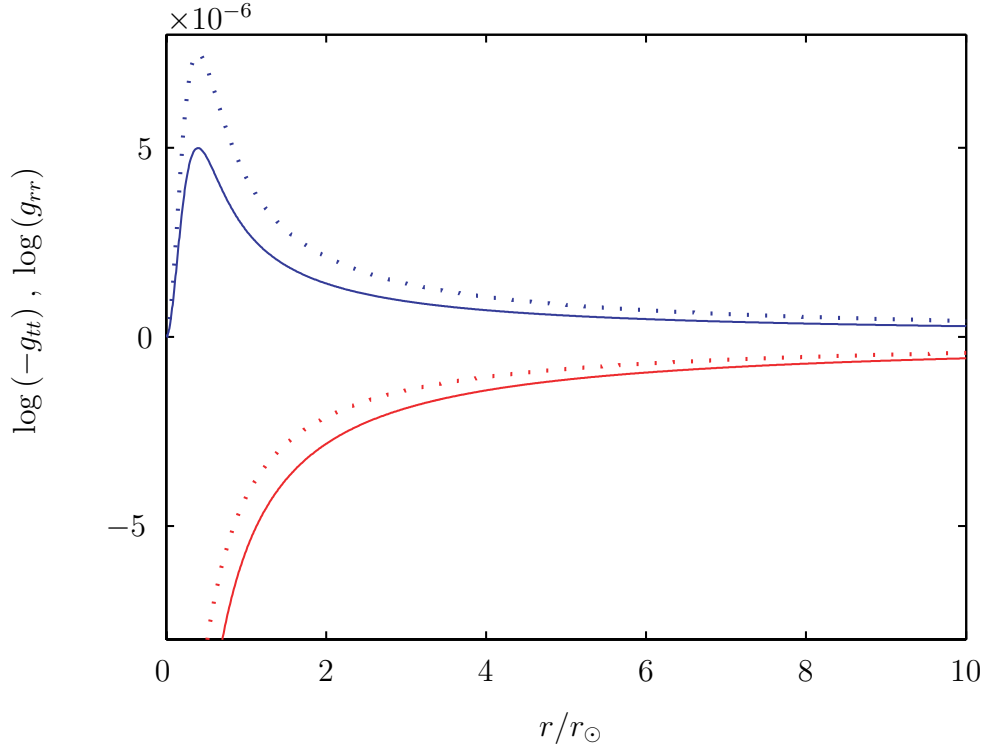


Figure 5.1: Shown are the temporal (lower curves) and radial (upper curves) components of $g_{\mu\nu}$ in the Solar System for the metric $f(R) = R - \mu^4/R$ model (solid) and for GR (dotted).

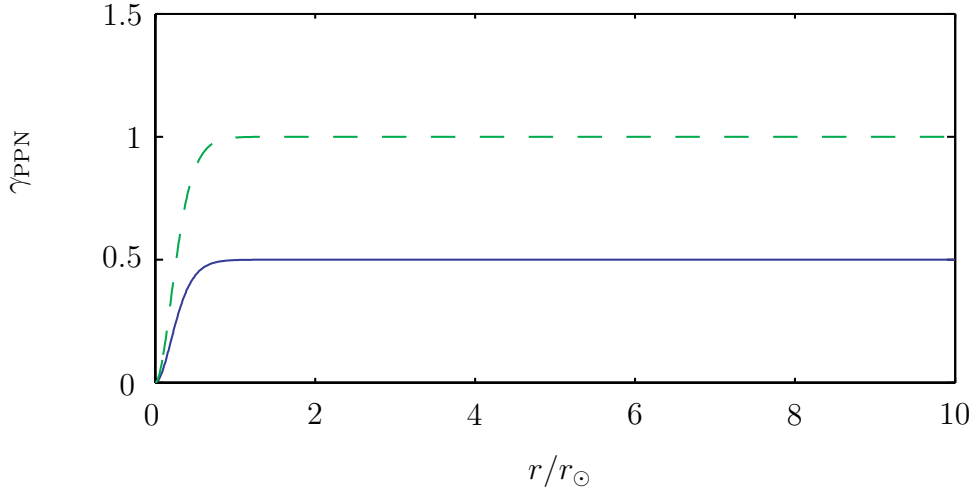


Figure 5.2: Shown is a typical solution for γ_{PPN} in metric $f(R)$ gravity (solid) and the corresponding solution in GR (dashed).

As was emphasized in Paper III, the exterior solution does however depend on the higher order boundary conditions set at the center of the star. While a large class of boundary conditions lead to solutions indistinguishable from the one shown in Fig. 5.1 and Fig. 5.2, there exists a small region of boundary conditions leading to $\gamma_{\text{PPN}} = 1$. For the models studied in Paper III, these solutions are nevertheless ruled out since they turn out to either be unstable in time or produce a much too weak gravitational field due to the non-standard relation between the gravitational mass and the density profile.

There exists a well-known time instability in metric $f(R)$ gravity, commonly referred to as the Dolgov-Kawasaki instability. In summary, perturbing around a static solution for the Ricci scalar in the weak field limit, $R(r) \rightarrow R(r) + \delta R(r, t)$, and expanding the field equations (5.3) to first order in small quantities, gives the following equation for a perturbation around a constant curvature background:

$$(\partial_t^2 - \vec{\nabla}^2)\delta R = -m_R^2\delta R, \quad (5.6)$$

where the parameter m_R^2 depends on the background curvature R only. Expanding in Fourier modes $\delta R_k(\vec{k}, t)$, we find the time dependence

$$\delta R_k(\vec{k}, t) \sim e^{\pm i\sqrt{k^2 + m_R^2}t}, \quad (5.7)$$

so that if $m_R^2 < 0$, all modes with $k < |m_R|$ are unstable. One can show that in order to obtain a solution with $\gamma_{\text{PPN}} \approx 1$, one needs $R \sim \kappa\rho$ [IV]. This typically gives a mass parameter $|m_R|^{-1} \ll 1$ second so that the system is violently unstable for a negative mass squared. This indeed turns out to be the case for the original scenario [217] and we refer to Paper IV for a detailed discussion of the stability issues in metric $f(R)$ gravity.

More complicated $f(R)$ functions has also been suggested which claim to be compatible with the Solar System constraints [226, 227, 247, 269]. While these models can indeed exhibit a stable solution with $\gamma_{\text{PPN}} = 1$, they do not elaborate on how such a configuration will be obtained. As discussed in Paper IV, a possible way to address this problem is to study the collapse of a protostellar dust cloud in order to determine which configurations are natural. While a full dynamical computation was beyond the scope of this paper, we studied how special the GR-like solutions are in the phase space. It turns out that the domain of GR-like solutions typically shrinks to a point in the phase space, surrounded by a continuum of equally acceptable solutions but with observationally excluded values of γ_{PPN} . Unless a physical reason to prefer such a particular configuration can be found, this poses a serious naturalness problem for the currently known metric $f(R)$ models for accelerating expansion of the Universe [IV].

5.2 Palatini $f(R)$ gravity

In Palatini $f(R)$ gravity one considers both the metric $g_{\mu\nu}$ and the affine connection $\Gamma_{\mu\nu}^\rho$ as independent variables. Since the Riemann tensor is defined solely

in terms of the affine connection, the Ricci scalar becomes a composite object in these theories, $R(g, \Gamma) \equiv g^{\mu\nu} R_{\mu\nu}(\Gamma) \equiv g^{\mu\nu} R_{\mu\sigma\nu}^{\sigma}(\Gamma)$. Varying the action (5.1) with respect to both $g_{\mu\nu}$ and $\Gamma_{\mu\nu}^{\rho}$ results in the equations of motion for Palatini $f(R)$ gravity:

$$F(\bar{R})\bar{R}_{\mu\nu} - \frac{1}{2}g_{\mu\nu}f(\bar{R}) = 8\pi GT_{\mu\nu}, \quad (5.8)$$

$$\bar{\nabla}_{\rho}(\sqrt{-g}F(\bar{R})g^{\mu\nu}) = 0, \quad (5.9)$$

where a bar reminds us that an object is indeed defined in terms of the affine connection and hence differs from the corresponding quantity in metric $f(R)$ gravity.

When deriving the above equations of motion we implicitly assumed that the Riemann tensor is torsionless and that the matter Lagrangian is independent of $\Gamma_{\mu\nu}^{\rho}$. The first of these assumptions is also made in General Relativity and it is in good agreement with the observational constraints on torsion [270]. The second assumption holds true for many applications in astrophysics and cosmology, where one often only considers perfect fluids, scalar fields, and the electromagnetic field. In these cases the matter Lagrangian will indeed only depend on $g_{\mu\nu}$ in addition to the matter fields. However, if one for example includes particles with spin, one either needs to make an *a priori* assumption that the matter Lagrangian depends on the Levi-Civita connection only, or face a more complicated theory. This issue has been discussed further in Refs. [173, 271, 272].

It is clear from the equation of motion for the affine connection, Eqn. (5.9), reduces to the metric compatibility equation for $g_{\mu\nu}$ if $F(\bar{R}) = \text{const.}$ In this case, and only in this case does the free affine connection reduce to the usual Levi-Civita connection, Eqn. (5.2). Now, by taking the trace of the equation of motion for the metric, Eqn. (5.8), we immediately see that the Ricci scalar is completely determined by the trace of the stress-energy tensor:

$$F(\bar{R})\bar{R} - 2f(\bar{R}) = 8\pi GT. \quad (5.10)$$

Hence, if T is constant so is \bar{R} and $F(\bar{R})$. As a result, the full field equations in vacuum reduce to the Einstein equations with a cosmological constant:

$$\bar{G}_{\mu\nu} = G_{\mu\nu} = -g_{\mu\nu}\Lambda_0, \quad (5.11)$$

where $\Lambda_0 \equiv \lambda(\bar{R}_0) = \lambda(R_0)$. Note that since Eqn. (5.9) now implies that the affine connection is given by Eqn. (5.2), both the Ricci scalar and the Einstein tensor will indeed depend on $g_{\mu\nu}$ only. The above reduced form of the field equations once again implies that the vacuum solution is of de Sitter type if the trace equation (5.10) has a nonzero positive root \bar{R}_0 . This makes the mechanism for obtaining an accelerating expansion very similar to the one in metric $f(R)$ gravity. However, there is an important difference. In metric $f(R)$ gravity the vacuum solution is less restricted since the Ricci scalar R corresponds to an additional degree of freedom, and the familiar Einstein equations are only

obtained in the relaxed vacuum limit $\nabla_\mu F \rightarrow 0$. In Palatini $f(R)$ gravity the second order nature of the field equations guarantees that \bar{R} is always completely determined by the trace of the stress-energy T and the vacuum equations always reduce to Eqn. (5.11). In particular, the spherically symmetric vacuum solution is unique and given by the Schwarzschild-dS metric (or the Schwarzschild-AdS metric for $\bar{R}_0 < 0$).

Let us now return to the equation of motion for the affine connection. By introducing a conformal metric $h_{\mu\nu} \equiv F g_{\mu\nu}$ we see that Eqn. (5.9) can be written as a metric compatibility equation for $h_{\mu\nu}$: $\bar{\nabla}_\rho(\sqrt{-h}h^{\mu\nu}) = 0$. That is, the affine connection corresponds to a Levi-Civita connection defined in terms of $h_{\mu\nu}$ and it follows that

$$\bar{R}_{\mu\nu}(\Gamma) = R_{\mu\nu}(g) + \frac{3}{2F^2}(\nabla_\mu F)(\nabla_\nu F) - \frac{1}{F} \left(\nabla_\mu \nabla_\nu - \frac{1}{2} g_{\mu\nu} \square \right) F, \quad (5.12)$$

where the covariant derivatives are given in terms of $g_{\mu\nu}$ only. We can hence rewrite the field equations, Eqn. (5.8) and Eqn. (5.9), as

$$\begin{aligned} G_{\mu\nu} = & \frac{8\pi G}{F} T_{\mu\nu} - g_{\mu\nu} \lambda(\bar{R}) + \frac{1}{F} (\nabla_\mu \nabla_\nu - g_{\mu\nu} \square) F \\ & - \frac{3}{2F^2} \left((\nabla_\mu F)(\nabla_\nu F) - \frac{1}{2} g_{\mu\nu} (\nabla F)^2 \right). \end{aligned} \quad (5.13)$$

Since f and F are still functions of the Ricci scalar \bar{R} , and thus algebraic functions of T via the trace equation (5.10), the right hand side is indeed completely determined by the stress-energy $T_{\mu\nu}$. That is, the above form explicitly separates spacetime geometry and matter content. Although not as compact as the original equations of motion, Eqn. (5.13) together with the trace equation (5.10) form the most useful set of equations in many applications.

5.2.1 Cosmology in Palatini $f(R)$ gravity

Cosmology in Palatini $f(R)$ gravity has received considerable attention [218, 248, 273–290] and is significantly different from the metric theory. For example, since a conformal term will not affect the trace equation (5.10), an R^2 correction to the gravitational action can not produce inflation in Palatini $f(R)$ gravity [291–293].

The expansion history of the Universe serves as a good discriminator between different models in Palatini $f(R)$ gravity since it is possible to reconstruct H as a function of z for an arbitrary choice of the function $f(R)$ [280]. By combining constraints from CMB, supernovae and baryonic acoustic oscillations, it is possible to put strong constraints on the allowed models. For $f(R) - R = \alpha(R/H_0^2)^\beta$, the best fit value of the parameter $\beta = 0.09$ and the allowed values are confined within a narrow band $|\beta| \lesssim 0.2$, so that only models similar to a cosmological constant are allowed [280]. In particular, the simple $-\mu^4/R$ scenario is ruled out also in Palatini $f(R)$ gravity. In contrast with the corresponding scenario in metric $f(R)$ gravity, this model is nevertheless in good agreement

with supernova data alone and it behaves like a cosmological constant also at higher orders in the Taylor expansion of the scale factor $a(t)$ around present time t_0 . The first and second order terms are naturally proportional to the Hubble rate H_0 and deceleration q_0 , but also the third order term corresponding to the jerk parameter $j_0 \equiv \ddot{a}_0/a_0 H_0^3$ is almost identical to that of a cosmological constant. The predicted value of the jerk parameter in the $-\mu^4/R$ model is $j_0 = 1.01 \pm 0.01$ [284, 287] which is very close to that of a flat Λ CDM model where $j = \text{const.} = 1$. Note that all cosmological scenarios with a transition from previous deceleration to acceleration at present have $j_0 > 0$ regardless of origin.

Strong constraints on Palatini $f(R)$ gravity can also be obtained from the matter power spectrum. The contributions from F in Eqn. (5.13) will introduce a gradient term corresponding to effective pressure fluctuations in matter in the equation governing the matter perturbations. In a sense, matter couples back on itself in Palatini $f(R)$ gravity, while no such term appears in the $f(R) = R - 2\Lambda$ limit. However, while other modifications may be small when F is close to 1, the gradient contribution will have significant impact on small scale structures also in this regime [278]. This tightens the constraint on the R^β model considerably and one obtains $|\beta| \lesssim 10^{-5}$ so that the scenario is virtually indistinguishable from a cosmological constant [282]. The non-standard behaviour of the perturbations will give rise to similar constraints also from the CMB angular power spectrum [288].

In summary, cosmological observations and in particular the matter power spectrum put very strong constraints on Palatini $f(R)$ gravity, only allowing models which are practically indistinguishable from a cosmological constant. It should nevertheless be noted that this is under the assumption that the Universe is filled with cold dark matter. In Ref. [290] it was shown that if one relaxes the assumption that dark matter is a pressureless perfect fluid, non-trivial functions $f(R)$ may still be allowed. The corresponding scenario seems somewhat contrived however.

5.2.2 Solar System constraints in Palatini $f(R)$ gravity

Since the spherically symmetric vacuum solution in Palatini $f(R)$ gravity is unique and given by the Schwarzschild-dS/AdS metric, the Solar System constraints seem to be trivially fulfilled. Indeed, given the desired value of the cosmological constant, γ_{PPN} is indistinguishable from 1 in Palatini $f(R)$ gravity. However, as discussed in Papers II and III, the modified source term for G_{00} will yield a non-standard relation between the gravitational mass and the density profile of the Sun. This becomes especially transparent when one considers a constant density object, for which the gravitational mass is given by [II]:

$$M = \int_0^{r_\odot} dr \frac{4\pi r^2 \rho}{F_\rho} + \frac{\Lambda_\rho - \Lambda_0}{6G} r_\odot^3, \quad (5.14)$$

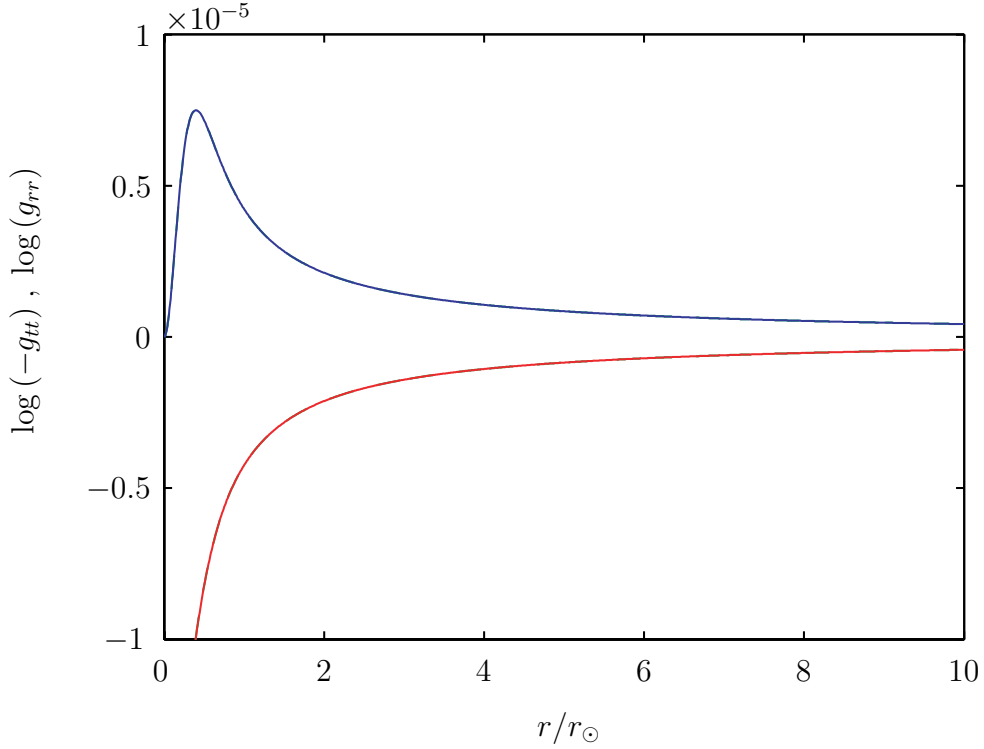


Figure 5.3: Shown is the temporal (lower curve) and radial (upper curve) component of $g_{\mu\nu}$ in the Solar System for a Palatini $f(R)$ model where $F \approx 1$ inside the Sun. The Palatini solution completely overlaps with the GR metric.

where F_ρ and Λ_ρ are the interior values determined by the density profile ρ and Λ_0 is the vacuum value of the cosmological constant. Although the small mass shift due to $\Lambda_\rho - \Lambda_0$ is usually completely negligible, the first term will strongly constrain the allowed forms of F . For example, while the original $-\mu^4/R$ model gives an F_ρ indistinguishable from 1, adding a conformal term $\sim R^2/\mu^2$ results in $F_\rho \sim 10^{31}$ at the center of the Sun. In such a case, the resulting gravitational field is so weak that spacetime is indistinguishable from a flat Minkowski background and no planets would revolve around the Sun. One could in principle imagine that F varies inside the Sun in such a way that it still gives rise to the observed gravitational field strength, but since the local density is what defines the local pressure and other thermodynamical properties of a star, it is obvious that F can not differ significantly from 1 inside the Sun without changing the predictions of Solar physics. In summary, Solar System observations demands that F is very close to 1 in the interior of the Sun. It is interesting to note that this constraint also applies to metric $f(R)$ gravity, since in order to obtain $\gamma_{\text{PPN}} = 1$ in these theories the solution must closely follow the corresponding Palatini solution [III,IV]. Fig. 5.3 shows the metric components g_{00} and g_{11} in the Solar System for a Palatini model where F is very close to 1 and we see that the solution is indeed indistinguishable from GR.

Finally, it should nevertheless be mentioned that Solar System constraints have been a matter of some debate also in Palatini $f(R)$ gravity [256, 264, 295–299]. However, as was shown in Refs. [264, 300], the analysis in some of these papers suffered from serious problems and it is fair to say that there seems to be no problems with the weak field limit in Palatini $f(R)$ gravity. This conclusion is also supported by Fig. 5.3 and the analysis in Paper III where we solved the full field equations (5.13) in the Solar System. As can be seen from Fig. 5.3, the metric in Palatini $f(R)$ gravity completely overlaps with the corresponding solution in GR so that $\gamma_{\text{PPN}} = 1$, unlike in metric $f(R)$ gravity where the metric deviates from the GR solution and yield $\gamma_{\text{PPN}} = 1/2$, see Fig. 5.1 and Fig. 5.2.

5.3 Scalar-tensor equivalence

There exists a well-known classical equivalence between $f(R)$ gravity and Jordan-Brans-Dicke theory. Consider the gravitational part S_f of the action (5.1) and rewrite it in terms of a new scalar field Φ which is fixed by a Lagrange multiplier:

$$S_\Phi = \frac{1}{2\kappa} \int d^4x \sqrt{-g} [f(\Phi) + F(\Phi)(R - \Phi)] . \quad (5.15)$$

Varying the above action with respect to Φ gives $(\partial F / \partial \Phi)(R - \Phi) = 0$ so that unless $\partial F / \partial \Phi = 0$ (corresponding to the Einstein-Hilbert action), $\Phi = R$ as desired and S_Φ is indeed equivalent to S_f . Note that the above holds regardless of how we define the Ricci scalar. Now, by defining

$$\phi \equiv F(\Phi) , \quad U(\phi) \equiv \phi \Phi - f(\Phi) , \quad (5.16)$$

the action (5.15) can be rewritten as

$$S_\Phi = \frac{1}{2\kappa} \int d^4x \sqrt{-g} [\phi R - U(\phi)] . \quad (5.17)$$

That is, it takes the form of a Jordans-Brans-Dicke theory. In metric $f(R)$ gravity, the Ricci scalar is defined in terms of the metric $g_{\mu\nu}$ only and it hence corresponds to a JBD theory with $\omega = 0$ as seen from above. However, in Palatini $f(R)$ gravity the Ricci scalar is instead a composite object, $\bar{R} \equiv g^{\mu\nu} \bar{R}_{\mu\nu}(\Gamma)$, which can be expressed in terms of the conformal metric $h_{\mu\nu} = \phi g_{\mu\nu}$. Thus, via the relation

$$\bar{R} \equiv g^{\mu\nu} \bar{R}_{\mu\nu} = R + \frac{3}{2\phi^2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi , \quad (5.18)$$

where $R \equiv g^{\mu\nu} R_{\mu\nu}$ and we have dropped total derivatives, we see that Palatini $f(R)$ gravity corresponds to a JBD theory with $\omega = -3/2$.

When written as scalar-tensor theories, the only thing which differs between metric and Palatini $f(R)$ gravity is the kinetic term of the scalar field. At first sight, the value $\omega = 0$ seems to imply a trivial behaviour for the scalar field

corresponding to metric $f(R)$ gravity. However, in the Jordan frame the kinetic terms of the scalar field and the metric mix so that the dynamics is not very transparent. It is easier to analyze the system in the Einstein frame where the Hamiltonian of the gravitational sector is diagonalized so that the spin 0 and spin 2 degrees of freedom separate. When the action is written in the Einstein frame, the conformal transformation will generate an additional piece to the kinetic term of the scalar field: $-(3/2\phi^2)g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$. Thus, the scalar field in metric $f(R)$ gravity is indeed dynamic, just like expected from the equations of motion (5.3) where F , *i.e.* R corresponds to an additional degree of freedom. On the contrary, in Palatini $f(R)$ gravity the contributions to the kinetic term of the scalar field will instead cancel, showing that ϕ is *not* a dynamical field in this theory. This can of course also be seen from the equations of motion in both frames. In the Jordan frame, it becomes apparent if we use the trace of Eqn. (3.11) to replace the Ricci scalar in the equation of motion for ϕ :

$$\begin{aligned} 0 &= 2\omega\Box\phi + \phi R - \frac{\omega}{\phi}(\partial\phi)^2 - \phi U'(\phi) \\ &= (3 + 2\omega)\Box\phi - \phi U'(\phi) + 2U(\phi) - \kappa T, \end{aligned} \quad (5.19)$$

where $U'(\phi) \equiv \partial U/\partial\phi$. That is, for $\omega = -3/2$ the above equation of motion will reduce to a purely algebraic relation between the scalar field and the stress-energy, just like in the Palatini trace equation (5.10) where \bar{R} is completely determined by T .

Let us finally note that while the equivalent scalar-tensor formulation of $f(R)$ gravity provides a useful alternative form of the equations of motion, it should also be used with some caution. In particular, the expressions for the post-Newtonian parameters in scalar-tensor theory, Eqn. (3.19) and Eqn. (3.20), are not valid in Palatini $f(R)$ gravity since the parametrization breaks down when $(3 + 2\omega) = 0$. Also, the boundary condition problem in metric $f(R)$ gravity is not alleviated by mapping it to a scalar-tensor theory. The corresponding scalar field has no straightforward physical interpretation and its boundary conditions are no less obscure than the higher order derivatives of $g_{\mu\nu}$.

Chapter 6

Summary and discussion

The main concern of this thesis has been the dark energy problem. We began with an overview of basic cosmology and discussed how present observations imply that the expansion rate of the Universe is accelerating. In a homogeneous and isotropic Universe filled with only regular matter this requires the presence of a nonzero positive cosmological constant Λ , assuming that gravitational dynamics is described by General Relativity also at very large scales. While there is yet no need to go beyond the Λ CDM model from an observational point of view, the severe theoretical problems associated with a cosmological constant $\Lambda \sim (10^{-3} \text{ eV})^2$ encourages us to seek for an alternative explanation. We reviewed the possibilities of including exotic fluids and effects from inhomogeneities in the Universe, but continued to explore a different approach. We considered a flat, homogeneous and isotropic Universe filled with regular matter but where modifications to gravity at cosmological distances lead to the observed acceleration at present. However, any extended theory of gravity may also alter the dynamics at much smaller scales and Solar System observations put strong constraints on deviations from General Relativity. In particular, we have studied these constraints in scalar-tensor theory and $f(R)$ gravity.

Scalar-tensor theories provide an attractive alternative for dark energy with good motivation from higher-dimensional theories. In particular, large extra dimensions (LED) not only offer a possible solution to the hierarchy problem but the associated scalar field can also give rise to extended quintessence. Moreover, when embedding the large extra dimensions in a supersymmetric bulk space, the corresponding SLED model will also predict that the effective four-dimensional cosmological constant vanishes at tree level so that this scenario offers a remarkably complete solution to the cosmological constant problem. The LED scenario of Albrecht *et al.* was explored in Paper I and we found that Solar System observations put strong constraints on the model. However, since the behaviour of an extended quintessence scenario can be very similar to that of regular quintessence, there is still enough freedom in the model to pass the cosmological tests. This is indeed the case in many extended quintessence models, but the solid theoretical foundation of the (S)LED model makes it an exception-

ally well founded scenario.

Non-linear corrections to the Einstein-Hilbert action in the form of $f(R)$ gravity have received much attention despite several difficulties within the scenario. The perhaps main criticism against $f(R)$ gravity as a solution to the dark energy problem is that models typically require the presence of a very small energy scale $\mu \sim \sqrt{\Lambda}$.¹ Such a small scale also appears in many quintessence models where it is considered a serious problem. However, this question has yet not received much attention in $f(R)$ gravity theories. Moreover, in metric $f(R)$ gravity, the small scale μ typically introduces a time instability which causes serious problems at Solar System scales. The most straightforward solution to the instability problem would be to set $\mu \sim M_{\text{Pl}}$ or some other large scale where classical gravity might break down. However, this would bring us right back where we started, since it would imply an asymptotic vacuum solution $R_0 \sim M_{\text{Pl}}^2$ which is just the cosmological constant problem in disguise.

While cosmological observations put particularly strong constraints on Palatini $f(R)$ gravity, metric $f(R)$ models should typically also behave similar to a cosmological constant. This makes the observational situation more difficult in $f(R)$ gravity than in a scalar-tensor theories based on large extra dimensions. Even if the corresponding extended quintessence scenario turns out to be practically indistinguishable from a cosmological constant, it will still have a characteristic behaviour at high energies so that constraints from particle accelerators may be used to further probe the model. In $f(R)$ gravity, the only possible way to constrain a model is simply via gravitational interactions.

In summary, Solar System observations put strong constraints on both scalar-tensor theories and $f(R)$ gravity. While an extended quintessence scenario and Palatini $f(R)$ gravity can in many cases fulfill these requirements, the situation is more problematic in metric $f(R)$ gravity. Although solutions giving the desired value $\gamma_{\text{PPN}} = 1$ do exist for a suitable choice of the function $f(R)$ also in the metric theory, it seems highly unlikely that such a configuration will be reached through the collapse of a protostellar dust cloud.

6.1 Summary of appended papers

Below is a brief summary of the work contained within the appended papers. Note that although we have tried to keep notation consistent throughout the first part of this thesis, it will sometimes depart in the appended papers. Most notably, Paper I uses opposite signature $(+, -, -, -)$ and the Jordan frame metric is denoted by $\hat{g}_{\mu\nu}$ while the Einstein frame metric is given by $g_{\mu\nu}$. In Papers II and III the quantities defined in terms of the affine connection in Palatini $f(R)$ gravity are not barred. Finally, in Paper IV the gravitational Lagrangian is written in the form $R + f(R)$ so that the function $f(R)$ parametrizes the

¹However, for an example of the contrary see the logarithmic toy model in Paper IV where $\mu^2 \gtrsim e^{100}\Lambda$.

deviation from General Relativity without a cosmological constant.

6.1.1 Paper I

We studied the cosmology of a dilatonic scalar-tensor theory obtained from the low energy limit of a six-dimensional brane world scenario, where large extra dimensions address the hierarchy problem and offer at least a partial solution for the naturalness problem in quintessence models. Our work was based on the LED model by Albrecht *et al.* [191, 192] which was later extended to the Super-symmetric Large Extra Dimensions (SLED) scenario [183–190]. We showed that the original scenario needs some correction when the Solar System constraints are more carefully accounted for. At first sight one appears to conclusively rule out the model. However, we also found that it is possible to salvage the overall scenario by adding new fields to the six-dimensional bulk space, and that this improved model also provides a new stabilization mechanism for the size of the extra dimensions. We showed that the corresponding cosmology not only allows for extended quintessence, but that it also can give rise to solutions similar to k -essence and possibly also phantom dark energy. Finally, we also showed that the observational imprint of the model is in principle detectable by the Super-Nova/Acceleration Probe (SNAP) in the upcoming Joint Dark Energy Mission (JDEM). While SNAP data alone will not be able distinguish it from a Λ CDM model with about 5% less dark energy, this degeneracy should be lifted when combining the data with constraints from the CMB.

Given the more recent developments of the SLED scenario, it would be interesting to revisit the above analysis in this context. While the overall cosmological evolution should remain the same since the effective four-dimensional action in the SLED scenario has the same form as in the LED model, the cosmological perturbations in this scenario are yet to be explored. Moreover, in the above study we only used the cosmological background value of the scalar field in order to estimate the matter coupling in the Solar System. A more proper analysis would of course be to solve the full field equations also in the Solar System.

6.1.2 Papers II-III

In General Relativity, the Einstein equations together with a perfect fluid in a static, spherically symmetric spacetime reduce to the Tolman-Oppenheimer-Volkov equations. These equations are the starting point when solving the problem of a realistic star and in these two papers we considered the corresponding problem in $f(R)$ gravity. Paper II considered $f(R)$ gravity in the Palatini formalism and we derived the corresponding generalized Tolman-Oppenheimer-Volkov equations. This showed that a large class of models in the Palatini formalism indeed pass the Solar System tests. In particular, models where $F(R) - 1$ is a decreasing function of R are typically compatible with the constraints. We also brought attention to the fact that extended theories of gravity such as Palatini

$f(R)$ gravity have a non-standard relation between the gravitational mass and the density profile of a star.

We extended the above study in Paper III, where we also included $f(R)$ gravity in the metric formalism. We derived the gravitational field in the Newtonian limit for a Sun-like star (both in the metric and in the Palatini formalism) and also obtained numerical solutions to the exact, generalized Tolman-Oppenheimer-Volkov equations in both formalisms. To our knowledge, no interior solutions of stars in $f(R)$ gravity had previously been computed in the literature. The numerical analysis performed in Paper III confirmed our earlier results for Palatini $f(R)$ gravity obtained in Paper II. In the metric case it was shown that the boundary conditions set at the center of the star will determine if metric $f(R)$ gravity is compatible with Solar System measurements. For a large class of boundary conditions, metric $f(R)$ gravity grossly violates the observations. Furthermore, although there exists boundary conditions for which the solution fulfills the constraints, these configurations typically turn out to be unstable and decay in time. We also showed that including effects from dark matter surrounding the star will not affect the predictions in either metric or Palatini $f(R)$ gravity.

6.1.3 Paper IV

Motivated by our findings in Paper III, we continued to study the exact requirements needed in order to have a stable configuration compatible with Solar System constraints in metric $f(R)$ gravity. We showed that the stability properties not only depend on the model but also on the specific configuration. Typically configurations giving the desired Post-Newtonian parameter $\gamma_{\text{PPN}} = 1$ are strongly constrained by the stability arguments. Furthermore, we found that even when these configurations are strictly stable in time, the domain of acceptable static spherical solutions typically shrinks to a point in the phase space. Unless a physical reason to prefer such a particular configuration can be found, this poses a naturalness problem for the currently known metric $f(R)$ models for accelerating expansion of the Universe.

In summary, our analysis strongly suggest that the Solar System constraints are not compatible with any form of metric $f(R)$ gravity which is at the same time designed to provide for the apparent acceleration of the Universe at present without a cosmological constant. However, it should be noted that our result do *not* rule out very small modifications to the Einstein-Hilbert action, such as might arise from quantum gravity corrections. The inherent problem with the models for accelerating expansion is that they typically introduce a very small energy scale μ which will either make the model violently unstable or incompatible with the high precision measurements in the Solar System.

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