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CHILDREN'S TALK ABOUT MATHEMATICS AND MATHEMATICAL TALK

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The relationship between mathematics and language is essential and complex. Language makes a connection between real life and formal mathematics. On the other hand mathematics can be seen as symbolic system. According to radical pedagogical view it is impossible for children to learn mathematics without language and connection to real life. In our paper we will highlight children's talk about mathematics and math talk they expressed in our research project. Children's talk is interpreted in semiotic and narrative spirit.

MATHEMATICS AND LANGUAGE

Why is Talk important in Mathematics?

Whether it is written, drawn, gestured, or spoken, the medium of mathematical expression is human language. Mathematics is a specialized language developed to communicate about particular aspects of the world. Mathematical knowledge develops through interactions and conversations between individuals and community. It is an intensely social activity. A major way of participating in a mathematics community is through talk. Children use language to present their ideas to each other, build theories together, share solution strategies, and generate definitions. By talking both to themselves and to others, children form, speak, test, and revise ideas. (Corwin et. al. 1995.)

Talk about mathematics

Hersh (1986) has answered to the question “What is mathematics?” as follows: “It would be that mathematics deals with ideas. Not pencil marks or chalk marks, not physical triangles or physical sets, but ideas (which may be presented or suggested by physical objects). What are the main properties of mathematical knowledge, as known to all of us from daily experience?”

- 1) Mathematical objects are invented or created by humans.*
- 2) They are created, not arbitrarily, but arise from activity with existing mathematical objects, and from the needs of science and daily life.*

3) *Once created, mathematical objects have properties which are well-determined, which we may have great difficulty in discovering, but which are possessed independently of our knowledge of them.*” (Hersh, 1986, 22.)

Malaty (1997, 53) points that there is mathematics in everything that humans have created and in everything that humans have not created. The nature of mathematics comes up especially then when you try to develop mathematical model from every day situation, and to apply mathematical system for example in the problem situation to another new every day situation (Ahtee & Pehkonen, 2000, 33-34). In school children have to learn formulas, exact proofs, or formalized definitions. Without real life connections this kind of math learning may restrict the talk about math in to formal mathematics.

According to Steinbring (2006, 136) mathematical knowledge cannot be revealed by a mere reading of mathematical signs, symbols, and principles. The signs have to be interpreted, and this interpretation requires experiences and implicit knowledge – one cannot understand these signs without any presuppositions. Such implicit knowledge, as well as attitudes and ways of using mathematical knowledge, are essential within a culture. Therefore, the learning and understanding of mathematics requires a cultural environment.

Mathematical talk: Children making spontaneous expressions and interpretations

According to Worthington & Carruthers (2003, 11) when children make actions, marks, draw, model and play, they make personal meaning. It is the child’s own meanings that should be the focus of the developing interest, rather than the child’s outcome of an adult’s planned piece of work, such as copied writing or representing a person ‘correctly’. Like Worthington & Carruthers (2003, 12) we see a child’s expression in spirit of Malaguzzi’s ‘hundred languages’, the theme of a poem that refers to diverse ways children can express themselves and that recognizes children’s amazing potential in making sense of their experiences: *“The child has a hundred languages, a hundred hands, a hundred thoughts, a hundred ways of thinking, of playing, of speaking. ...”*

Worthington & Carruthers (2003, 84) defined five common forms of children’s graphical marks: dynamic (marks that are lively and suggestive of action), pictographic (representative marks), iconic (discrete marks of children’s own devising), written, and symbolic. These forms can be seen in our data. According to Saarnivaara (1993, 103-104) children interpreting pictures and photos expect of them a resemblance to reality. It is essential that the picture creates a strong feeling of reality in the child. The condition for this is that the work imitates reality faithfully, and is a more or less “perfect” analogy of it. However, it is not only a question of the skillful imitation of reality. The child assumes that the subject matter is also true. We

as researchers share this view, and we think that all the children's emotional and mathematical expressions are true.

About semiotics

The Peircean sign-relation consists of “a triple connection of sign, thing signified and cognition produced in the mind.” A sign, or representamen, is something which stands to somebody for something in some respect or capacity. It addresses somebody, that is, creates in the mind of that person an equivalent sign, or perhaps a more developed sign. That sign which it creates Peirce calls the interpretant of the first sign. The sign stands for something, its object. It stands for that object not in all respects, but in reference to a sort of idea, which he has sometimes called the ground of the representamen. (Hoopes, 1991; Steinbring 2006, 141.)

The reflection on the interrelationship between sign and meaning is called semiotic activity. Semiotic activity consists in every inter- or intra-personal reflection on the interrelationships between a sign and its meaning(s) in order to investigate and improve mutual correspondence. Signs can be words or graphics (e.g. symbols, drawings, diagrams or schemata). To make a sign and its meaning match optimally, the sign, the meaning, or both can be adjusted: sometimes people modify the sign to make it more adequate for the expression of the meaning, sometimes they elaborate the meaning in order to adjust it to the sign. (van Oers & Wardekker 1999, 234.)

The primary focus in a semiotic perspective is on communicative activity in mathematics utilizing signs. This involves both sign reception and comprehension via listening and reading, and sign production via speaking and writing or sketching. While these two directions of sign communication are conceptually distinct, in practice these types of activity overlap and are mutually shaping in conversations, i.e., semiotic exchanges between persons within a social context. Sign production or utterance is primarily an agentic act and often has a creative aspect. (Ernest 2006, 69.)

Vygotsky (1978) argues that all semiotic functioning is first developed in the young human being through the convergence of several modes of representation, including spoken language, bodily movements associated with drawing and painting, and the use of physical objects as signs, standing for imagined objects in play. Through such modes of expression the power and general properties of the semiotic relation between sign and object, representation and meaning, signifier and signified is first learned and developed.

For example, pictures represent as iconic objects the real world. Children's spontaneous reactions to the pictures are signs, representamen. Children's symbolic relations to mathematics can be interpreted of these signs.

Bruner (1960, 1964) has presented a synthesis of Piaget and Peirce. Bruner claims that root meanings for signs are often constructed by individuals on the basis of enactive, bodily experiences. Subsequently, Bruner argues, these meanings are

further developed through internalisations of iconic representations before being fully represented symbolically. While defining the narrative construal of reality, Bruner (1999, 147) use a metaphorical interpretation: “We live in a sea of stories, and like the fish who will be the last to discover water, we have our own difficulties grasping what it is like to swim in stories. It is not that we lack competence in creating our narrative accounts of reality – far from it. We are, if anything, too expert. Our problem, rather, is achieving consciousness of what we so easily do automatically, an ancient problem of *prise de conscience*.”

According to Doxiadis (2003, 20) mathematical narrative must enter the school curriculum, in both primary and secondary education. The aim is: a) to increase the appeal of the subject, b) to give it a sense of intellectual, historical and social relevance and a place in our culture, c) to give students a better sense of the scope of the field, beyond the necessarily limited technical mathematics that can be taught within the constraints of the school system. In the Finnish National Core Curriculum of Mathematics (2004, 157) is emphasized that children should learn mathematics by talking, modelling, explaining, and presenting their ideas to each other.

The early years of schooling are crucial, as it is often here that the dislike of mathematics is planted. The main cause of this is the difficulty of a young child accepting abstraction and irrelevance – which mostly peaks with the introduction of the concept of number and arithmetic operations. At age five or six, a child lives in a storied internal environment, i.e. an environment cognitively organized by stories of all kinds, of family, of home, of daytime routine, of behaviour, of neighbourhood, of games, of friends, of animals, of dream. The main characteristics of the storied world are integration and emotional richness. With the introduction to mathematics, the child is *de-storied*, a neologism that sounds suspiciously close to “destroyed”. We must be very careful when we provide the first bites of the fruit of the Tree of Abstract Knowledge. (Doxiadis 2003, 20.)

One of the basic aims of our research is to help children to create their own spontaneous narratives about mathematics.

AIMS OF RESEARCH

The main aims of this article are:

1. To describe children’s talk about mathematics: How children are defining limits of mathematics?
2. To describe children’s mathematical talk: What kind of spontaneous mathematical expressions children produce?

DEVELOPMENT OF RESEARCH METHODS

In order to describe and understand meanings of mathematics during childhood, and to study children’s emotions towards mathematics and mathematics learning we had

to find the way to develop a hermeneutic phenomenological method especially for children aged 6 to 8. The method should be developed also for those children who can not read and write yet. After some common reflections and conversations we started to develop a pictorial test. The basic idea of this test is in the Harter & Pike's (1984) Pictorial Scale of Perceived Competence and Social Acceptance for Young Children which was presented by Byrne (1996). The other theoretical backgrounds are the theoretical viewpoint of children's spontaneous marks and meaning making (Worthington & Carruthers, 2003), and gestalt psychology (see e.g. Donderi, 2006). For the pictorial test we gathered 37 pictures of mathematical world in a wide sense. The picture sets are:

1. mathematical issues (11) (4 comparisons, 2 one to one correspondence, 5 problems)
2. human beings (7)
3. culture products (7)
4. toys and fairy-tale creatures (6)
5. nature and nature products (3)
6. built environment (3)

The first three sets are most essential in our research. Because the test is developed for children there are pictures about toys and fairy-tale creatures. Children's developing environments consist either nature or built environments or both of them. Picture types are mathematical tasks (9), drawings (12) and photos (16). We have copyright owners' permissions to use their pictures in our test. There are examples of our pictorial test in the next two pictures (1. & 2.) representing the two mathematical worlds.



Picture 1: Bunches of rowanberries



Picture 2: Euro problem

The layout of the pictorial test book is based on gestalt psychology: pictures are bright, sharp and large enough; around the pictures there is enough space for a child to concentrate on one picture at a time and to write spontaneously down her/his ideas. The double pages are harmonious considering the content and style. Mathematical issues are surrounded by the real world mathematics.

In order to make the emotional expression easy to children we used a familiar three point's smiley-face Likert-scale (happy, neutral, and sad).

Children were asked to evaluate all pictures from three viewpoints: 1) Is there any kind of mathematics in the picture? 2) How did you felt the mathematics in the picture? 3) Please, write down your own mathematical ideas about the pictures.

DATA GATHERING

We sent our research message via 12 teachers who were studying in our institution. These teachers presented our appeal to both their preschool and the first and second grade colleagues. Twenty volunteer teachers from different parts of Finland announced their and their pupils' willing to take part in our research. So we call this sample as quasi-random. The pictorial test was presented in 23 classes to 299 children from preschool to grade 2. We have got research permissions from children, their parents, teachers, school head masters and chief education officers. Data gathering was organised during the period from January to March 2006. There are the numbers of subjects by grades and by gender in the next two tables (1. & 2.).

Groups	Frequency	%
Preschool	93	31,1
Grade 1	158	52,8
Grade 2	48	16,1
Total	299	100,0

Table 1: Subjects by the grade

Groups	Frequency	%
Girls	154	51,5
Boys	145	48,5
Total	299	100,0

Table 2: Subjects by the gender

DATA ANALYSIS AND RESULTS

The pictorial test was coded as follows:

- 1) smiley-face Likert-scale: 1 = sad, 2 = neutral, 3 = happy.
- 2) Children's mathematical expressions (under the pictures): 0 = nothing, 1 = numbers, 2 = exercises (e.g., $2 + 3$), 3 = solved exercises (e.g., $2 + 3 = 5$), 4 = amount expressions and comparisons, 5 = word problems, 6 = mental models.
- 3) Children's verbal expressions about mathematics: 0 = no mathematical content, 1 = words, 2 = sentences, (besides these contents we also looked for children's emotions from their verbal expressions: 3 = happy, 4 = sad).

Talk about mathematics

The pictures of pictorial test were grouped into two sets: the traditional school math pictures (18), and the 'everyday' math pictures (19). Children's emotional and verbal opinions of mathematics were described with two scales: school math (formal math), and 'everyday math' (informal math). Then we analysed by medians and quartiles children's positions in math world. Mathematics symbolically meant just school mathematics for a part of the children (ca. 10 %). Some children (ca. 10 %)

symbolically were attached to ‘everyday’ math. We are wondering if they have taken up negative attitude towards school math. Most children had very strict opinions like: “You can not find any math in this picture.” or “Oh, this is great! This is mathematics!” Some children were considering the limits of math like: “You can not count anything of this picture because it is music!” As Hersh (1986, 22) has argued well-determined mathematical objects may effect some difficulties in rediscovering if the connection to daily life activities is broken. Children’s early life experiences form their conceptions about mathematics and about themselves as mathematics learners.

In classroom interactions, the learners are to become familiar with different forms of mathematical signs in the interaction to acquire their use by means of social participation, and not to use finished given signs according to strict rules. For the learners the signs and the forms of their interpretation develop by means of mediations to reference objects. Further, a generalizing use of the signs in a certain way develops only gradually in the temporal, interactive development; one cannot give the learner finished mathematical signs in their essential meaning at the beginning of her or his process of learning. (Steinbring, 2006, 145.) We wonder if children’s strict conceptions of mathematics are expressions about finished mathematical signs and strict rules. These conceptions could be expanded in the spirit of Steinbring.

Mathematical talk

We grouped children’s spontaneous mathematical expressions into three sets: amount expressions and comparisons, word problems, and mental models. Frequencies are presented in table 3.

Math talk	% (n=299)
Amounts	10.7 %
Word problems	22.7 %
Mental models	22.1 %

Table 3: Frequencies of math talk

The traditional model of mathematics learning often is understood as silent counting. Rather small frequencies of math expressions may be a sign of this tradition. On the other hand children had to write down their mathematical remarks and some children may have had troubles in writing.

Some examples of children’s spontaneous expressions:

Amount expressions and comparisons:

“You can count berries though it could be rather difficult.” (A second-grade boy, Tuomas, about the picture with bunches of rowanberries)

“There is more lemonade in the other bottle.” (A second-grade girl, Ida, about the picture of forest)

“There are trees behind the long river.” (A second-grade girl, Elisa, about the picture of a bridge scenery)

Word problems:

“How much water you need for the ice sculpture?” (A first-grade girl, Maria, about the picture of an ice sculpture)

“How many yellow papers you find in the picture?” (A first-grade girl, Oona, about the picture of three children and a teacher)

“There are five hundred trees in the forest. A woodcutter comes and fells four hundred and fifty trees. Then there are only fifty trees left.” (A second-grade boy, Eero, about the picture of forest)

Mental models:

“You can count the black stripes of the bee.” (A first-grade girl, Emilia, about the picture of a bee)

“The berries symbolize the task of dividing them in two equal groups.” (A second-grade boy, Wili, about the picture with bunches of rowanberries)

“I really do not know but you still can count: pot + pot + pot + pot + pot = 5 pots.” (A second-grade girl, Jenni, about the picture of five honey pots)

“This picture is just like a problem task, and I like them very much.” (A second-grade boy, Topias, about the picture of euro problem)

“The cat has plenty of stripes to count.” (A pre-school girl, Jessica, about the picture of a cat)

While interpreting the pictures children have created in their minds signs and the corresponding meanings which they express in words or graphics. In the spirit of Bruner children have expressed their narrative construal of mathematical reality. Finnish children’s good performance in mathematics and science is well-known. Still we are concerned about those children who expressed only numbers and number exercises. We do not know if these children have living senses of numbers and number exercises as Bussi and Bazzini (2003, 216-217) have written about learning algebra.

Conclusions

Our starting point of this research is to highlight the meaning of real life experiences and signs and meanings most children learn in their early years. According to Presmeg (1998) there is strong evidence that traditional mathematics teaching does not facilitate a view of mathematics that encourages students to see the potential of mathematics outside the classroom. Although some reports indicate that children are

involved in many life activities with mathematical aspects, they continue to see mathematics as an isolated subject without much relevance to their lives.

From our point of view, our pictorial test may be seen as a method for children to find mediations between signs and reference objects by means of examples as Steinbring (2006, 141, 157) has argued. The pictures of our test are examples of cultural environment which Steinbring (2006, 136) sees as a basic requirement for learning and understanding of mathematics.

On the basis of children's mathematical talk presented in our results we can conclude that children have a need to express in words their mathematical ideas and interpretations. Children should have much more opportunities for these expressions – even before they learn to read and write. For example, one preschool teacher wrote down her group's authentic expressions during our data gathering.

We wonder if real life narratives could form an ideal basis for early math learning in school. We wonder if in mathematics learning environments we should respect children's life orientations and action contexts. We want to conclude with the words of Doxiadis (2003, 20): "Save time for narrative, use it to embed mathematics in the soul."

References

Ahtee, M. & Pehkonen, E.: 2000, *Introduction to the didactics of mathematical subjects*, (Publ. in Finnish), Edita, Helsinki.

Bussi, M. G. & Bazzini, L.: 2003, Research, practice and theory in didactics of mathematics: towards dialogue between different fields, *Educational Studies in Mathematics* 54, 203-223.

Bruner, J.: 1960, *The Process of Education*, Cambridge, Massachusetts, Harvard University Press.

Bruner, J.: 1964, *Towards a Theory of Instruction*, Cambridge, Massachusetts, Harvard University Press.

Bruner, J.: 1999, *The culture of education*, Cambridge, Massachusetts, Harvard University Press.

Byrne, B.M.: 1996, *Measuring Self-Concept Across the Life Span*, US Washington, American Psychological Association.

Corwin, R. B. et al.: 1995, Supporting Mathematical Talk in Classrooms. *Hands On!* 18 (1).

http://www2.terc.edu/handsonIssues/spring_95/suptalk.html

Donderi, D. C.: 2006, Visual Complexity: A Review, *Psychological Bulletin*, 132 (1), 73-97.

Doxiadis, A.: 2003, Embedding mathematics in the soul: narrative as a force in mathematics education, *Opening address to the Third Mediterranean Conference of Mathematics Education*, Athens, January 3, 2003.

<http://www.apostolosdoxiadis.com/files/essays/embeddingmath.pdf>

Ernest, P.: 2006, A semiotic perspective of mathematical activity the case of number, *Educational Studies in Mathematics* 61, 67-101.

Finnish National Core Curriculum 2004, The Finnish National Board of Education, Helsinki.

Harter, S. & Pike, R.: 1984, The Pictorial Perceived Competence Scale for Young Children, *Child Development*, 55, 1969-1982.

Hersh, R.: 1986, Some proposals for reviving the philosophy of mathematics, in T. Tymoczko, (ed.) *New directions in the philosophy of mathematics*, Birkhäuser, Boston, pp. 9-28.

Hoopes, J. (ed.): 1991, *Peirce on signs: Writings on semiotic*, University of North Carolina Press, Chapel Hill.

Malaty, G.: 1997, A child on the way to the world of mathematics, in M. Siniharju, M. (ed.) *New directions of the pre-school and the elementary instruction*, Publ. in Finnish, The Government of Education, Helsinki, pp. 51-90.

van Oers, B. & Wardekker, W.: 1999, On becoming an authentic learner: semiotic activity in the early grades, *Journal of Curriculum Studies*, 31 (2), 229-249.

Presmeg, N. C.: 1998, A semiotic Analysis of Students' Own Cultural mathematics, *Paper presented at the Annual Meeting of the International Group for the Psychology of Mathematics Education (PME)*, Stellenbosch, South Africa, July, 12-17, 1998.

Saarnivaara, M.: 1993, *The child and art interpretation*, (Publ. in Finnish), Institute for Educational Research, Jyväskylä.

Steinbring, H.: 2006, What makes a sign a mathematical sign? – An epistemological perspective on mathematical interaction, *Educational Studies in mathematics*, 61 (1-2), 133-162.

Vygotsky, L.: 1978, *Mind in Society*, Harvard University Press, Cambridge, Massachusetts.

Worthington, M. & Carruthers, E.: 2003, *Children's Mathematics: Making Marks, Making Meaning*, Sage, Thousand Oaks, CA.