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The aim of our research is to explore 6-9-year-old children's spontaneous relation to mathematics. The first aim of this article was to describe a new pictorial method to test children's mathematical emotions and expressions as authentic and wide as possible. The second aim of this article was to study children's expressions inspired by the pictorial test. We found that girls and boys expressed different emotions towards pictures. The pictures inspired children to produce both traditional math expressions (numbers and simple arithmetical tasks) and more creative math problems and models. In conclusions we suggest some improvements concerning early mathematics teaching.

INTRODUCTION

What is mathematics?

Hersh (1986) has answered to the question "What is mathematics?" as follows: "It would be that mathematics deals with ideas. Not pencil marks or chalk marks, not physical triangles or physical sets, but ideas (which may be presented or suggested by physical objects). What are the main properties of mathematical knowledge, as known to all of us from daily experience?

1) *Mathematical objects are invented or created by humans.*
2) *They are created, not arbitrarily, but arise from activity with existing mathematical objects, and from the needs of science and daily life.*
3) *Once created, mathematical objects have properties which are well-determined, which we may have great difficulty in discovering, but which are possessed independently of our knowledge of them." (Hersh, 1986, 22.)*

Malaty (1997, 53) points that there is mathematics in everything that humans have created and in everything that humans have not created. The nature of mathematics comes up especially then when you try to develop mathematical model from every day situation, and to apply mathematical system for example in the problem situation to another new every day situation (Ahtee & Pehkonen, 2000, 33-34).

When we are thinking about the nature of mathematics and the use of mathematics we notice that mathematics is not just a school subject but it has also interests in common with every direction in the world. Mathematics is like music, sports and science the product of culture and we can define it on the basis of culture.

THEORETICAL FRAMEWORK

Early mathematical development

The development of mathematical skills and concepts of children at the age of 2 to 8 years has been described as being highly individual, both in the rate at which children reach is essential mathematical skills and in substance within mathematical concepts, as well as in relationships among contextualized knowledge concerning different aspects of numbers (Fuson, 1988). According to Hannula (2005, 21-22) socio-culturally ways of utilizing numerosity in action play a role in learning to utilize the subitizing mechanism for exact enumeration. In utilizing numerosity in action, intentional focusing on numerosity is a necessary intermediate process. How easily the children consider numerosity in their actions indicates the amount of practice the children are acquiring in their surroundings. Aunola et al. (2004) showed that children's mathematical skills develop in a cumulative manner from the preschool to the first years of school, even to the extent that the initial mathematical skills in beginning of preschool were positively associated with their later growth rate: the growth of mathematical skills was faster among those who entered preschool with already higher mathematical skills. Aunola et al. (2004) also showed that by the end of grade 2 children have problems both in attachment for mathematics and in math learning. According to Berry & Sahlberg (1995, 54) many children have preconceptions about modelling which are based on interpretations of real models. They argue that it is worth to utilize these preconceptions in school mathematics.

Self-relevant emotions

All human emotions are, in a loose sense, "self-relevant". Emotions arise when something self-relevant happens or is about to happen. In the language of appraisal theory (Lazarus, 1966), we experience emotions when we judge that events have positive or negative significance for our well-being. The specific type of emotional response is shaped both by primary appraisals of the positive versus negative implications of the event for the individual and by secondary appraisals (e.g., of one's ability to cope with the events). But all emotions arise from events that in some way have relevance for oneself. (Tangney, 2003, 384.) So the emotions which arise on the basis of mathematical experiences are relevant for children's conceptions of themselves as mathematics learners.

According to Roberts et. al (2002, 665) four sources for the origins of optimism are:

1. genetics,
2. child's environment, parents' model,

3. child’s environment, the feedback child receives from parents, teachers and from other significant persons,
4. life experiences that promote either mastery or helplessness.

The last two issues are most relevant at preschool and at primary school. Concerning mathematics teaching and mathematics learning environments children should get more positive feedback and learning experiences that promote mastery not helplessness.

Children making spontaneous expressions and interpretations

According to Worthington & Carruthers (2003, 11) when children make actions, marks, draw, model and play, they make personal meaning. It is the child’s own meanings that should be the focus of the developing interest, rather than the child’s outcome of an adult’s planned piece of work, such as copied writing or representing a person ‘correctly’. Like Worthington & Carruthers (2003, 12) we see a child’s expression in spirit of Malaguzzi’s ‘hundred languages’, the theme of a poem that refers to diverse ways children can express themselves and that recognizes children’s amazing potential in making sense of their experiences: “*The child has a hundred languages, a hundred hands, a hundred thoughts, a hundred ways of thinking, of playing, of speaking. …*”

Worthington & Carruthers (2003, 84) defined five common forms of children’s graphical marks: dynamic, pictographic, iconic, written, and symbolic. These forms can be seen also in our data. According to Saarnivaara (1993, 103-104) children interpreting pictures and photos expect of them a resemblance to reality. It is essential that the picture creates a strong feeling of reality in the child. The condition for this is that the work imitates reality faithfully, and is a more or less “perfect” analogy of it. However, it is not only a question of the skillful imitation of reality. The child assumes that the subject matter is also true. We as researcher share this view and we will also think that all the children’s emotional and mathematical expressions are true.

METHODOLOGY: HERMENEUTIC PHENOMENOLOGY

Hyde (2005) suggests hermeneutic phenomenology as a theoretical framework for reflecting, interpreting and gaining insight into children’s mind. Hermeneutic phenomenology offers possibilities to explore using van Manen’s notion of life world existentials as guides to reflection upon the life expression of the child. The four life world existentials are lived space (spatiality), lived body (corporeality), lived time (temporality) and lived human relation (relationality). In using these as a means by which to interpret the life expression of children, we could hope that some insights into their mathematical minds can be gleaned.

When Edmund Husserl (1970) founded phenomenology about 100 years ago, his starting point was the experience that science was preoccupied with explaining natural objects or events, whereas the understandable meaning of these objects and events was taken for granted within the framework of natural research and received little attention. When, for example, a biologist investigates trees, they are objects to be scientifically explained, and their meaning, with which we are familiar in lived experience, needs no particular attention. Even mathematics, as a stream of mental events, was regarded as an object susceptible to natural explanation: How must the brain and psyche function to conclude that 2 added to 3 give 5? Husserl was a mathematician before he became a philosopher, and it was obvious to him that mathematics is about understandable meaning. The mathematician understands that in every single case of adding 2 and 3 the answer must be 5 and explanations of the functioning of brain, psyche, mind or intellectual behaviour are of little interest to him as a mathematician. Mathematics is not a science based on collected evidence for (or against) explanatory hypotheses, but a science expressing mathematical experience as lived experience, i.e. an experience from within, not from without, an experience of a logical coherence constituting meaning. As mathematicians develop mathematics similarly phenomenologists develop phenomenology. The starting point is lived experience. Within this experience we are already familiar with the meaning of all kinds of phenomena.

Shortly we want to understand hermeneutically what kind of expressions children produce inspired by the pictorial test. In a phenomenological way we want to reach children's mathematical world of experiences as far it is ethically possible.

AIMS OF RESEARCH

The main aims of this article are:

1. To describe the development of the pictorial test.
2. To study children's spontaneous mathematical and emotional expressions inspired by the pictorial test.

DEVELOPMENT OF RESEARCH METHODS

In order to describe and understand meanings of mathematics during childhood, and to study children's emotions towards mathematics and mathematics learning we had to find the way to develop a hermeneutic phenomenological method especially for children aged 6 to 9. The method should be developed also for those children who can not read and write yet. After some common reflections and conversations we started to develop a pictorial test. The basic idea of this test is in the Harter & Pike's (1984) Pictorial Scale of Perceived Competence and Social Acceptance for Young Children which was presented by Byrne (1996). The other theoretical backgrounds are the theoretical viewpoint of children's spontaneous marks and meaning making (Worthington & Carruthers, 2003), and gestalt psychology (see e.g. Donderi, 2006). For the pictorial test we gathered 37 pictures of mathematical world in a wide sense. The picture sets are:

1. mathematical issues (11) (4 comparisons, 2 one to one correspondence, 5 problems)

2. human beings (7)
3. culture products (7)
4. toys and fairy-tale creatures (6)
5. nature and nature products (3)
6. built environment (3)

The first three sets are most essential in our research. Because the test is developed for children there are pictures about toys and fairy-tale creatures. Children's developing environments consist either nature or built environments or both of them. Picture types are mathematical tasks (9), drawings (12) and photos (16). We have copyright owners' permissions to use their pictures in our test.

The layout of the pictorial test book is based on gestalt psychology: pictures are bright, scarp and large enough; around the pictures there is enough space for a child to concentrate on one picture at time and to write spontaneously down her/his ideas. The double pages are harmonious considering the content and style. Mathematical issues are surrounded by the real world mathematics.

In order to make the emotional expression easy to children we used a familiar three point's smiley-face Likert-scale (happy, neutral, and sad).

Children were asked to evaluate all pictures from three viewpoints: 1) Is there any kind of mathematics in the picture?, 2) How did you felt the mathematics in the picture?, 3) Please, write down your own mathematical ideas about the pictures.

DATA GATHERING

We sent our research message via 12 teachers who were studying in our institution. These teachers presented our appeal to both their pre-school and the first and second grade colleagues. Twenty volunteer teachers from different parts of Finland announced their and their pupils' willing to take part in our research. So we call this sample as quasi-random. The pictorial test was presented in 23 classes to 299 children from preschool to grade 2. We have got research permissions from children, their parents, teachers, school head masters and chief education officers. Data gathering was organised during the period from January to March 2006. In the next two tables (1. & 2.) there are the numbers of subjects by grades and by gender.

Groups	Frequency	%
Preschool	93	31,1
Grade 1	158	52,8
Grade 2	48	16,1
Total	299	100,0

Table 1: Subjects by the grade

Groups	Frequency	%
Girls	154	51,5
Boys	145	48,5
Total	299	100,0

Table 2: Subjects by the gender

DATA ANALYSIS

The pictorial test was coded as follows:

1) smiley-face Likert-scale: 1 = sad, 2 = neutral, 3 = happy.
2) Children's mathematical expressions (under the pictures): 0 = nothing, 1 = numbers, 2 = exercises (e.g., 2 + 3), 3 = solved exercises (e.g., 2 + 3 = 5), 4 = amount expressions and comparisons, 5 = word problems, 6 = mental models.
3) Children's verbal expressions about mathematics: 0 = no mathematical content, 1 = words, 2 = sentences, (besides these contents we also looked for children's emotions from their verbal expressions: 3 = happy, 4 = sad).

RESULTS: CHILDREN'S MATHEMATICAL AND EMOTIONAL EXPRESSIONS

Children's mathematical expressions

In table 3 there is children's mathematical productivity of all 37 pictures (percentages of children who produced any mathematical issues). Many children wrote down more than one mathematical expression. Girls and boys produced most numbers and math tasks. The differences between girls and boys were highest in expressions and comparisons of amounts. The preschoolers were more eager to produce numbers while the first and the second graders produced more eagerly math tasks. The second graders produced also 54.2 % word tasks and mental tasks of all the 37 pictures. These results are mostly natural from the developmental viewpoint.

There are only small differences in girls' and boys' mathematical productivity favouring girls (see table 3). According to Aunio's (2006, 10) research review there are contradictory research results in children's mathematical performance and gender. For example Dehaene's (1997), Nunes & Bryant's (1996) research results show that girls and boys possess identical primary numerical abilities. Carr and Jessup (1997) have reported that during the first school year, boys and girls may use different strategies for solving mathematical problems, but there is no difference in the level of performance. Whereas Jordan, Kaplan et al. (2006) found in their research small but reliable gender effects favouring boys on overall number sense performance as well as on nonverbal calculation.

Groups	Numbers (%)	Math tasks (%)	Amounts (%)	Word tasks (%)	Mental models (%)
Girls	42.2	57.1	12.3	22.7	21.4
Boys	41.4	55.9	9.3	20.7	20.7
Preschoolers	59.1	8.6	2.2	1.1	2.2
First graders	32.3	77.8	10.1	24.1	22.2
Second graders	39.6	79.2	19.2	54.2	54.2

Table 3: Children's mathematical productivity

Mathematically the most "productive" pictures (see table 4) among the preschoolers and the first graders were "Five honey pots" while mathematically the most "productive" picture among the second graders was "Toy cars on a carpet". The second graders seem to have adapted to school mathematics because one of their mathematically most "productive" picture included a Math task about amounts and comparisons. However it is interesting that one of the least “productive” pictures among the second graders was “Math lesson” (see table 5). Considering the differences between girls and boys we found that boys were mathematically most “productive” with the picture “Two dices (5 & 6 dots)”, and the girls were most “productive” with pictures with social issues like the pictures “Seven children in a boat” and “Two children reading”. For example:

Joel (first grader) wrote about two dices: *“The dices show the numbers that make together eleven”.*

Milja (first grader) wrote about seven children in a boat: *“You can count the children’s heads.”*

Girls	Five honey pots	Seven children in a boat	Two children reading
Boys	Five honey pots	Five laughing children	Two dices (5 & 6 dots)
Preschoolers	Five honey pots	Two bikes	Two dices (5 & 6 dots)
First graders	Five honey pots	Seven children in a boat	City and street view
Second graders	Toy cars on a carpet	Seven children in a boat	Mathematical task

Table 4: Mathematically the most “productive” pictures

Girls	Picture domino playing	Sheet music	Ice sculpture
Boys	Book shelves	Sheet music	Ice sculpture
Preschoolers	Forrest view	Sheet music	Ice sculpture
First graders	Book shelves	Sheet music	Hippo knitting
Second graders	Math lesson	Sheet music	Girl reading a book

Table 5: Mathematically the least “productive” pictures

All children groups were least productive with the picture about sheet music (see table 5). It is obvious that children do not see mathematics in music because they might think that music is a different school subject. For example Emilia (preschooler) told about sheet music: “*No mathematics because you sing from music.”*

Emotional expressions

Most pleasant pictures (table 6) among all groups were mathematical tasks with common school problems, for example about money or other amounts and comparisons. Girls found mathematically most pleasant picture with social mathematics like “Five laughing children”. Only first graders found this picture one of the most pleasant. This gives reference to the conclusion that girls are socially more extrovert. Boys found one of the most pleasant pictures “Toy cars on a carpet” while girls found this picture as one of the most unpleasant pictures (see table 7). Second graders found “Math lesson” as an unpleasant picture.

Girls	Five laughing children	Two dices (5 & 6 dots)	Mathematical task (€)
Boys	Toy cars on a carpet	Two dices (5 & 6 dots)	Mathematical task (€)
Preschoolers	Fruit classification	Two dices (5 & 6 dots)	Mathematical task (€)
First graders	Stones	Five laughing children	Mathematical task (stars)
Second graders	Two dices (5 & 6 dots)	Mathematical task (€)	Patchwork quilt

Table 6: Mathematically most pleasant pictures

Girls	Spider and web	Human figures	Toy cars on a carpet
Boys	Bee	Cat	Sheet music
Preschoolers	Bee	Spider and web	Human figures
First graders	Spider and web	Cat	Problem solving
Second graders	Math lesson	Human figures	Sheet music

Table 7: Mathematically most unpleasant pictures

When children are thinking if there is some mathematics in the picture they somehow are solving a mathematical problem. According to Hannula's (2004, 56-57) review affects are not merely 'noise' of human behaviour in problem solving, but a representational system. This affective representational system has been divided into four facets of affective stage: emotional stage, attitudes, beliefs and values.

CONCLUSIONS

The first aim of this article was to describe a new method to test children's mathematical emotions and expressions as authentic and wide as possible. Our pictorial test seems to have offered enough space for children to produce their expressions. When we arranged data gathering teachers told us how children had enjoyed the test. Some preschool teachers told us that they had started to develop mathematics teaching on the basis of the test.

The second aim of this article was to study children's spontaneous mathematical and emotional expressions inspired by the pictorial test. We found that girls and boys expressed different emotions towards pictures. After they had found their favourite contents they were eager and capable to produce mathematical issues and verbal expressions. Almost 60 % of preschool children produced numbers in the pictorial test. It is interesting that second graders selected school mathematics pictures among both most pleasant and most unpleasant pictures. This result may have connections to Aunola's et al. (2004) result that by the end of grade two some children may have problems both in attachment for mathematics and in math learning.

The results of this article give some implications towards school mathematics. Perhaps there should be different kind of textbooks for girls and boys. Commonly typical textbooks do not have space for children's spontaneous expressions. Because in our research children produced plenty of numbers and easy arithmetic tasks it is obvious that early mathematics education emphasizes too much learning about numbers. Sophian (2004) has developed mathematics preschool curriculum consistently with Davidov's (the influential Soviet educator from 1970's) ideas. Both Davidov and Sophian view mathematics learning as primarily the matter of learning to reason effectively about quantities (particularly, for young children, tangible, manipulable, physical quantities) and only within that broader objective as a matter of learning about numbers.

The future plans of our research are to study more deeply children's emotions and core beliefs towards mathematics, and challenging groups of children. Mathematics learning should produce joy and positive challenges. At the end of our article we want to introduce a happy small mathematician Sanni (first grader) who in a very creative way wrote down all kind of issues which could be counted: *"stripes of the blouse, coloured drawings, and bricks; feet, eyes, blouses, mouths, and trousers; people and the boards of the fence; car tyres and windows"*.

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